

Whether Government intervention and Vocational training increases Education and Employment? : A mathematical model

G.Madhan Kumar¹, M.Mullai²

¹*Department of Mathematics, Alagappa University,
Karaikudi-630004, India.*

email: madhan001kumar@gmail.com

²*Department of Mathematics, Alagappa University,
Karaikudi-630004, India.*

email: mullaim@alagappauniversity.ac.in

Abstract: This paper develops a non-linear mathematical model about the education and employment for both male and female population. We divide the model into two aspects. one, by considering both the male and female as a single population and the other by keep them as a separate population. Government intervention in education and the impact of vocational training to uneducated are discussed, analyzed and the results are shown graphically. The basic reproduction number is found to analyze the transmission dynamics of education and employment in the population. The stability and equilibria of the model are found and analyzed. Numerical simulation is carried out to show the stability results numerically.

Keywords: Education, Employment, Stability, Equilibria, Basic reproduction number, Vocational training

1. Introduction

Education is defined as the process of acquiring the knowledge. The person who teaches the subject / skills is called as teacher / master and the person who listens the subject / learns the skills is called student. In the ancient period, the teacher-student relationship is like the father-son relationship. Students went to teachers home for learning. But now, at present everything changes, the parents are not willing to send their children to school instead they send their children to factories/ petty shops for daily wages. Due to overpopulation, the cost of every material raises even the basic thing like medicine. The poor people are not able to run their family. In order to eat three times food a day, every person in the economically backward family has go to daily work. Due to this, illiteracy rate as well as the child labor rate grows higher. So, government initiate the 'Right to Education Act (RTE) to increase the literacy rate and abolish the child labor. According to RTE, free and compulsory education is provided to all children between 6 to 14 age group. Even though, illiteracy rate are not been totally abolished. Most of the parents refuse to send their girl children to school due to lack of basic facilities and lack of easy access ability to schools. Therefore, when compared to males, females illiteracy rate are always higher[4]. Somehow, certain population is educated but there arises a solution less problem called 'youth unemployment'. Generally unemployment occurs when an educated / uneducated person, whom searching for the employment but unable to find the job. Youth unemployment specifically denotes the unemployed persons of 15 to 24 age group. Youth unemployment is mainly due to overpopulation and underpayment. It leads the unemployed persons to the stage of frustration and even to commit suicide[3]. The universal truth is "Great leaders always have divergent thinking". Mahatma Gandhi, the father of our Indian nation already quoted and stressed about Vocational education / training. Vocational education is defined as carrier education. If suppose everyone have some technical skills, then they don't need to worry about

their jobs. Even though, they are not able to find a suitable job, then they can work on their own technical skills.

2. Model formulation

The total population N is divided into ten compartments such as

$$N = S_m + S_f + E_m + E_f + U_m + U_f + P_m + P_f + E + U$$

where

- S_m and S_f are the susceptible male and female population who may / may not prone to education / uneducation / partial education.
- E_m and E_f are the educated male and female population.
- U_m and U_f represents the uneducated male and female population.
- P_m and P_f denotes the partially educated male and female population.
- E and U expresses the employed and unemployed (both male and female) population.

The parents who know the value of education send their children (S_m and S_f) to school in order to perceive education. so, once they completed upto any degree then they are called as educated (E_m and E_f). Some parents don't know about the value of education. So due to government intervention (θ and θ_x) they send their children to school. Some parents wont send their children (S_m and S_f) to school, so they move to uneducated compartment (U_m and U_f). Some parents who belong to lower economic status / orthodox stops their child's (S_m and S_f) education upto eighth or ninth standard and hence, they move to partially educated (P_m and P_f) compartment. Certain population from (E_m and E_f) moves to the employed (E) compartment due to good educational skills. Eventhough, they are appointed if salary is not sufficient they quit their jobs. Those who did not have good educational skills, move to unemployed (U) compartment. The uneducated persons (U_m and U_f) and partially educated persons (P_m and P_f) whom have willing to find a job reach the employed (E) compartment otherwise, they move to unemployed (U) compartment. With these assumption, the non-linear mathematical model is formulated as follows

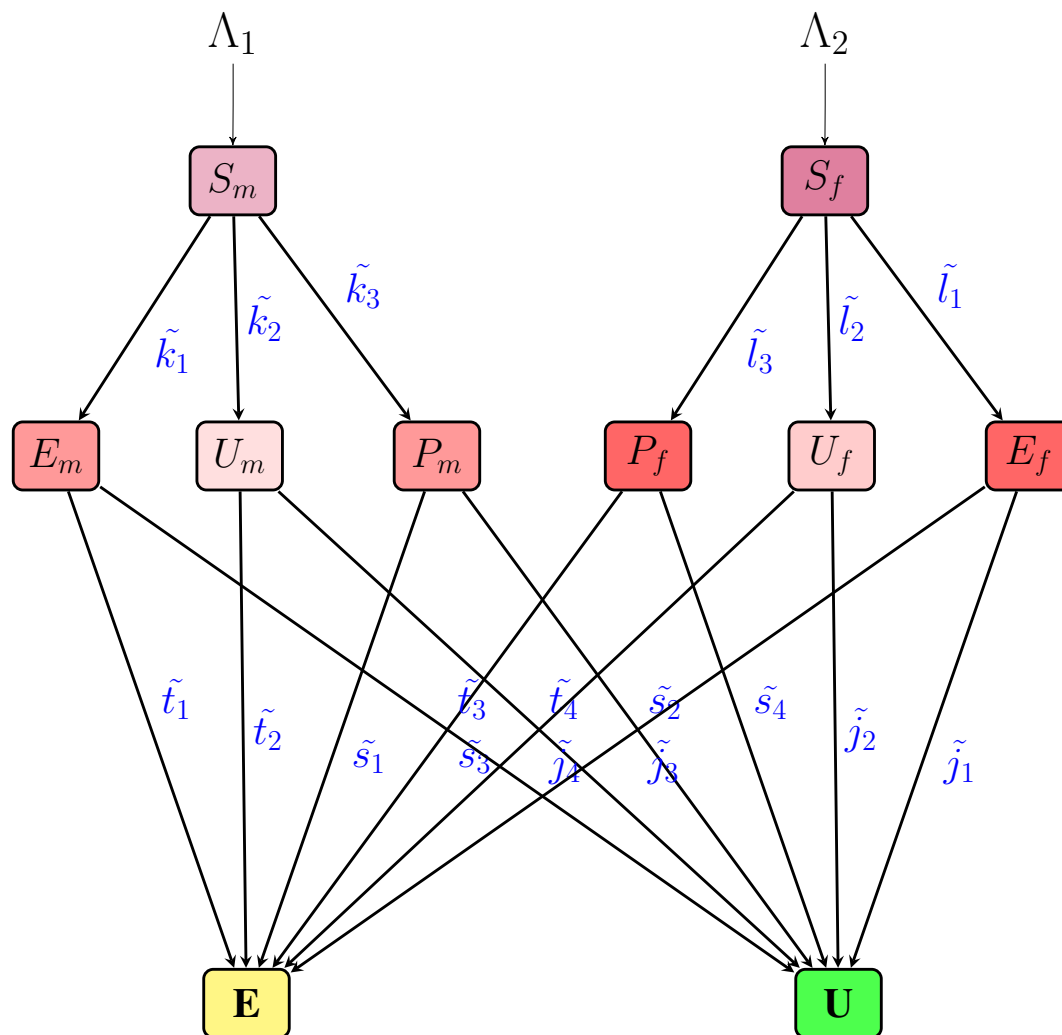
$$\begin{aligned}
 \frac{dS_m}{dt} &= \tilde{\Lambda}_1 - (\tilde{k}_h + \tilde{k}_i + \tilde{k}_j)S_m - \tilde{\mu}_1 S_m \\
 \frac{dS_f}{dt} &= \tilde{\Lambda}_2 - (\tilde{l}_h + \tilde{l}_i + \tilde{l}_j)S_f - \tilde{\mu}_2 S_f \\
 \frac{dE_m}{dt} &= \tilde{k}_h S_m - (\tilde{t}_1 + \tilde{t}_3)E_m - \tilde{\mu}_1 E_m \\
 \frac{dE_f}{dt} &= \tilde{l}_h S_f - (\tilde{j}_1 + \tilde{j}_3)E_f - \tilde{\mu}_2 E_f \\
 \frac{dU_m}{dt} &= \tilde{k}_i S_m - (\tilde{t}_2 + \tilde{t}_4)U_m - \tilde{\mu}_1 U_m \\
 \frac{dU_f}{dt} &= \tilde{l}_i S_f - (\tilde{j}_2 + \tilde{j}_4)U_f - \tilde{\mu}_2 U_f \\
 \frac{dP_m}{dt} &= \tilde{k}_j S_m - (\tilde{s}_1 + \tilde{s}_2)P_m - \tilde{\mu}_1 P_m \\
 \frac{dP_f}{dt} &= \tilde{l}_j S_f - (\tilde{s}_3 + \tilde{s}_4)P_f - \tilde{\mu}_2 P_f \\
 \frac{dE}{dt} &= \tilde{t}_1 E_m + \tilde{j}_1 E_f + \tilde{t}_2 U_m + \tilde{j}_2 U_f + \tilde{s}_1 P_m + \tilde{s}_3 P_f - \tilde{\mu} E \\
 \frac{dU}{dt} &= \tilde{t}_3 E_m + \tilde{j}_3 E_f + \tilde{t}_4 U_m + \tilde{j}_4 U_f + \tilde{s}_2 P_m + \tilde{s}_4 P_f - \tilde{\mu} U
 \end{aligned} \tag{2.1}$$

where

$$\begin{aligned}
 \tilde{k}_h &= \tilde{k}_1 E_m = [\tilde{\alpha}(1 + \tilde{\theta})] E_m \\
 \tilde{l}_h &= \tilde{l}_1 E_f = [\tilde{\alpha}_x(1 + \tilde{\theta}_x)] E_f \\
 \tilde{k}_i &= \tilde{k}_2 U_m = [\tilde{\beta}(1 - \tilde{\theta})] U_m \\
 \tilde{l}_i &= \tilde{l}_2 U_f = [\tilde{\beta}_x(1 - \tilde{\theta}_x)] U_f \\
 \tilde{k}_j &= \tilde{k}_3 P_m = [\tilde{\gamma}(1 - \tilde{\omega})] P_m \\
 \tilde{l}_j &= \tilde{l}_3 P_f = [\tilde{\gamma}_x(1 - \tilde{\omega}_x)] P_f \\
 \tilde{t}_1 &= \tilde{\rho} \\
 \tilde{j}_1 &= \tilde{\rho}_x \\
 \tilde{t}_3 &= \tilde{m}(1 - \tilde{\rho}) \\
 \tilde{j}_3 &= \tilde{m}_x(1 - \tilde{\rho}_x) \\
 \tilde{t}_2 &= \tilde{\pi} \\
 \tilde{j}_2 &= \tilde{\pi}_x \\
 \tilde{t}_4 &= \tilde{n}(1 - \tilde{\pi}) \\
 \tilde{j}_4 &= \tilde{n}_x(1 - \tilde{\pi}_x) \\
 \tilde{s}_1 &= \tilde{\delta} \\
 \tilde{s}_3 &= \tilde{\delta}_x \\
 \tilde{s}_2 &= \tilde{r}(1 - \tilde{\delta}) \\
 \tilde{s}_4 &= \tilde{r}_x(1 - \tilde{\delta}_x)
 \end{aligned}$$

