Progressive Failure Analysis of a Tapered Thick Composite Plate

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Abstract

This study investigates the progressive failure analysis of a tapered thick composite plate. The governing differential equation of motion of the various tapered configurations of a thickness tapered reinforced composite plate are presented in the finite element method using first order shear deformation theory. The failure analysis is carried out by considering the fact that the crack exits parallel to the fibers when the ply fails and the cracked ply is being replaced by a hypothetical ply that has no transverse stiffness, transverse tensile strength and shear strength. However, the longitudinal modulus and strength is considered to remain unchanged. First the local stress and strains in each ply is found out under various loading conditions. Then by employing the ply-by-ply stresses and strains in failure theories, the strength ratio is calculated. Multiplying the strength ratio to the applied load yields the load level of the failure of the first ply. Once the first ply failure load is evaluated, the stiffness of the damaged ply or plies is degraded and the actual load level of the previous failure is considered in further calculation. The strength ratios of the remaining undamaged plies are evaluated at each level of loadings. Thus the degraded stiffness of the tapered composite plate from first ply failure to last ply failure under various loading condition are investigated.

Key Words— Tapered laminated composite thick plate, Vibration, Finite Element Method.

1. INTRODUCTION

High strength to density ratio of composite materials makes them more desirable materials over the conventional materials for structural applications. Composite structures are widely subjected to a various loading conditions during their service life. It is very important to understand the response of those structures for various loading conditions to exploit their full strength. As the composite laminate consists of number of lamina stacked together at various orientations or ply angles, understanding the failure response of a composite laminate is a pre-requisite for understanding the failure response of a composite structure [1]. Failure of a composite structure can be explained in various ways such as first ply failure (FPF), last ply failure (LPF) and progressive ply failure (PPF). The first ply failure occurs when the failure initiates in a single layer in a laminate in either the fiber direction or in the direction perpendicular to the fibers. Last ply failure occurs after the structure has degraded to the point where it is no longer capable of carrying additional load and most of the plies fail progressively [2].
Tolson and Zabaras [1] developed a numerical model and investigated experimentally to predict the first-fly failure and last-ply failure using progressive stiffness reduction technique. Reddy et al. [2] developed a progressive failure algorithm using generalized layer-wise plate theory to predict the first-ply and ultimate failure load of a uniform composite laminate and suggested a new stiffness reduction technique for the failed element. Cheung et al. [3] used the higher-order shear deformation theory and Lee’s strength criterion coupled with finite strip method to investigate the progressive failure analysis of anisotropic composite plates. The effects of fiber orientation and the number of plies on the load carrying capacity were also investigated. Tay and Lim [4] analyzed the stiffness-loss of uniform composite laminates with general balanced lay-up sequences and containing distributed transverse cracks. It was shown that the constitutive theory of damage applicable for a cross-ply laminate could be extended to the general ply-lay-ups using finite element formulation. Padhi et al. [5] studied theoretically and experimentally the non-linear behavior, first-ply failure and ultimate collapse of laminated composite plates with clamped edges subjected to transverse pressure. Sun et al. [6] examined the progressive failure of delaminated composite plates using the Reissner-Mindlin plate theory and the Von Karman’s non-linear plate theory and concluded that the delamination growth is significantly affected by the boundary condition and the stiffness degradation plays an important role in the strength analysis of delaminated plate. Knight et al. [7] studied the progressive failure analysis of composite structures using structural analysis of general shells (STAGS) and demonstrated the capability of the STAGS to analyze the progressive damage. Bulent and Karakuzu [8] investigated the failure load, the failure mode, and the propagation of failure of composite plate with different fiber orientation, different material properties and different geometries under pin-loaded conditions.

Watkins et al. [9] investigated experimentally the delamination in composite beams using fiber optic sensor and neural network algorithm. The finite element formulation of the uniform composite beam was developed using classical beam theory to validate the results obtained experimentally. Chen et al. [10] studied the dynamic response and the progressive failure of the delaminated uniform composite plates using the first-order shear deformation theory. They investigated the effects of frequency of dynamic load, delamination length and location, and reduction of structure stiffness during the progressive failure.

Ganesan and Liu [11] developed non-linear finite element formulation based on first order shear deformation theory and investigated the first-ply failure load, the ultimate failure load, the buckling load, the maximum transverse displacements, and locations and modes of failure of tapered laminated plates under the action of uni-axial compression. The design optimization of composite structures to yield the optimal natural frequencies and damping factor is becoming an important aspects of research. Nagaraj and Vasudevan [12] presented the review of recent development in progressive failure analysis for various structural elements such as beams, plates, panels and shells in laminated composites. Even though few works have been focused on the the static and dynamic properties of uniform and tapered composite beam structures, the progressive failure analysis of a tapered composite plate under various loading conditions including in-plane and out-of-plane loading conditions is yet to be investigated. In this present study, the progressive failure analysis of the various configurations of tapered thick composite plates with ply drop-off is being sought to find the first ply failure and last ply failure using the finite element formulation developed. Furthermore, as the progressive failure analysis is the framework that accounts for the continuous stiffness degradation of materials and indicates the decrease in load carrying capacity and localization of the stress, the failure analysis of the structure would provide an idea to understand from the first-ply failure to last-ply failure.

