Application of Unscented Kalman Filter for States Estimation in Evaporator

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Abstract - An evaporator is a nonlinear Multi Input and Multi Output (MIMO) system. The feed flow and steam flow are needed to control in order to obtain the required dry matter content and product flow of evaporator. With the feed flow, steam flow, dry matter content and product flow data, the system identification has been done by using nonlinear autoregressive exogenous model (NARX). To achieve the required concentration and feed flow of the product, the states of the system must be known. The Unscented Kalman Filter (UKF) algorithm is applied to estimate both states in the existence of random noise in measurements. The Extended Kalman filter (EKF) estimates the states by using predictor and corrector mechanism. For linearizing the process and measurement model it uses Jacobian matrix. When compared to the nonlinear function the Gaussian distribution is simpler to approximate and this is the idea behind the UKF. UKF applied deterministic sampling approach to identify the mean and covariance estimates. In this work the both states dry matter content and product flow of the nonlinear system (Evaporator) should be estimated by using UKF and EKF. Also, the estimation accuracy of this method is analyzed by comparing the Mean Square Error (MSE) of UKF and EKF.

Keywords: Evaporator, Unscented Kalman Filter (UKF), Extended Kalman Filter (EKF), Mean Square Error (MSE), Integral Square Error (ISE), Integral Absolute Error (IAE).

I. INTRODUCTION

Many process control strategies are available, but the knowledge about the process state is important for giving the control input. For that an approximate model of the process is need to be found. By using the collected data from the evaporator, the system identification is done. Then the filter algorithms UKF and EKF are applied to predict the states in the presence of random noise in measurements.

Instead of EKF other method was presented using the UKF. The accuracy of the UKF is excellent than the EKF even for a highly nonlinear system. Dual UKF was used to predict and estimate the states [1]. The various methods of Kalman filter for the estimate both weights and states using dual estimation of a noisy time series were demonstrated. To improve the estimation performance utilized both process and observation noise model [2]. The chemical process is taken and use unscented filter for state estimation. Its implementation is simpler than EKF because there is no need of Jacobian matrix [3]. Application of dual-UKF for simultaneous state and parameter estimation in the problems of surface atmosphere exchange [4]. The CSTR model was taken and states were estimated using both UKF and EKF. It concludes if the operating region was small both performs well but in the case of large operating region EKF did not give satisfied result like UKF [5]. Introduced two methods were En-KF and UKF for state estimation in an autonomous hybrid system. Then, implement nonlinear Model Predictive Controller for a designed system [6]. Both the states of dynamic systems and its parameters were estimated simultaneously, in noise observations. The main contribution was to show the efficiency of a proposed simplified dual-EKF technique was compared with the conventional joint EKF [7]. KF methods were used to predict and estimates both states and parameters of the Hodgkin Huxley
neuronal model. All the methods have been mathematically justified and presented [8]. The Kalman, Extended Kalman and Unscented Kalman filter dynamics were implemented and design of the optimal filters were carried out. The filter algorithms were used to estimate the states in the presence of random noise in measurement and it was used for the development of control [9]. A new algorithm called Kalman randomized joint UKF (KR-JUKF) was introduced and applied to CSTR. The performance evaluation shows how KR-JUKF is superior to joint and dual-EKF and joint-UKF [11]. For prediction nonlinear unscented transforms (UTs) was used in the UKF. Various UTs, were discussed, evaluated, and compared. It says when compared to simplex and spherical UTs, the much better result were given by basic and general UTs. The proposed methodology was tested to estimate four states of the nonlinear plant [12].

The paper is discussed in the below manner: Section 2 discusses the Evaporator model. Section 3 explains about the Extended Kalman Filter. Section 4 details the Unscented Kalman Filter. Section 5 includes the simulation results of EKF and UKF, the performance criteria were evaluated. Finally Section 6 concludes how the UKF is better than EKF.

II. MODELING OF AN EVAPORATOR

An evaporator is a widely used device in food, chemical, cement and so many process industries. It is used to convert the liquid form present in the substance into its vapor-form. The liquid from the target substance get evaporated into vapor. The objective is to concentrate the non-volatile solute like bases, acids, inorganic salts or organic compounds from a solvent. Sodium sulfate, phosphoric acid, caustic soda, sodium chloride, caustic potash, and urea are the common solutes. Mostly water is the common solvent in the evaporation system. There are different types of evaporators available in industries. In this falling film evaporator was taken. The evaporator consists of many tubes and it is surrounded by cylinder. The input liquid is fed into the tube and the steam is passed through the cylinder. When the steam is passed the liquid present in the feed get evaporated and the concentrate product collected data the bottom of the evaporator. The figure 1 shows the cross-sectional diagram of an evaporator.