Parameter	Description
$\tilde{\Lambda}_1$	Male recruitment rate
$\tilde{\Lambda}_2$	Female recruitment rate
$\tilde{\alpha}$	Rate of transmission from susceptible male to educated male
$\tilde{\alpha}_x$	Rate of transmission from susceptible female to educated female
$\tilde{\beta}$	Rate of transmission from susceptible male to uneducated male
$\tilde{\beta}_x$	Rate of transmission from susceptible female to uneducated female
$\tilde{\gamma}$	Rate of transmission from susceptible male to partially educated male
$\tilde{\gamma}_x$	Rate of transmission from susceptible female to partially educated female
$\tilde{\theta}, \tilde{\theta}_x$	Rate of transmission through government intervention
$\tilde{\omega}, \tilde{\omega}_x$	Rate of transmission through higher socio-economic status
$\tilde{\rho}, \tilde{\rho}_x$	Educated move to employed through good skills and communication
$\tilde{m}, \tilde{m}_x, \tilde{n}, \tilde{n}_x, \tilde{r}, \tilde{r}_x$	Modification parameters
$\tilde{\pi}, \tilde{\pi}_x$	Uneducated move to employed through their ability to find a job
$\tilde{\delta}, \tilde{\delta}_x$	Partially educated move to employed through their ability to find a job

Table 1. Description of parameters



Transfer diagram of the model

3. Full model

In this section, let us consider the model (2.1) and investigate about their education and employment by both analytically and numerically. To make the investigation more broader, let us think about certain questions.

- If there exists strong government intervention in education, then what happens??
- If there is no government intervention in education, then what happens ??
- If Vocational training is provided to uneducated, then what happens??
- If there is no vocational training to uneducated persons, then what happens??

3.1. Government intervention and no vocational training

In the model (2.1), already the government intervention (θ) presents. So, let us find equilibria and stability of the model. The equilibria is found by keeping the right hand side of the equation zero. Therefore, the model (2.1) has a

educated-free, uneducated-free and partially educated-free equilibrium (E_0) is given by

$$\bar{E}_0 = (S_m^0, S_f^0, E_m^0, E_f^0, U_m^0, U_f^0, P_m^0, P_f^0, E^0, U^0) = \left(\frac{\tilde{\Lambda}_1}{\tilde{\mu}_1}, \frac{\tilde{\Lambda}_2}{\tilde{\mu}_2}, 0, 0, 0, 0, 0, 0, 0, 0 \right).$$

By using the notations mentioned in [1], the basic reproductive number R_0 of the model (2.1) is given by

$$f = \begin{pmatrix} \tilde{k}_1 S_m E_m \\ \tilde{l}_1 S_f E_f \\ \tilde{k}_2 S_m U_m \\ \tilde{l}_2 S_f U_f \\ \tilde{k}_3 S_m P_m \\ \tilde{l}_3 S_f P_f \end{pmatrix}$$

and

$$v = \begin{pmatrix} (\tilde{t}_1 + \tilde{t}_3 + \tilde{\mu}_1) E_m \\ (\tilde{j}_1 + \tilde{j}_3 + \tilde{\mu}_2) E_f \\ (\tilde{t}_2 + \tilde{t}_4 + \tilde{\mu}_1) U_m \\ (\tilde{j}_2 + \tilde{j}_4 + \tilde{\mu}_2) U_f \\ (\tilde{s}_1 + \tilde{s}_2 + \tilde{\mu}_1) P_m \\ (\tilde{s}_3 + \tilde{s}_4 + \tilde{\mu}_2) P_f \end{pmatrix}$$

Now,

$$F = \begin{pmatrix} \tilde{k}_1 S_m^0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \tilde{l}_1 S_f^0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \tilde{k}_2 S_m^0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \tilde{l}_2 S_f^0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \tilde{k}_3 S_m^0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \tilde{l}_3 S_f^0 \end{pmatrix}$$

and

$$V = \begin{pmatrix} (\tilde{t}_1 + \tilde{t}_3 + \tilde{\mu}_1) & 0 & 0 & 0 & 0 & 0 \\ 0 & (\tilde{j}_1 + \tilde{j}_3 + \tilde{\mu}_2) & 0 & 0 & 0 & 0 \\ 0 & 0 & (\tilde{t}_2 + \tilde{t}_4 + \tilde{\mu}_1) & 0 & 0 & 0 \\ 0 & 0 & 0 & (\tilde{j}_2 + \tilde{j}_4 + \tilde{\mu}_2) & 0 & 0 \\ 0 & 0 & 0 & 0 & (\tilde{s}_1 + \tilde{s}_2 + \tilde{\mu}_1) & 0 \\ 0 & 0 & 0 & 0 & 0 & (\tilde{s}_3 + \tilde{s}_4 + \tilde{\mu}_2) \end{pmatrix}$$

Then,

$$FV^{-1} = \begin{pmatrix} \frac{\tilde{k}_1 \tilde{\Lambda}_1}{(\tilde{t}_1 + \tilde{t}_3 + \tilde{\mu}_1) \tilde{\mu}_1} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\tilde{l}_1 \tilde{\Lambda}_2}{(\tilde{j}_1 + \tilde{j}_3 + \tilde{\mu}_2) \tilde{\mu}_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\tilde{k}_2 \tilde{\Lambda}_1}{(\tilde{t}_2 + \tilde{t}_4 + \tilde{\mu}_1) \tilde{\mu}_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\tilde{l}_2 \tilde{\Lambda}_2}{(\tilde{j}_2 + \tilde{j}_4 + \tilde{\mu}_2) \tilde{\mu}_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\tilde{k}_3 \tilde{\Lambda}_1}{(\tilde{s}_1 + \tilde{s}_2 + \tilde{\mu}_1) \tilde{\mu}_1} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\tilde{l}_3 \tilde{\Lambda}_2}{(\tilde{s}_3 + \tilde{s}_4 + \tilde{\mu}_2) \tilde{\mu}_2} \end{pmatrix}$$

$$\begin{aligned}
 R_0 &= \max(R_{em}, R_{ef}, R_{um}, R_{uf}, R_{pm}, R_{pf}) \\
 &= \max\left(\frac{\tilde{k}_1 \tilde{\Lambda}_1}{(\tilde{t}_1 + \tilde{t}_3 + \tilde{\mu}_1) \tilde{\mu}_1}, \frac{\tilde{l}_1 \tilde{\Lambda}_2}{(\tilde{j}_1 + \tilde{j}_3 + \tilde{\mu}_2) \tilde{\mu}_2}, \frac{\tilde{k}_2 \tilde{\Lambda}_1}{(\tilde{t}_2 + \tilde{t}_4 + \tilde{\mu}_1) \tilde{\mu}_1}, \right. \\
 &\quad \left. \frac{\tilde{l}_2 \tilde{\Lambda}_2}{(\tilde{j}_2 + \tilde{j}_4 + \tilde{\mu}_2) \tilde{\mu}_2}, \frac{\tilde{k}_3 \tilde{\Lambda}_1}{(\tilde{s}_1 + \tilde{s}_2 + \tilde{\mu}_1) \tilde{\mu}}, \frac{\tilde{l}_3 \tilde{\Lambda}_2}{(\tilde{s}_3 + \tilde{s}_4 + \tilde{\mu}_2) \tilde{\mu}_2}\right)
 \end{aligned} \tag{3.1}$$

Here, $R_{em}, R_{ef}, R_{um}, R_{uf}, R_{pm}$ and R_{pf} represents the reproductive number of educated males, educated females, uneducated males, uneducated females, partially educated males and partially educated females. If any one among these six increased, then $R_0 > 1$.