2. NUMERICAL MODELLING OF A TAPERED COMPOSITE PLATE

A tapered composite plate is considered for the development of the finite element model. These taper structures are formed by dropping off plies internally and are denoted as taper configuration 1 (TS1), Taper configuration 2 (TS2) and Taper Configuration 3 (TS3) as shown in figure 1. In figure
1. ‘$D$’ represent the number of domains along the longitudinal directions and the left end thickness as $H_L$ and $H_R$ as the right end thickness and Length ‘$L$’ and breadth ‘$B$’ are considered as the planar dimensions. The first order shear deformation theory (FSDT) is used to model the layers of a composite plate into an equivalent single layer.

![Figure 1. Representation of various tapered configurations](image)

(a) Tapered Configuration 1 (TS1) (b) Tapered Configuration 2 (TS2) and (c) Tapered Configuration 3 (TS3).

The strain energy of the tapered composite plate is expressed as:

$$U = \frac{1}{2} \sum_{i} \left[ \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial u}{\partial y} \right)^2 + \left( \frac{\partial w}{\partial x} \right)^2 + \left( \frac{\partial w}{\partial y} \right)^2 \right]$$

where $L$ and $B$ are the length and breadth of the plate in $x$ and $y$ directions of the tapered composite plate. The strain energy equation is substituted in Lagrange's equation to formulate the governing equation of motion in finite element form.
A rectangular element with four nodes and with one node in each corner is considered for the development of finite element of the tapered composite plate. Each node contains five degrees of freedom (DOF) which includes \( u_m, v_m, w_m, \Psi_x, \text{ and } \Psi_y \). The governing equations of motion for the tapered composite plate in the finite element form can then be obtained as:

\[
[k'](d') = (f')
\]

where \([k']\) is the element stiffness matrices and \( \{d'\} = \{u_m, v_m, w_m, \Psi_x, \Psi_y\}^T \) is the displacement vector at each node and \( \{f'\} \) is the element force vector.

**Progressive Failure Analysis and Its Iterative Procedure**

The progressive failure analysis is carried out by considering the fact that the crack exists parallel to the fibers when a ply fails. However, this ply is still capable of taking load parallel to the fibers. Hence the cracked ply is being replaced by a hypothetical ply that has no transverse stiffness, transverse tensile strength, and shear strength. However, the longitudinal modulus and strength is considered to remain unchanged. To investigate the ply failure, the local stresses and strains in each ply is found first out under the loading conditions considered. Then by employing the ply-by-ply stresses and strains in failure theories, the strength ratio is calculated. Multiplying the strength ratio to the applied load gives the load level of the failure of the first ply. Once the first ply failure load is evaluated, the stiffness of the damaged ply or plies is degraded and the actual load level of the previous failure is considered for further calculation. The strength ratios of the remaining undamaged plies are evaluated at each level of loadings. The procedure is repeated until all the plies in the laminate have failed. The procedure has been briefed as shown in Figure 2.

**3. Results and Discussions**

The numerical simulation on tapered composite plates is performed by considering a Glass-Epoxy laminated composite plate of length \((L)\) 300 mm, breadth \((B)\) 300 mm and 12 ply’s and each ply thickness of a lamina as 0.19 mm. The tapered composite plates are investigated by considering various taper configurations with ply orientations \([0^\circ/90^\circ]_6s\) at thick section of the tapered composite plates. at the ply orientation at the thin end of the various tapered configurations are TS1, TS2 and TS3 are \([0^\circ/90^\circ]_3s, [0^\circ/90^\circ]_3s, \text{ and } [0^\circ]_6s\). The simulated results in terms of the first ply failure load and the progressive ply failure and the corresponding ply orientation are presented in Table 1 for the various tapered configurations. It can be seen that the failure varies for the various tapered configuration. It is due to the fact that the stiffness of the tapered configuration varies according to the ply drop off and the resin location. It can also be seen that the load at which the unidirectional plies fail is much higher than the angle plies. This can also be related to the fact that the unidirectional
plies provide higher stiffness compared to the angle ply. Further, the order of the ply failure differs with various tapered configurations. Hence, it can be concluded that the progressive failure analysis provides the designer few important guidelines on selection of proper tapered configurations and the ply orientation and their angle.

Table 1. Progressive failure analysis for taper configurations TS1, TS2 and TS3 composite plates.

<table>
<thead>
<tr>
<th>Taper config.</th>
<th>Ply orientation</th>
<th>Load (MPa)</th>
<th>Ply No.</th>
<th>Failure Order</th>
<th>Failure Ply orientation</th>
</tr>
</thead>
<tbody>
<tr>
<td>TS1 [0°/90°]_9</td>
<td>9</td>
<td>645</td>
<td>2,4,5,7</td>
<td>90°,90°,90°,90°</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>670.77</td>
<td>1,3,6,8</td>
<td>0°,0°,0°,0°</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>905</td>
<td>1,2,3,4,5,6,7,8</td>
<td>90°,90°,90°,90°,90°,90°,90°,90°</td>
<td></td>
</tr>
</tbody>
</table>

5. CONCLUSIONS

In this paper, the first-ply and progressive failure analysis of the configuration of the tapered composite plate are investigated. It was demonstrated that the load carrying capacity of the structure depends on the ply orientation and the taper configuration. It can be seen that the ply orientations play an important role in load carrying capacity in the structure. The fundamental work for the progressive failure analysis has been carried out using the presented finite element analysis and the results were presented on the failure load, order of the ply failure and its corresponding ply angle. This study provides the important guidelines for the designers to consider the tapered configuration for their applications.

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REFERENCES