Fig. 1.Cross-sectional Diagram of Evaporator

The mass balance of the evaporator

\[ \text{[Feed]} + \text{[Steam]} = \text{[Product]} + \text{[Vapor]} + \text{[Condensate]} \]

Assume \( \text{Steam} = \text{condensate} \)

\[ \text{[Feed]} = \text{[Product]} + \text{[Vapor]} \]

\[ F \cdot x_f = P \cdot x_p + V \cdot x_v \quad (1) \]

Where 
- \( F \) = Feed flow rate
- \( x_f \) = Molar fraction of liquid in feed
- \( P \) = Product flow rate
- \( x_p \) = Molar fraction of liquid in product
- \( V \) = Vapor flow rate
- \( x_v \) = Molar fraction of liquid in vapor

III. EXTENDED KALMAN FILTER

The Extended Kalman filter (EKF) is one of the nonlinear versions of Kalman filter. The EKF linearizes the nonlinear model. The Kalman filter is optimal for linear state transition model and the linear observation model with additive noise. But almost all the systems are nonlinear in nature.
Kalman filter did not applicable for nonlinear system model. For the estimation of nonlinear system model the more familiar approach is EKF.

The Extended Kalman Filter estimates the states of the nonlinear system by linearizing the state transition model and observation model. The linearization is done by Taylor series. Then the estimated state is corrected by comparing with sensor measurement state. The EKF is done by predictor and corrector mechanism. The flow chart of EKF is given in the figure 2.

\[
\begin{align*}
X_k &= f(X_{k-1}, U_{k-1}) + W_k \\
Z_k &= h(X_k) + V_k
\end{align*}
\]

Where \(X_k\) = system state \\
\(U_k\) = system input \\
\(Z_k\) = measured state \\
\(W_k\) = process noise \\
\(V_k\) = observation noise \\
f, h = nonlinear mapping

The EKF gives better result for system with less nonlinearity. But it produces estimation error in the case of highly nonlinear systems. The error is due to approximating (linearize) the transition state and observation model.

IV. UNSCENTED KALMAN FILTER

The UKF solves the linearization problem of the EKF. This filter more accurately estimates the original mean and covariance in the presence of noise. When compared to EKF, UKF is more robust. By using unscented transform technique the minimum set of points have been chosen around mean. These points are called sigma points. Then apply these points to the nonlinear functions. Then in UKF there is no need of Jacobian matrix. The figure 3 shows the flow chart of UKF.
The augmented state and covariance matrix is given below

\[
\begin{align*}
\mathbf{x}_{k|k-1}^a &= \left[ \mathbf{x}_{k|k-1} \ E_2 \left[ \mathbf{w}_k^T \right] \right]^T \\
\mathbf{P}_{k|k-1}^a &= \begin{bmatrix} \mathbf{P}_{k|k-1} & 0 \\ 0 & \mathbf{Q}_k \end{bmatrix}
\end{align*}
\]
(13) (14)

Derivation of \(2L + 1\) sigma points from an augmented state and covariance
Where \(L\) = augmented state dimension.

\[
\begin{align*}
\chi_{k|k-1}^{i} &= \mathbf{x}_{k|k-1}^a + \sqrt{(L+\lambda)\mathbf{P}_{k|k-1}^a}, & i = 1, \ldots, L \\
\chi_{k|k-1}^{i} &= \mathbf{x}_{k|k-1}^a - \sqrt{(L+\lambda)\mathbf{P}_{k|k-1}^a}, & i = L + 1, \ldots, 2L
\end{align*}
\]
(15) (16) (17)

Propagation of sigma points through \(f\).
Where \(f\) = transition function

\[
\chi_{k|k-1}^{i} = f \left( \chi_{k-1|k-1}^{i} \right) \quad i = 0, \ldots, 2L
\]
(18)

State and covariance prediction.

\[
\begin{align*}
\hat{\mathbf{x}}_{k|k-1} &= \sum_{i=0}^{2L} W_i^f \chi_{k|k-1}^{i} \\
\mathbf{P}_{k|k-1} &= \sum_{i=0}^{2L} W_i^f \left[ \chi_{k|k-1}^{i} - \hat{\mathbf{x}}_{k|k-1} \right] \left[ \chi_{k|k-1}^{i} - \hat{\mathbf{x}}_{k|k-1} \right]^T
\end{align*}
\]
(19) (20)

The weights are given by

\[
\begin{align*}
W_0^f &= \frac{\lambda}{L+\lambda} \\
W_\beta^f &= \frac{\lambda}{L+\lambda} + (1 - \alpha^2 + \beta) \\
W_\lambda^f &= \frac{2(L+\lambda)}{L+\lambda} - L
\end{align*}
\]
(21) (22) (23)

The \(\alpha\), \(\beta\) and \(K\) are primary, secondary and tertiary parameters
Where \(\alpha = 10^{-3}\)
\(K = 0\)
\(\beta = 2\)

The augmented state and covariance

\[
\begin{align*}
\mathbf{x}_{k|k-1}^a &= \left[ \mathbf{x}_{k|k-1}^a \ \mathbf{P}_{k|k-1}^a \right]^T \\
\mathbf{P}_{k|k-1}^a &= \begin{bmatrix} \mathbf{P}_{k|k-1} & 0 \\ 0 & \mathbf{R}_k \end{bmatrix}
\end{align*}
\]
(24) (25) (26)