The educated, uneducated and partially educated equilibrium (E_*) is given by

$$\begin{aligned}
 S_m^* &= \frac{\tilde{\Lambda}_1}{(\tilde{k}_h + \tilde{k}_i + \tilde{k}_j + \tilde{\mu}_1)}, \\
 S_f^* &= \frac{\tilde{\Lambda}_2}{(\tilde{l}_h + \tilde{l}_i + \tilde{l}_j + \tilde{\mu}_2)}, \\
 E_m^* &= \frac{\tilde{k}_h S_m^*}{(\tilde{t}_1 + \tilde{t}_3 + \tilde{\mu}_1)}, \\
 E_f^* &= \frac{\tilde{l}_h S_f^*}{(\tilde{j}_1 + \tilde{j}_3 + \tilde{\mu}_2)}, \\
 U_m^* &= \frac{\tilde{k}_i S_m^*}{(\tilde{t}_2 + \tilde{t}_4 + \tilde{\mu}_1)}, \\
 U_f^* &= \frac{\tilde{l}_i S_f^*}{(\tilde{j}_2 + \tilde{j}_4 + \tilde{\mu}_2)}, \\
 P_m^* &= \frac{\tilde{k}_j S_m^*}{(\tilde{s}_1 + \tilde{s}_2 + \tilde{\mu}_1)}, \\
 P_f^* &= \frac{\tilde{l}_j S_f^*}{(\tilde{s}_3 + \tilde{s}_4 + \tilde{\mu}_2)}, \\
 E^* &= \frac{\tilde{t}_1 E_m^* + \tilde{j}_1 E_f^* + \tilde{t}_2 U_m^* + \tilde{j}_2 U_f^* + \tilde{s}_1 P_m^* + \tilde{s}_3 P_f^*}{\tilde{\mu}} \text{ and} \\
 U^* &= \frac{\tilde{t}_3 E_m^* + \tilde{j}_3 E_f^* + \tilde{t}_4 U_m^* + \tilde{j}_4 U_f^* + \tilde{s}_2 P_m^* + \tilde{s}_4 P_f^*}{\tilde{\mu}}.
 \end{aligned}$$

The jacobian matrix of the model (2.1) is given as

$$J = \begin{pmatrix}
 a_{11} & 0 & -\tilde{k}_1 S_m & 0 & -\tilde{k}_2 S_m & 0 & -\tilde{k}_3 S_m & 0 & 0 & 0 \\
 0 & a_{22} & 0 & -\tilde{l}_1 S_f & 0 & -\tilde{l}_2 S_f & 0 & -\tilde{l}_3 S_f & 0 & 0 \\
 \tilde{k}_1 E_m & 0 & a_{33} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & \tilde{l}_1 E_f & 0 & a_{44} & 0 & 0 & 0 & 0 & 0 & 0 \\
 \tilde{k}_2 U_m & 0 & 0 & 0 & a_{55} & 0 & 0 & 0 & 0 & 0 \\
 0 & \tilde{l}_2 U_f & 0 & 0 & 0 & a_{66} & 0 & 0 & 0 & 0 \\
 \tilde{k}_3 P_m & 0 & 0 & 0 & 0 & 0 & a_{77} & 0 & 0 & 0 \\
 0 & \tilde{l}_3 P_f & 0 & 0 & 0 & 0 & 0 & a_{88} & 0 & 0 \\
 0 & 0 & \tilde{t}_1 & \tilde{j}_1 & \tilde{t}_2 & \tilde{j}_2 & \tilde{s}_1 & \tilde{s}_3 & -\tilde{\mu} & 0 \\
 0 & 0 & \tilde{t}_3 & \tilde{j}_3 & \tilde{t}_4 & \tilde{j}_4 & \tilde{s}_2 & \tilde{s}_4 & 0 & -\tilde{\mu}
 \end{pmatrix}$$

where $a_{11} = -(\tilde{k}_1 E_m + \tilde{k}_2 U_m + \tilde{k}_3 P_m + \tilde{\mu}_1)$, $a_{22} = -(\tilde{l}_1 E_f + \tilde{l}_2 U_f + \tilde{l}_3 P_f + \tilde{\mu}_2)$, $a_{33} = \tilde{k}_1 S_m - (\tilde{t}_1 + \tilde{t}_3 + \tilde{\mu}_1)$, $a_{44} = \tilde{l}_1 S_f - (\tilde{j}_1 + \tilde{j}_3 + \tilde{\mu}_2)$, $a_{55} = \tilde{k}_2 S_m - (\tilde{t}_2 + \tilde{t}_4 + \tilde{\mu}_1)$, $a_{66} = \tilde{l}_2 S_f - (\tilde{j}_2 + \tilde{j}_4 + \tilde{\mu}_2)$, $a_{77} = \tilde{k}_3 S_m - (\tilde{s}_1 + \tilde{s}_2 + \tilde{\mu}_1)$,

$$a_{88} = \tilde{l}_3 S_f - (\tilde{s}_3 + \tilde{s}_4 + \tilde{\mu}_2).$$

The evaluated jacobian matrix of the model (2.1) at E_* is given by

$$J = \begin{pmatrix} a_{11} & 0 & a_{13} & 0 & a_{15} & 0 & a_{17} & 0 & 0 & 0 \\ 0 & a_{22} & 0 & a_{24} & 0 & a_{26} & 0 & a_{28} & 0 & 0 \\ a_{31} & 0 & a_{33} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & a_{42} & 0 & a_{44} & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{51} & 0 & 0 & 0 & a_{55} & 0 & 0 & 0 & 0 & 0 \\ 0 & a_{62} & 0 & 0 & 0 & a_{66} & 0 & 0 & 0 & 0 \\ a_{71} & 0 & 0 & 0 & 0 & 0 & a_{77} & 0 & 0 & 0 \\ 0 & a_{82} & 0 & 0 & 0 & 0 & 0 & a_{88} & 0 & 0 \\ 0 & 0 & t_1 & j_1 & t_2 & j_2 & s_1 & s_3 & -\tilde{\mu} & 0 \\ 0 & 0 & t_3 & j_3 & t_4 & j_4 & s_2 & s_4 & 0 & -\tilde{\mu} \end{pmatrix}$$

where

$$\begin{aligned} a_{11} &= -(\tilde{k}_1 \frac{\tilde{k}_h S_m^*}{(\tilde{t}_1 + \tilde{t}_3 + \tilde{\mu}_1)} + \tilde{k}_2 \frac{\tilde{k}_i S_m^*}{(\tilde{t}_2 + \tilde{t}_4 + \tilde{\mu}_1)} + \tilde{k}_3 \frac{\tilde{k}_j S_m^*}{(\tilde{s}_1 + \tilde{s}_2 + \tilde{\mu}_1)} + \tilde{\mu}_1), \\ a_{13} &= -\tilde{k}_1 \frac{\tilde{\Lambda}_1}{(\tilde{k}_h + \tilde{k}_i + \tilde{k}_j + \tilde{\mu}_1)}, \\ a_{15} &= -\tilde{k}_2 \frac{\tilde{\Lambda}_1}{(\tilde{k}_h + \tilde{k}_i + \tilde{k}_j + \tilde{\mu}_1)}, \\ a_{17} &= -\tilde{k}_3 \frac{\tilde{\Lambda}_1}{(\tilde{k}_h + \tilde{k}_i + \tilde{k}_j + \tilde{\mu}_1)}, \\ a_{22} &= -(\tilde{l}_1 \frac{\tilde{l}_h S_f^*}{(\tilde{j}_1 + \tilde{j}_3 + \tilde{\mu}_2)} + \tilde{l}_2 \frac{\tilde{l}_i S_f^*}{(\tilde{j}_2 + \tilde{j}_4 + \tilde{\mu}_2)} + \tilde{l}_3 \frac{\tilde{l}_j S_f^*}{(\tilde{s}_3 + \tilde{s}_4 + \tilde{\mu}_2)}), \\ a_{24} &= -\tilde{l}_1 \frac{\tilde{\Lambda}_2}{(\tilde{l}_h + \tilde{l}_i + \tilde{l}_j + \tilde{\mu}_2)}, \\ a_{26} &= -\tilde{l}_2 \frac{\tilde{\Lambda}_2}{(\tilde{l}_h + \tilde{l}_i + \tilde{l}_j + \tilde{\mu}_2)}, \\ a_{28} &= -\tilde{l}_3 \frac{\tilde{\Lambda}_2}{(\tilde{l}_h + \tilde{l}_i + \tilde{l}_j + \tilde{\mu}_2)}, \\ a_{31} &= \tilde{k}_1 \frac{\tilde{k}_h S_m^*}{(\tilde{t}_1 + \tilde{t}_3 + \tilde{\mu}_1)}, \\ a_{33} &= \tilde{k}_1 \frac{\tilde{\Lambda}_1}{(\tilde{k}_h + \tilde{k}_i + \tilde{k}_j + \tilde{\mu}_1)} - (\tilde{t}_1 + \tilde{t}_3 + \tilde{\mu}_1), \\ a_{42} &= \tilde{l}_1 \frac{\tilde{l}_h S_m^*}{\tilde{j}_1 + \tilde{j}_3 + \tilde{\mu}_2}, \\ a_{44} &= \tilde{l}_1 \frac{\tilde{\Lambda}_2}{(\tilde{l}_h + \tilde{l}_i + \tilde{l}_j + \tilde{\mu}_2)} - (\tilde{j}_1 + \tilde{j}_3 + \tilde{\mu}_2), \\ a_{51} &= \tilde{k}_2 \frac{\tilde{k}_i S_m^*}{(\tilde{t}_2 + \tilde{t}_4 + \tilde{\mu}_1)}, \end{aligned} \tag{3.2}$$

$$\begin{aligned}
a_{55} &= \tilde{k}_2 \frac{\tilde{\Lambda}_1}{(\tilde{k}_h + \tilde{k}_i + \tilde{k}_j + \tilde{\mu}_1)} - (\tilde{t}_2 + \tilde{t}_4 + \tilde{\mu}_1), \\
a_{62} &= \tilde{l}_2 \frac{\tilde{l}_i S_f^*}{(\tilde{j}_2 + \tilde{j}_4 + \tilde{\mu}_2)}, \\
a_{66} &= \tilde{l}_2 \frac{\tilde{\Lambda}_2}{(\tilde{l}_h + \tilde{l}_i + \tilde{l}_j + \tilde{\mu}_2)} - (\tilde{j}_2 + \tilde{j}_4 + \tilde{\mu}_2), \\
a_{71} &= \tilde{k}_3 \frac{\tilde{k}_j S_m^*}{(\tilde{s}_1 + \tilde{s}_2 + \tilde{\mu}_1)}, \\
a_{77} &= \tilde{k}_3 \frac{\tilde{\Lambda}_1}{(\tilde{k}_h + \tilde{k}_i + \tilde{k}_j + \tilde{\mu}_1)} - (\tilde{s}_1 + \tilde{s}_2 + \tilde{\mu}_1), \\
a_{82} &= \tilde{l}_3 \frac{\tilde{l}_j S_f^*}{(\tilde{s}_3 + \tilde{s}_4 + \tilde{\mu}_2)}, \\
a_{88} &= \tilde{l}_3 \frac{\tilde{\Lambda}_2}{(\tilde{l}_h + \tilde{l}_i + \tilde{l}_j + \tilde{\mu}_2)} - (\tilde{s}_3 + \tilde{s}_4 + \tilde{\mu}_2)
\end{aligned}$$

Theorem 3.1. *The educated-free, uneducated-free and partially educated-free equilibrium (E_0) exists whenever $R_0 < 1$.*

Proof. The jacobian of the model (2.1) at E_0 is given by

$$J_{E_0} = \begin{pmatrix} -\tilde{\mu}_1 & 0 & -\tilde{k}_1 \frac{\tilde{\Lambda}_1}{\tilde{\mu}_1} & 0 & -\tilde{k}_2 \frac{\tilde{\Lambda}_1}{\tilde{\mu}_1} & 0 & -\tilde{k}_3 \frac{\tilde{\Lambda}_1}{\tilde{\mu}_1} & 0 & 0 & 0 \\ 0 & -\tilde{\mu}_2 & 0 & -\tilde{l}_1 \frac{\tilde{\Lambda}_2}{\tilde{\mu}_2} & 0 & -\tilde{l}_2 \frac{\tilde{\Lambda}_2}{\tilde{\mu}_2} & 0 & -\tilde{l}_3 \frac{\tilde{\Lambda}_2}{\tilde{\mu}_2} & 0 & 0 \\ 0 & 0 & a_{33} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_{44} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_{55} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & a_{66} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & a_{77} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{88} & 0 & 0 \\ 0 & 0 & \tilde{t}_1 & \tilde{j}_1 & \tilde{t}_2 & \tilde{j}_2 & \tilde{s}_1 & \tilde{s}_3 & -\tilde{\mu} & 0 \\ 0 & 0 & \tilde{t}_3 & \tilde{j}_3 & \tilde{t}_4 & \tilde{j}_4 & \tilde{s}_2 & \tilde{s}_4 & 0 & -\tilde{\mu} \end{pmatrix} \quad (3.3)$$

where $a_{33} = \tilde{k}_1 \frac{\tilde{\Lambda}_1}{\tilde{\mu}_1} - (\tilde{t}_1 + \tilde{t}_3 + \tilde{\mu}_1)$, $a_{44} = \tilde{l}_1 \frac{\tilde{\Lambda}_2}{\tilde{\mu}_2} - (\tilde{j}_1 + \tilde{j}_3 + \tilde{\mu}_2)$, $a_{55} = \tilde{k}_2 \frac{\tilde{\Lambda}_1}{\tilde{\mu}_1} - (\tilde{t}_2 + \tilde{t}_4 + \tilde{\mu}_1)$, $a_{66} = \tilde{l}_2 \frac{\tilde{\Lambda}_2}{\tilde{\mu}_2} - (\tilde{j}_2 + \tilde{j}_4 + \tilde{\mu}_2)$, $a_{77} = \tilde{k}_3 \frac{\tilde{\Lambda}_1}{\tilde{\mu}_1} - (\tilde{s}_1 + \tilde{s}_2 + \tilde{\mu}_1)$, $a_{88} = \tilde{l}_3 \frac{\tilde{\Lambda}_2}{\tilde{\mu}_2} - (\tilde{s}_3 + \tilde{s}_4 + \tilde{\mu}_2)$.

After linearization, the evaluated jacobian matrix at E_0 becomes

$$J_{E_0} = \begin{vmatrix} -\tilde{\mu}_1 - \lambda & 0 & -\tilde{k}_1 \frac{\tilde{\Lambda}_1}{\tilde{\mu}_1} & 0 & -\tilde{k}_2 \frac{\tilde{\Lambda}_1}{\tilde{\mu}_1} & 0 & -\tilde{k}_3 \frac{\tilde{\Lambda}_1}{\tilde{\mu}_1} & 0 & 0 & 0 \\ 0 & -\tilde{\mu}_2 - \lambda & 0 & -\tilde{l}_1 \frac{\tilde{\Lambda}_2}{\tilde{\mu}_2} & 0 & -\tilde{l}_2 \frac{\tilde{\Lambda}_2}{\tilde{\mu}_2} & 0 & -\tilde{l}_3 \frac{\tilde{\Lambda}_2}{\tilde{\mu}_2} & 0 & 0 \\ 0 & 0 & a_{33} - \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_{44} - \lambda & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_{55} - \lambda & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & a_{66} - \lambda & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & a_{77} - \lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{88} - \lambda & 0 & 0 \\ 0 & 0 & \tilde{t}_1 & \tilde{j}_1 & \tilde{t}_2 & \tilde{j}_2 & \tilde{s}_1 & \tilde{s}_3 & -\tilde{\mu} - \lambda & 0 \\ 0 & 0 & \tilde{t}_3 & \tilde{j}_3 & \tilde{t}_4 & \tilde{j}_4 & \tilde{s}_2 & \tilde{s}_4 & 0 & -\tilde{\mu} - \lambda \end{vmatrix} \quad (3.4)$$

where,

$$\begin{aligned}
 \lambda_1 &= -\tilde{\mu}_1, \\
 \lambda_2 &= -\tilde{\mu}_2, \\
 \lambda_3 &= \tilde{k}_1 \frac{\tilde{\Lambda}_1}{\tilde{\mu}_1} - (\tilde{t}_1 + \tilde{t}_3 + \tilde{\mu}_1), \\
 \lambda_4 &= \tilde{l}_1 \frac{\tilde{\Lambda}_2}{\tilde{\mu}_2} - (\tilde{j}_1 + \tilde{j}_3 + \tilde{\mu}_2), \\
 \lambda_5 &= \tilde{k}_2 \frac{\tilde{\Lambda}_1}{\tilde{\mu}_1} - (\tilde{t}_2 + \tilde{t}_4 + \tilde{\mu}_1), \\
 \lambda_6 &= \tilde{l}_2 \frac{\tilde{\Lambda}_2}{\tilde{\mu}_2} - (\tilde{j}_2 + \tilde{j}_4 + \tilde{\mu}_2), \\
 \lambda_7 &= \tilde{k}_3 \frac{\tilde{\Lambda}_1}{\tilde{\mu}_1} - (\tilde{s}_1 + \tilde{s}_2 + \tilde{\mu}_1), \\
 \lambda_8 &= \tilde{l}_3 \frac{\tilde{\Lambda}_2}{\tilde{\mu}_2} - (\tilde{s}_3 + \tilde{s}_4 + \tilde{\mu}_2), \\
 \lambda_9 &= -\tilde{\mu}, \\
 \lambda_{10} &= -\tilde{\mu}.
 \end{aligned}$$

From the above $\lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7, \lambda_8$ are the largest eigen values. If

$$\begin{aligned}
 \lambda_3 &= \tilde{k}_1 \frac{\tilde{\Lambda}_1}{\tilde{\mu}_1} - (\tilde{t}_1 + \tilde{t}_3 + \tilde{\mu}_1) > 0, \\
 \lambda_4 &= \tilde{l}_1 \frac{\tilde{\Lambda}_2}{\tilde{\mu}_2} - (\tilde{j}_1 + \tilde{j}_3 + \tilde{\mu}_2) > 0, \\
 \lambda_5 &= \tilde{k}_2 \frac{\tilde{\Lambda}_1}{\tilde{\mu}_1} - (\tilde{t}_2 + \tilde{t}_4 + \tilde{\mu}_1) > 0, \\
 \lambda_6 &= \tilde{l}_2 \frac{\tilde{\Lambda}_2}{\tilde{\mu}_2} - (\tilde{j}_2 + \tilde{j}_4 + \tilde{\mu}_2) > 0, \\
 \lambda_7 &= \tilde{k}_3 \frac{\tilde{\Lambda}_1}{\tilde{\mu}_1} - (\tilde{s}_1 + \tilde{s}_2 + \tilde{\mu}_1) > 0, \\
 \lambda_8 &= \tilde{l}_3 \frac{\tilde{\Lambda}_2}{\tilde{\mu}_2} - (\tilde{s}_3 + \tilde{s}_4 + \tilde{\mu}_2) > 0
 \end{aligned}$$

then,

$$\begin{aligned}
 \tilde{k}_1 \frac{\tilde{\Lambda}_1}{(\tilde{t}_1 + \tilde{t}_3 + \tilde{\mu}_1)\tilde{\mu}_1} &< 1, \\
 \tilde{l}_1 \frac{\tilde{\Lambda}_2}{(\tilde{j}_1 + \tilde{j}_3 + \tilde{\mu}_2)\tilde{\mu}_2} &< 1, \\
 \tilde{k}_2 \frac{\tilde{\Lambda}_1}{(\tilde{t}_2 + \tilde{t}_4 + \tilde{\mu}_1)\tilde{\mu}_1} &< 1, \\
 \tilde{l}_2 \frac{\tilde{\Lambda}_2}{(\tilde{j}_2 + \tilde{j}_4 + \tilde{\mu}_2)\tilde{\mu}_2} &< 1, \\
 \tilde{k}_3 \frac{\tilde{\Lambda}_1}{(\tilde{s}_1 + \tilde{s}_2 + \tilde{\mu}_1)\tilde{\mu}_1} &< 1, \\
 \tilde{l}_3 \frac{\tilde{\Lambda}_2}{(\tilde{s}_3 + \tilde{s}_4 + \tilde{\mu}_2)\tilde{\mu}_2} &< 1
 \end{aligned}$$

10

i.e,

$$\begin{aligned}
 R_{em} &< 1, \\
 R_{ef} &< 1, \\
 R_{um} &< 1, \\
 R_{uf} &< 1, \\
 R_{pm} &< 1, \\
 R_{pf} &< 1
 \end{aligned}$$

By equation (5.1), $R_0 < 1$. Therefore, (E_0) exists whenever $R_0 < 1$. ■

Theorem 3.2. *The educated-free, uneducated-free and partially educated-free equilibrium (E_0) of the model (2.1) is locally asymptotically stable (LAS) only when $R_0 < 1$.*

Proof. From equation (3.3), the characteristic equation is given by

$$(\tilde{\mu} + \lambda)^4(a_{33} + \lambda)(\lambda^5 + q_1\lambda^4 + q_2\lambda^3 + q_3\lambda^2 + q_4\lambda + q_5) = 0$$