The derivation of sigma points from augmented state \((\chi_{k|k-1}^a)\) and covariance \((\mathbf{P}_{k|k-1}^a)\)

\[
\begin{align*}
\chi_{k|k-1}^a &= \chi_{k|k-1}^a \\
\chi_{k|k-1}^{i} &= \chi_{k|k-1}^a + \sqrt{(L+\lambda)\mathbf{P}_{k|k-1}^a}, & i = 1, \ldots, L \\
\chi_{k|k-1}^{i} &= \chi_{k|k-1}^a - \sqrt{(L+\lambda)\mathbf{P}_{k|k-1}^a}, & i = L + 1, \ldots, 2L
\end{align*}
\]
(27) (28) (29)

Propagation of sigma points through \(h\).
Where \(h\) = observation function

\[
\gamma_k^{i} = h(\chi_{k|k-1}^a) \quad i = 0, \ldots, 2L
\]
(30)

UKF Kalman gain.

\[
\mathbf{K}_k = \mathbf{P}_{\chi_k,\mathbf{x}_k} \mathbf{P}_{\mathbf{x}_k,\mathbf{x}_k}^{-1}
\]
(31)

Where

\[
\mathbf{P}_{\chi_k,\mathbf{x}_k} = \sum_{i=0}^{2L} W_i^f \left[ \chi_{k|k-1}^{i} - \mathbf{\tilde{X}}_{k|k-1} \right] \left[ \gamma_k^{i} - \mathbf{\tilde{X}}_k \right]^T
\]
(32)

\[
\mathbf{P}_{\mathbf{x}_k,\mathbf{x}_k} = \sum_{i=0}^{2L} W_i^f \left[ \gamma_k^{i} - \mathbf{\tilde{X}}_k \right] \left[ \gamma_k^{i} - \mathbf{\tilde{X}}_k \right]^T
\]
(33) (34)
State and covariance updation

\[
\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k (z_k - \hat{z}_k) \tag{35}
\]

\[
P_{k|k} = P_{k|k-1} - K_k P_{x_k|x_k} K_k^T \tag{36}
\]

V. RESULTS AND DISCUSSION

The nonlinear ARX model has been found by using collected input and output data. Then UKF has been applied to estimate the nonlinear MIMO system states. By using the MATLAB simulation, result of the UKF is analyzed and compared with the EKF.

1) Simulation results of state-1

The figure 4 shows the simulation result of UKF for state 1. From this figure it is clear that as the iteration increases the estimated state become nearer to original state and the error decreases slowly. The figure 5 gives the simulation result of EKF for state 1. This figure shows that even the number of iteration increases the estimated state gets deviated from the original state. Hence the UKF method gives better result than the EKF method.

2) Performance comparison for state 1

![Fig. 5. Simulation result for state-1 using EKF](image)

<table>
<thead>
<tr>
<th>ALGORITHMS</th>
<th>UKF</th>
<th>EKF</th>
</tr>
</thead>
<tbody>
<tr>
<td>MSE</td>
<td>0.0332</td>
<td>66.7</td>
</tr>
<tr>
<td>ISE</td>
<td>2.5036</td>
<td>6743.3</td>
</tr>
<tr>
<td>IAE</td>
<td>1.5823</td>
<td>81.69</td>
</tr>
</tbody>
</table>

The Table I shows the performance criteria with respect to MSE, ISE and IAE of both the estimation methods at the time period 1000. The UKF error is less when compared to EKF.

3) Simulation results of state-2

The figure 6 shows the simulation result of UKF for state 1. From this figure it is clear that the
estimated state closer to an original state. The figure 7 gives the simulation result of EKF for state 1. This figure shows that even the number of iteration increase the estimated state gets deviated from the original state and the error also increases. Hence the UKF method gives better result than the EKF method.

4) Performance comparison for state 2

<table>
<thead>
<tr>
<th>ALGORITHMS</th>
<th>UKF</th>
<th>EKF</th>
</tr>
</thead>
<tbody>
<tr>
<td>MSE</td>
<td>0.0039</td>
<td>361.89</td>
</tr>
<tr>
<td>ISE</td>
<td>4.0940</td>
<td>31691</td>
</tr>
<tr>
<td>IAE</td>
<td>2.0234</td>
<td>601.58</td>
</tr>
</tbody>
</table>

The Table II shows the performance criteria with respect to MSE, ISE and IAE of both the estimation methods at the time period 1000. The UKF error is less when compared to EKF.

VI. CONCLUSION

In this work an evaporator with a nonlinear Multi Input and Multi Output (MIMO) system was taken. The input data like feed flow, steam flow and output data like dry matter content and product flow were obtained. With the help of these data the system identification has been done by using nonlinear autoregressive exogenous model (NARX). To achieve the required concentration and feed flow of the product, the states of the system must be known. Then the filter algorithms like UKF and EKF were applied to estimate the state in the presence of random noise in measurements. The UKF and EKF were compared by using simulation in MATLAB. The estimated state becomes nearer to the original state as the iteration increases and the error decreases slowly. But in EKF the estimated state was not nearer to the original state. The MSE, ISE and IAE were used to evaluate UKF and EKF.
REFERENCES


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