Here, $q_1 = -[a_{44} + a_{55} + a_{66} + a_{77} + a_{88}]$, $q_2 = [a_{44}a_{55} + a_{55}a_{66} + a_{66}a_{77} + a_{77}a_{88}]$,
 $q_3 = -[a_{44}a_{55}a_{66} + a_{55}a_{66}a_{77} + a_{66}a_{77}a_{88}]$,

$$\begin{aligned}
 q_4 &= \begin{vmatrix} a_{44} & a_{45} & a_{46} & a_{47} \\ a_{54} & a_{55} & a_{56} & a_{57} \\ a_{64} & a_{65} & a_{66} & a_{67} \\ a_{74} & a_{75} & a_{76} & a_{77} \end{vmatrix} \\
 q_5 &= - \begin{vmatrix} a_{44} & a_{45} & a_{46} & a_{47} & a_{48} \\ a_{54} & a_{55} & a_{56} & a_{57} & a_{58} \\ a_{64} & a_{65} & a_{66} & a_{67} & a_{68} \\ a_{74} & a_{75} & a_{76} & a_{77} & a_{78} \\ a_{84} & a_{85} & a_{86} & a_{87} & a_{88} \end{vmatrix}
 \end{aligned}$$

By Routh-hurwitz criterion [2], the equilibrium point M_0 is locally asymptotically stable when it satisfies the conditions

$$q_1 > 0, \quad (3.5)$$

$$q_2 > 0, \quad (3.6)$$

$$q_3 > 0, \quad (3.7)$$

$$q_4 > 0, \quad (3.8)$$

$$q_5 > 0, \quad (3.9)$$

$$q_1q_2q_3 > q_3^2 + q_1^2q_4, \quad (3.10)$$

$$(q_1q_4 - q_5)(q_1q_2q_3 - q_3^2 - q_1^2q_4) > q_5(q_1q_2 - q_3)^2 + q_1q_5^2 \quad (3.11)$$

at $R_m < 1$. ■

Theorem 3.3. *The educated, uneducated and partially educated equilibrium (E_*) of the model (2.1) is locally asymptotically stable (LAS) only when $R_0 > 1$.*

Proof. Since $R_0 > 1$, the educated, uneducated and partially educated state starts to emerge and grow rapidly. The conditions mentioned in equations (3.4) to (3.10) are satisfied at $R_0 > 1$, then we can say that the equilibrium point (E_1) of the model (2.1) is locally asymptotically stable. ■

Theorem 3.4. *The educated-free, uneducated-free and partially educated-free equilibrium (E_0) of the model (2.1) is globally asymptotically stable (GAS) only when $R_0 < 1$.*

Proof. By using comparison theorem [1], from the model (2.1)

$$\begin{pmatrix} d/dt \\ d/dt \\ d/dt \\ d/dt \\ d/dt \\ d/dt \\ d/dt \end{pmatrix} \begin{pmatrix} E_m \\ E_f \\ U_m \\ U_f \\ P_m \\ P_f \\ E \\ U \end{pmatrix} = (F - V) \begin{pmatrix} E_m \\ E_f \\ U_m \\ U_f \\ P_m \\ P_f \\ E \\ U \end{pmatrix} - \begin{pmatrix} \tilde{k}_h(S_m^0 - S_m) \\ \tilde{l}_h(S_f^0 - S_f) \\ \tilde{k}_i(S_m^0 - S_m) \\ \tilde{l}_i(S_f^0 - S_f) \\ \tilde{k}_j(S_m^0 - S_m) \\ \tilde{l}_j(S_f^0 - S_f) \\ 0 \\ 0 \end{pmatrix}$$

which leads to

$$\begin{pmatrix} d/dt \\ d/dt \\ d/dt \\ d/dt \\ d/dt \\ d/dt \\ d/dt \end{pmatrix} \begin{pmatrix} E_m \\ E_f \\ U_m \\ U_f \\ P_m \\ P_f \\ E \\ U \end{pmatrix} \leq (F - V) \begin{pmatrix} E_m \\ E_f \\ U_m \\ U_f \\ P_m \\ P_f \\ E \\ U \end{pmatrix}$$

From theorem(3.1), we showed that the eigen values of F-V are negative and the model (2.1) is stable at $R_m < 1$. Thus, $(E_m, E_f, U_m, U_f, P_m, P_f, E, U) \rightarrow (0,0,0,0,0,0,0,0)$ and $S_m \rightarrow \frac{\tilde{\Lambda}_1}{\mu_1}$, $S_f \rightarrow \frac{\tilde{\Lambda}_2}{\mu_2}$ as $t \rightarrow \infty$ (i.e) $(E_m, E_f, U_m, U_f, P_m, P_f, E, U) \rightarrow M_0$ as $t \rightarrow \infty$. Hence, we proved that the educated-free, uneducated-free and partially educated-free equilibrium point is globally asymptotically stable only when $R_m < 1$. ■

Theorem 3.5. *The educated, uneducated and partially educated equilibrium (E_*) of the model (2.1) is globally asymptotically stable (GAS) only when $R_0 > 1$.*

3.2. No government intervention and no vocational training

If there is no government intervention ($\theta = 0$) and no vocational training, then the model 2.1 becomes

$$\begin{aligned} \frac{dS_m}{dt} &= \tilde{\Lambda}_1 - (\tilde{\alpha}E_m + \tilde{\beta}U_m + [\tilde{\gamma}(1 - \tilde{\omega})]P_m)S_m - \tilde{\mu}_1S_m \\ \frac{dS_f}{dt} &= \tilde{\Lambda}_2 - (\tilde{\alpha}_xE_f + \tilde{\beta}_xU_f + [\tilde{\gamma}_x(1 - \tilde{\omega}_x)]P_f)S_f - \tilde{\mu}_2S_f \\ \frac{dE_m}{dt} &= \tilde{\alpha}E_mS_m - (\tilde{t}_1 + \tilde{t}_3)E_m - \tilde{\mu}_1E_m \\ \frac{dE_f}{dt} &= \tilde{\alpha}_xE_fS_f - (\tilde{j}_1 + \tilde{j}_3)E_f - \tilde{\mu}_2E_f \\ \frac{dU_m}{dt} &= \tilde{\beta}U_mS_m - (\tilde{t}_2 + \tilde{t}_4)U_m - \tilde{\mu}_1U_m \\ \frac{dU_f}{dt} &= \tilde{\beta}_xU_fS_f - (\tilde{j}_2 + \tilde{j}_4)U_f - \tilde{\mu}_2U_f \\ \frac{dP_m}{dt} &= [\tilde{\gamma}(1 - \tilde{\omega})]P_mS_m - (\tilde{s}_1 + \tilde{s}_2)P_m - \tilde{\mu}_1P_m \\ \frac{dP_f}{dt} &= [\tilde{\gamma}_x(1 - \tilde{\omega}_x)]P_fS_f - (\tilde{s}_3 + \tilde{s}_4)P_f - \tilde{\mu}_2P_f \\ \frac{dE}{dt} &= \tilde{\rho}E_m + \tilde{\rho}_xE_f + \tilde{\pi}U_m + \tilde{\pi}_xU_f + \tilde{\delta}P_m + \tilde{\delta}_xP_f - \mu E \\ \frac{dU}{dt} &= \tilde{m}(1 - \tilde{\rho})E_m + \tilde{m}_1(1 - \tilde{\rho}_x)E_f + \tilde{n}(1 - \tilde{\pi})U_m + \tilde{n}_1(1 - \tilde{\pi}_x)U_f + \tilde{r}(1 - \tilde{\delta})P_m + \tilde{r}_1(1 - \tilde{\delta}_x)P_f - \tilde{\mu}U \end{aligned} \quad (3.12)$$

The equilibrium point (E_1) of the model 2.1 is given by

$$E_1 = (S_m^0, S_f^0, E_m^0, E_f^0, U_m^0, U_f^0, P_m^0, P_f^0, E^0, U^0) = \left(\frac{\tilde{\Lambda}_1}{\tilde{\mu}_1}, \frac{\tilde{\Lambda}_2}{\tilde{\mu}_2}, 0, 0, 0, 0, 0, 0, 0, 0 \right).$$

The reproductive number (R_1) of the model 2.1 is given by

$$\begin{aligned} R_1 &= \max(R_{em}, R_{ef}, R_{um}, R_{uf}, R_{pm}, R_{pf}) \\ &= \max\left(\frac{\tilde{\alpha}\tilde{\Lambda}_1}{(\tilde{\rho} + \tilde{m}(1 - \tilde{\rho}) + \tilde{\mu}_1)\tilde{\mu}_1}, \frac{\tilde{\alpha}_x\tilde{\Lambda}_2}{(\tilde{\rho}_x + \tilde{m}_1(1 - \tilde{\rho}_x) + \tilde{\mu}_2)\tilde{\mu}_2}, \frac{\tilde{\beta}\tilde{\Lambda}_1}{(\tilde{\pi} + \tilde{n}(1 - \tilde{\pi}) + \tilde{\mu}_1)\tilde{\mu}_1}, \right. \\ &\quad \left. \frac{\tilde{\beta}_x\tilde{\Lambda}_2}{(\tilde{\pi}_x + \tilde{n}_1(1 - \tilde{\pi}_x) + \tilde{\mu}_2)\tilde{\mu}_2}, \frac{[\tilde{\gamma}(1 - \tilde{\omega})]\tilde{\Lambda}_1}{(\tilde{\delta} + \tilde{r}(1 - \tilde{\delta}) + \tilde{\mu}_1)\tilde{\mu}_1}, \frac{[\tilde{\gamma}_x(1 - \tilde{\omega}_x)]\tilde{\Lambda}_2}{(\tilde{\delta}_x + \tilde{r}_1(1 - \tilde{\delta}_x) + \tilde{\mu}_2)\tilde{\mu}_2}\right) \end{aligned} \quad (3.13)$$

The equilibrium point (E_2) of the model 2.1 is given by

$$\begin{aligned} S_m^* &= \frac{\tilde{\Lambda}_1}{(\tilde{k}_h + \tilde{k}_i + \tilde{k}_j + \tilde{\mu}_1)}, \\ S_f^* &= \frac{\tilde{\Lambda}_2}{(\tilde{l}_h + \tilde{l}_i + \tilde{l}_j + \tilde{\mu}_2)}, \\ E_m^* &= \frac{\tilde{k}_h S_m^*}{(\tilde{\rho} + \tilde{m}(1 - \tilde{\rho}) + \tilde{\mu}_1)}, \\ E_f^* &= \frac{\tilde{l}_h S_f^*}{(\tilde{\rho}_x + \tilde{m}_1(1 - \tilde{\rho}_x) + \tilde{\mu}_2)}, \\ U_m^* &= \frac{\tilde{k}_i S_m^*}{(\tilde{\pi} + \tilde{n}(1 - \tilde{\pi}) + \tilde{\mu}_1)}, \\ U_f^* &= \frac{\tilde{l}_i S_f^*}{(\tilde{\pi}_x + \tilde{n}_1(1 - \tilde{\pi}_x) + \tilde{\mu}_2)}, \\ P_m^* &= \frac{\tilde{k}_j S_m^*}{(\tilde{\delta} + \tilde{r}(1 - \tilde{\delta}) + \tilde{\mu}_1)}, \\ P_f^* &= \frac{\tilde{l}_j S_f^*}{(\tilde{\delta}_x + \tilde{r}_1(1 - \tilde{\delta}_x) + \tilde{\mu}_2)}, \\ E^* &= \frac{\tilde{\rho} E_m^* + \tilde{\rho}_x E_f^* + \tilde{\pi} U_m^* + \tilde{\pi}_x U_f^* + \tilde{\gamma} P_m^* + \tilde{\gamma}_x P_f^*}{\tilde{\mu}}, \\ U^* &= \frac{\tilde{m}(1 - \tilde{\rho}) E_m^* + \tilde{m}_1(1 - \tilde{\rho}_x) E_f^* + \tilde{n}(1 - \tilde{\pi}) U_m^* + \tilde{n}_1(1 - \tilde{\pi}_x) U_f^* + \tilde{r}(1 - \tilde{\delta}) P_m^* + \tilde{r}_1(1 - \tilde{\delta}_x) P_f^*}{\tilde{\mu}}. \end{aligned}$$

where

$$\tilde{k}_h = \tilde{\alpha} E_m, \tilde{k}_i = \tilde{\beta} U_m, \tilde{k}_j = \tilde{\gamma}(1 - \tilde{\omega}) P_m, \tilde{l}_h = \tilde{\alpha}_x E_f, \tilde{l}_i = \tilde{\beta}_x U_f, \tilde{l}_j = \tilde{\gamma}_x(1 - \tilde{\omega}_x) P_f$$

3.3. Government intervention and Vocational training exists

In this section, consider that when vocational training is provided to uneducated persons then, the model 2.1 becomes

$$\begin{aligned}
 \frac{dS_m}{dt} &= \tilde{\Lambda}_1 - (\tilde{\alpha}(1 + \tilde{\theta})E_m + \tilde{\beta}(1 - \tilde{\theta})U_m + [\tilde{\gamma}(1 - \tilde{\omega})]P_m)S_m - \tilde{\mu}_1 S_m \\
 \frac{dS_f}{dt} &= \tilde{\Lambda}_2 - (\tilde{\alpha}_x(1 + \tilde{\theta}_x)E_f + \tilde{\beta}_x(1 - \tilde{\theta}_x)U_f + [\tilde{\gamma}_x(1 - \tilde{\omega}_x)]P_f)S_f - \tilde{\mu}_2 S_f \\
 \frac{dE_m}{dt} &= \tilde{\alpha}(1 + \tilde{\theta})E_m S_m - (\tilde{\rho} + \tilde{m}(1 - \tilde{\rho}))E_m - \tilde{\mu}_1 E_m \\
 \frac{dE_f}{dt} &= \tilde{\alpha}_x(1 + \tilde{\theta}_x)E_f S_f - (\tilde{\rho}_x + \tilde{m}_1(1 - \tilde{\rho}_x))E_f - \tilde{\mu}_2 E_f \\
 \frac{dU_m}{dt} &= \tilde{\beta}(1 - \tilde{\theta})U_m S_m - \left(\frac{\tilde{\pi}}{a_1} + \frac{\tilde{n}(1 - \tilde{\pi})}{a_1}\right)U_m - \tilde{\mu}_1 U_m \\
 \frac{dU_f}{dt} &= \tilde{\beta}_x(1 - \tilde{\theta}_x)U_f S_f - \left(\frac{\tilde{\pi}_x}{a_2} + \frac{\tilde{n}_1(1 - \tilde{\pi}_x)}{a_2}\right)U_f - \tilde{\mu}_2 U_f \\
 \frac{dP_m}{dt} &= [\tilde{\gamma}(1 - \tilde{\omega})]P_m S_m - (\tilde{\delta} + \tilde{r}(1 - \tilde{\delta}))P_m - \tilde{\mu}_1 P_m \\
 \frac{dP_f}{dt} &= [\tilde{\gamma}_x(1 - \tilde{\omega}_x)]P_f S_f - (\tilde{\delta}_x + \tilde{r}_1(1 - \tilde{\delta}_x))P_f - \tilde{\mu}_2 P_f \\
 \frac{dE}{dt} &= \tilde{\rho}E_m + \tilde{\rho}_xE_f + \frac{\tilde{\pi}}{a_1}U_m + \frac{\tilde{\pi}_x}{a_2}U_f + \tilde{\delta}P_m + \tilde{\delta}_xP_f - \tilde{\mu}E \\
 \frac{dU}{dt} &= \tilde{m}(1 - \tilde{\rho})E_m + \tilde{m}_1(1 - \tilde{\rho}_x)E_f + \frac{\tilde{n}(1 - \tilde{\pi})}{a_1}U_m + \frac{\tilde{n}_1(1 - \tilde{\pi}_x)}{a_2}U_f + \tilde{r}(1 - \tilde{\delta})P_m + \tilde{r}_1(1 - \tilde{\delta}_x)P_f - \tilde{\mu}U
 \end{aligned} \tag{3.14}$$

The model (3.14) has equilibrium (E_3) given by

$$E_3 = (S_m^0, S_f^0, E_m^0, E_f^0, U_m^0, U_f^0, P_m^0, P_f^0, E^0, U^0) = \left(\frac{\tilde{\Lambda}_1}{\tilde{\mu}_1}, \frac{\tilde{\Lambda}_2}{\tilde{\mu}_2}, 0, 0, 0, 0, 0, 0, 0, 0 \right).$$

$$\begin{aligned}
 R_2 &= \max(R_{em}, R_{ef}, R_{um}, R_{uf}, R_{pm}, R_{pf}) \\
 &= \max\left(\frac{\tilde{\alpha}\tilde{\Lambda}_1}{(\tilde{\rho} + \tilde{m}(1 - \tilde{\rho}) + \tilde{\mu}_1)\tilde{\mu}_1}, \frac{\tilde{\alpha}_x\tilde{\Lambda}_2}{\tilde{\rho}_x + \tilde{m}_1(1 - \tilde{\rho}_x) + \tilde{\mu}_2}\tilde{\mu}_2, \frac{\tilde{\beta}\tilde{\Lambda}_1}{\left(\frac{\tilde{\pi} + \tilde{n}(1 - \tilde{\pi})}{a_1} + \tilde{\mu}_1\right)\tilde{\mu}_1}, \right. \\
 &\quad \left. \frac{\tilde{\beta}_x\tilde{\Lambda}_2}{\left(\frac{\tilde{\pi}_x + \tilde{n}_1(1 - \tilde{\pi}_x)}{a_2} + \tilde{\mu}_2\right)\tilde{\mu}_2}, \frac{[\tilde{\gamma}(1 - \tilde{\omega})]\tilde{\Lambda}_1}{(\tilde{\delta} + \tilde{r}(1 - \tilde{\delta}) + \tilde{\mu}_1)\tilde{\mu}_1}, \frac{[\tilde{\gamma}_x(1 - \tilde{\omega}_x)]\tilde{\Lambda}_2}{(\tilde{\delta}_x + \tilde{r}_1(1 - \tilde{\delta}_x) + \tilde{\mu}_2)\tilde{\mu}_2}\right)
 \end{aligned} \tag{3.15}$$

The model 3.14 has (E_4) equilibrium given by

$$\begin{aligned}
 S_m^* &= \frac{\tilde{\Lambda}_1}{(\tilde{k}_h + \tilde{k}_i + \tilde{k}_j + \tilde{\mu}_1)}, \\
 S_f^* &= \frac{\tilde{\Lambda}_2}{(\tilde{l}_h + \tilde{l}_i + \tilde{l}_j + \tilde{\mu}_2)}, \\
 E_m^* &= \frac{\tilde{k}_h S_m^*}{(\tilde{\rho} + \tilde{m}(1 - \tilde{\rho}) + \tilde{\mu}_1)}, \\
 E_f^* &= \frac{\tilde{l}_h S_f^*}{(\tilde{\rho}_x + \tilde{m}_1(1 - \tilde{\rho}_x) + \tilde{\mu}_2)},
 \end{aligned}$$

$$\begin{aligned}
U_m^* &= \frac{\tilde{k}_i S_m^*}{\left(\frac{\tilde{\pi} + \tilde{n}(1 - \tilde{\pi})}{a_1} + \tilde{\mu}_1\right)}, \\
U_f^* &= \frac{\tilde{l}_i S_f^*}{\left(\frac{\tilde{\pi}_x + \tilde{n}_1(1 - \tilde{\pi}_x)}{a_2} + \tilde{\mu}_2\right)}, \\
P_m^* &= \frac{\tilde{k}_j S_m^*}{(\tilde{\delta} + \tilde{r}(1 - \tilde{\delta}) + \tilde{\mu}_1)}, \\
P_f^* &= \frac{\tilde{l}_j S_f^*}{(\tilde{\delta}_x + \tilde{r}_1(1 - \tilde{\delta}_x) + \tilde{\mu}_2)}, \\
E^* &= \frac{\tilde{\rho} E_m^* + \tilde{\rho}_x E_f^* + \frac{\tilde{\pi}}{a_1} U_m^* + \frac{\tilde{\pi}_x}{a_2} U_f^* + \tilde{\gamma} P_m^* + \tilde{\gamma}_x P_f^*}{\tilde{\mu}}, \\
U^* &= \frac{\tilde{m}(1 - \tilde{\rho}) E_m^* + \tilde{m}_1(1 - \tilde{\rho}_x) E_f^* + \frac{\tilde{n}(1 - \tilde{\pi})}{a_1} U_m^* + \frac{\tilde{n}_1(1 - \tilde{\pi}_x)}{a_2} U_f^* + \tilde{r}(1 - \tilde{\delta}) P_m^* + \tilde{r}_1(1 - \tilde{\delta}_x) P_f^*}{\tilde{\mu}}.
\end{aligned}$$

where

$$\tilde{k}_h = \tilde{\alpha} E_m, \tilde{k}_i = \tilde{\beta} U_m, \tilde{k}_j = \tilde{\gamma}(1 - \tilde{\omega}) P_m, \tilde{l}_h = \tilde{\alpha}_x E_f, \tilde{l}_i = \tilde{\beta}_x U_f, \tilde{l}_j = \tilde{\gamma}_x(1 - \tilde{\omega}_x) P_f$$

4. Males only

In this section, we discussed about the nonlinear transmission of educated, uneducated and partially educated males to employed and unemployed males by neglecting the female population ($S_f=0$, $E_f=0$, $U_f=0$, $P_f=0$) in the model. Then, the model (2.1) is simplified as

$$\begin{aligned}
\frac{dS_m}{dt} &= \tilde{\Lambda}_1 - (\tilde{k}_h + \tilde{k}_i + \tilde{k}_j) S_m - \tilde{\mu}_1 S_m \\
\frac{dE_m}{dt} &= \tilde{k}_h S_m - (\tilde{t}_1 + \tilde{t}_3) E_m - \tilde{\mu}_1 E_m \\
\frac{dU_m}{dt} &= \tilde{k}_i S_m - (\tilde{t}_2 + \tilde{t}_4) U_m - \tilde{\mu}_1 U_m \\
\frac{dP_m}{dt} &= \tilde{k}_j S_m - (\tilde{s}_1 + \tilde{s}_2) P_m - \tilde{\mu}_1 P_m \\
\frac{dE}{dt} &= \tilde{t}_1 E_m + \tilde{t}_2 U_m + \tilde{s}_1 P_m - \tilde{\mu} E \\
\frac{dU}{dt} &= \tilde{t}_3 E_m + \tilde{t}_4 U_m + \tilde{s}_2 P_m - \tilde{\mu} U
\end{aligned} \tag{4.1}$$

The equilibria and stability analysis regarding the education in the male population are analyzed and explained here. The equilibria of the model (4.1) can be found by making the right hand side of the equations equal to zero.

The educated-free, uneducated-free and partially educated-free equilibrium ($E_m U_m P_m F E$) point of (4.1) is given by

$$M_0 = (S_m^0, E_m^0, U_m^0, P_m^0, E^0, U^0) = \left(\frac{\tilde{\Lambda}_1}{\tilde{\mu}_1}, 0, 0, 0, 0, 0 \right).$$

The jacobian matrix for model (4.1) is written as

$$J = \begin{pmatrix}
-(\tilde{k}_1 E_m + \tilde{k}_2 U_m + \tilde{k}_3 P_m + \tilde{\mu}_1) & -\tilde{k}_1 S_m & -\tilde{k}_2 S_m & -\tilde{k}_3 S_m & 0 & 0 \\
\tilde{k}_1 E_m & \tilde{k}_1 S_m - (\tilde{t}_1 + \tilde{t}_3 + \tilde{\mu}) & 0 & 0 & 0 & 0 \\
\tilde{k}_2 U_m & 0 & \tilde{k}_2 S_m - (\tilde{t}_2 + \tilde{t}_4 + \tilde{\mu}_1) & 0 & 0 & 0 \\
\tilde{k}_3 P_m & 0 & 0 & \tilde{k}_3 S_m - (\tilde{s}_1 + \tilde{s}_2 + \tilde{\mu}_1) & 0 & 0 \\
0 & \tilde{t}_1 & \tilde{t}_2 & \tilde{s}_1 & -\tilde{\mu} & 0 \\
0 & \tilde{t}_3 & \tilde{t}_4 & \tilde{s}_2 & 0 & -\tilde{\mu}
\end{pmatrix}$$

After linearization, the above jacobian matrix becomes

$$J_{M_0} = \begin{vmatrix} -\tilde{\mu}_1 - \lambda & -\tilde{k}_1 \frac{\tilde{\Lambda}_1}{\tilde{\mu}_1} & -\tilde{k}_2 \frac{\tilde{\Lambda}_1}{\tilde{\mu}_1} & -\tilde{k}_3 \frac{\tilde{\Lambda}_1}{\tilde{\mu}_1} & 0 & 0 \\ 0 & [\tilde{k}_1 \frac{\tilde{\Lambda}_1}{\tilde{\mu}} - (\tilde{t}_1 + \tilde{t}_3 + \tilde{\mu}_1)] - \lambda & 0 & 0 & 0 & 0 \\ 0 & 0 & [\tilde{k}_2 \frac{\tilde{\Lambda}_1}{\tilde{\mu}} - (\tilde{t}_2 + \tilde{t}_4 + \tilde{\mu}_1)] - \lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & [\tilde{k}_3 \frac{\tilde{\Lambda}_1}{\tilde{\mu}_1} - (\tilde{s}_1 + \tilde{s}_2 + \tilde{\mu}_1)] - \lambda & 0 & 0 \\ 0 & \tilde{t}_1 & \tilde{t}_2 & \tilde{s}_1 & -\tilde{\mu} - \lambda & 0 \\ 0 & \tilde{t}_3 & \tilde{t}_4 & \tilde{s}_2 & 0 & -\tilde{\mu} - \lambda \end{vmatrix}$$

Here, the largest eigen value may be $\tilde{k}_1 \frac{\tilde{\Lambda}_1}{\tilde{\mu}_1} - (\tilde{t}_1 + \tilde{t}_3 + \tilde{\mu}_1)$ or $\tilde{k}_2 \frac{\tilde{\Lambda}_1}{\tilde{\mu}_1} - (\tilde{t}_2 + \tilde{t}_4 + \tilde{\mu}_1)$ or $\tilde{k}_3 \frac{\tilde{\Lambda}_1}{\tilde{\mu}_1} - (\tilde{s}_1 + \tilde{s}_2 + \tilde{\mu}_1)$. The basic reproduction number (R_m) is defined as the number of educated / uneducated / partially educated males created by the single educated / uneducated / partially educated male in the susceptible male population. Thus basic reproduction number for the model (4.1) is given by

$$R_m = \max(R_{em}, R_{um}, R_{pm}) = \max\left(\frac{\tilde{k}_1 \tilde{\Lambda}_1}{(\tilde{t}_1 + \tilde{t}_3 + \tilde{\mu}_1) \tilde{\mu}_1}, \frac{\tilde{k}_2 \tilde{\Lambda}_1}{(\tilde{t}_2 + \tilde{t}_4 + \tilde{\mu}_1) \tilde{\mu}_1}, \frac{\tilde{k}_3 \tilde{\Lambda}_1}{(\tilde{s}_1 + \tilde{s}_2 + \tilde{\mu}_1) \tilde{\mu}_1}\right) \quad (4.2)$$

where R_{em} , R_{um} and R_{pm} represents the basic reproduction of educated males, uneducated males and partially educated males respectively.

The educated, uneducated and partially educated equilibrium ($E_m U_m P_m E$) point of the model (4.1) is given by $M_* = (S_m^*, E_m^*, U_m^*, P_m^*, E^*, U^*)$ where, $S_m^* = \frac{\tilde{\Lambda}_1}{(\tilde{k}_h + \tilde{k}_i + \tilde{k}_j + \tilde{\mu}_1)}$, $E_m^* = \frac{\tilde{k}_h S_m^*}{(\tilde{t}_1 + \tilde{t}_3 + \tilde{\mu}_1)}$, $U_m^* = \frac{\tilde{k}_i S_m^*}{(\tilde{t}_2 + \tilde{t}_4 + \tilde{\mu}_1)}$, $P_m^* = \frac{\tilde{k}_j S_m^*}{(\tilde{s}_1 + \tilde{s}_2 + \tilde{\mu}_1)}$, $E^* = \frac{(\tilde{t}_1 E_m^* + \tilde{t}_2 U_m^* + \tilde{s}_1 P_m^*)}{\mu}$ and $U^* = \frac{\tilde{t}_3 E_m^* + \tilde{t}_4 U_m^* + \tilde{s}_2 P_m^*}{\mu}$.

The evaluated jacobian matrix at M_* is given by

$$J_{M_*} = \begin{pmatrix} a_{11} & -\tilde{k}_1 \left(\frac{\tilde{\Lambda}_1}{\tilde{k}_h + \tilde{k}_i + \tilde{k}_j + \tilde{\mu}_1} \right) & -\tilde{k}_2 \left(\frac{\tilde{\Lambda}_1}{\tilde{k}_h + \tilde{k}_i + \tilde{k}_j + \tilde{\mu}_1} \right) & -\tilde{k}_3 \left(\frac{\tilde{\Lambda}_1}{\tilde{k}_h + \tilde{k}_i + \tilde{k}_j + \tilde{\mu}_1} \right) & 0 & 0 \\ \tilde{k}_1 \left(\frac{\tilde{k}_h S_m^*}{\tilde{t}_1 + \tilde{t}_3 + \tilde{\mu}_1} \right) & a_{22} & 0 & 0 & 0 & 0 \\ \tilde{k}_2 \left(\frac{\tilde{k}_i S_m^*}{\tilde{t}_2 + \tilde{t}_4 + \tilde{\mu}_1} \right) & 0 & a_{33} & 0 & 0 & 0 \\ \tilde{k}_3 \left(\frac{\tilde{k}_j S_m^*}{\tilde{s}_1 + \tilde{s}_2 + \tilde{\mu}_1} \right) & 0 & 0 & a_{44} & 0 & 0 \\ 0 & \tilde{t}_1 & \tilde{t}_2 & \tilde{s}_1 & -\tilde{\mu} & 0 \\ 0 & \tilde{t}_3 & \tilde{t}_4 & \tilde{s}_2 & 0 & -\tilde{\mu} \end{pmatrix}$$

where $a_{11} = -(\tilde{k}_1 \left(\frac{\tilde{k}_h S_m^*}{\tilde{t}_1 + \tilde{t}_3 + \tilde{\mu}_1} \right) + \tilde{k}_2 \left(\frac{\tilde{k}_i S_m^*}{\tilde{t}_2 + \tilde{t}_4 + \tilde{\mu}_1} \right) + \tilde{k}_3 \left(\frac{\tilde{k}_j S_m^*}{\tilde{s}_1 + \tilde{s}_2 + \tilde{\mu}_1} \right) + \tilde{\mu}_1)$, $a_{22} = \tilde{k}_1 \left(\frac{\tilde{\Lambda}_1}{\tilde{k}_h + \tilde{k}_i + \tilde{k}_j + \tilde{\mu}_1} \right) - (\tilde{t}_1 + \tilde{t}_3 + \tilde{\mu}_1)$, $a_{33} = \tilde{k}_2 \left(\frac{\tilde{\Lambda}_1}{\tilde{k}_h + \tilde{k}_i + \tilde{k}_j + \tilde{\mu}_1} \right) - (\tilde{t}_2 + \tilde{t}_4 + \tilde{\mu}_1)$ and $a_{44} = \tilde{k}_3 \left(\frac{\tilde{\Lambda}_1}{\tilde{k}_h + \tilde{k}_i + \tilde{k}_j + \tilde{\mu}_1} \right) - (\tilde{s}_1 + \tilde{s}_2 + \tilde{\mu}_1)$.

5. Females only

In this section, we investigated about the education and employment in the female population by considering $S_m = 0$, $E_m = 0$, $U_m = 0$ and $P_m = 0$ in model (2.1). Then, the model becomes

$$\begin{aligned}
\frac{dS_f}{dt} &= \tilde{\Lambda}_2 - (\tilde{l}_h + \tilde{l}_i + \tilde{l}_j)S_f - \tilde{\mu}_2 S_f \\
\frac{dE_f}{dt} &= \tilde{l}_h S_f - (\tilde{j}_1 + \tilde{j}_3)E_f - \tilde{\mu}_2 E_f \\
\frac{dU_f}{dt} &= \tilde{l}_i S_f - (\tilde{j}_2 + \tilde{j}_4)U_f - \tilde{\mu}_2 U_f \\
\frac{dP_f}{dt} &= \tilde{l}_j S_f - (\tilde{s}_3 + \tilde{s}_4)P_f - \tilde{\mu}_2 P_f \\
\frac{dE}{dt} &= \tilde{j}_1 E_f + \tilde{j}_2 U_f + \tilde{s}_3 P_f - \mu E \\
\frac{dU}{dt} &= \tilde{j}_3 E_f + \tilde{j}_4 U_f + \tilde{s}_4 P_f - \mu U
\end{aligned} \tag{5.1}$$

The model (5.1) has a educated-free, uneducated-free and partially educated-free equilibrium F_0 given by

$$F_0 = (S_f^0, E_f^0, U_f^0, P_f^0, E^0, U^0) = \left(\frac{\tilde{\Lambda}_2}{\tilde{\mu}_2}, 0, 0, 0, 0, 0 \right).$$

Following [1], the basic reproductive number R_f of the model (5.1) is given by

$$R_f = \max(R_{ef}, R_{uf}, R_{pf}) = \max\left(\frac{\tilde{l}_1 \tilde{\Lambda}_2}{(\tilde{j}_1 + \tilde{j}_3 + \tilde{\mu}_2)\tilde{\mu}_2}, \frac{\tilde{l}_2 \tilde{\Lambda}_2}{(\tilde{j}_2 + \tilde{j}_4 + \tilde{\mu}_2)\tilde{\mu}_2}, \frac{\tilde{l}_3 \tilde{\Lambda}_2}{(\tilde{s}_3 + \tilde{s}_4 + \tilde{\mu}_2)\tilde{\mu}_2}\right)$$

R_{ef} , R_{uf} and R_{pf} denotes the basic reproductive number of educated females, uneducated females and partially educated females respectively. Here, R_f evaluates the number of newly educated / uneducated / partially educated females generated by a single educated / uneducated / partially educated female in her entire infectious (denotes education) lifetime when she is introduced in the susceptible female S_f population.

The evaluated jacobian matrix at F_0 is given by

$$J_{F_0} = \begin{pmatrix} -\tilde{\mu}_2 & -\tilde{l}_1(\frac{\tilde{\Lambda}_2}{\tilde{\mu}_2}) & -\tilde{l}_2(\frac{\tilde{\Lambda}_2}{\tilde{\mu}_2}) & -\tilde{l}_3(\frac{\tilde{\Lambda}_2}{\tilde{\mu}_2}) & 0 & 0 \\ 0 & \tilde{l}_1 \frac{\tilde{\Lambda}_2}{\tilde{\mu}_2} - (\tilde{j}_1 + \tilde{j}_3 + \tilde{\mu}_2) & 0 & 0 & 0 & 0 \\ 0 & 0 & \tilde{l}_2 \frac{\tilde{\Lambda}_2}{\tilde{\mu}_2} - (\tilde{j}_2 + \tilde{j}_4 + \tilde{\mu}_2) & 0 & 0 & 0 \\ 0 & 0 & 0 & \tilde{l}_3 \frac{\tilde{\Lambda}_2}{\tilde{\mu}_2} - (\tilde{s}_3 + \tilde{s}_4 + \tilde{\mu}_2) & 0 & 0 \\ 0 & \tilde{j}_1 & \tilde{j}_2 & \tilde{s}_3 & -\mu & 0 \\ 0 & \tilde{j}_3 & \tilde{j}_4 & \tilde{s}_4 & 0 & -\mu \end{pmatrix} \tag{5.2}$$

From (5.2), the local stability of F_0 is satisfied only at

$$\begin{aligned}
\tilde{l}_1 \frac{\tilde{\Lambda}_2}{\tilde{\mu}_2} &< (\tilde{j}_1 + \tilde{j}_3 + \tilde{\mu}_2), \\
\tilde{l}_2 \frac{\tilde{\Lambda}_2}{\tilde{\mu}_2} &< (\tilde{j}_2 + \tilde{j}_4 + \tilde{\mu}_2), \\
\tilde{l}_3 \frac{\tilde{\Lambda}_2}{\tilde{\mu}_2} &< (\tilde{s}_3 + \tilde{s}_4 + \tilde{\mu}_2)
\end{aligned}$$

i.e $R_f < 1$. In case, if $R_f > 1$, then F_* exists.

The model (5.1) has a educated, uneducated and partially educated equilibrium F_* given by

$$F_* = (S_f^*, E_f^*, U_f^*, P_f^*, E^*, U^*)$$

where,

$$\begin{aligned} S_f^* &= \frac{\tilde{\Lambda}_2}{(\tilde{l}_h + \tilde{l}_i + \tilde{l}_j + \tilde{\mu}_2)}, \\ E_f^* &= \frac{\tilde{l}_h S_f^*}{(\tilde{j}_1 + \tilde{j}_3 + \tilde{\mu}_2)}, \\ U_f^* &= \frac{\tilde{l}_i S_f^*}{(\tilde{j}_2 + \tilde{j}_4 + \tilde{\mu}_2)}, \\ P_f^* &= \frac{\tilde{l}_j S_f^*}{(\tilde{s}_3 + \tilde{s}_4 + \tilde{\mu}_2)}, \\ E^* &= \frac{\tilde{j}_1 E_f^* + \tilde{j}_2 U_f^* + \tilde{s}_3 P_f^*}{\tilde{\mu}} \text{ and} \\ U^* &= \frac{\tilde{j}_3 E_f^* + \tilde{j}_4 U_f^* + \tilde{s}_4 P_f^*}{\tilde{\mu}}. \end{aligned}$$

The evaluated jacobian matrix at F_* is given by

$$J_{F_*} = \begin{pmatrix} a_{11} & -\tilde{l}_1(\frac{\tilde{\Lambda}_2}{\tilde{l}_h + \tilde{l}_i + \tilde{l}_j + \tilde{\mu}_2}) - \tilde{l}_2(\frac{\tilde{\Lambda}_2}{\tilde{l}_h + \tilde{l}_i + \tilde{l}_j + \tilde{\mu}_2}) - \tilde{l}_3(\frac{\tilde{\Lambda}_2}{\tilde{l}_h + \tilde{l}_i + \tilde{l}_j + \tilde{\mu}_2}) & 0 & 0 & 0 \\ \tilde{l}_1(\frac{\tilde{l}_h S_f^*}{\tilde{j}_1 + \tilde{j}_3 + \tilde{\mu}_2}) & a_{22} & 0 & 0 & 0 \\ \tilde{l}_2(\frac{\tilde{l}_i S_f^*}{\tilde{j}_2 + \tilde{j}_4 + \tilde{\mu}_2}) & 0 & a_{33} & 0 & 0 \\ \tilde{l}_3(\frac{\tilde{l}_j S_f^*}{\tilde{s}_3 + \tilde{s}_4 + \tilde{\mu}_2}) & 0 & 0 & a_{44} & 0 \\ 0 & \tilde{j}_1 & \tilde{j}_2 & \tilde{s}_3 & -\tilde{\mu} \\ 0 & \tilde{j}_3 & \tilde{j}_4 & \tilde{s}_4 & 0 \end{pmatrix} \quad (5.3)$$

where $a_{11} = -(\tilde{l}_1(\frac{\tilde{l}_h S_f^*}{\tilde{j}_1 + \tilde{j}_3 + \tilde{\mu}_2}) + \tilde{l}_2(\frac{\tilde{l}_i S_f^*}{\tilde{j}_2 + \tilde{j}_4 + \tilde{\mu}_2}) + \tilde{l}_3(\frac{\tilde{l}_j S_f^*}{\tilde{s}_3 + \tilde{s}_4 + \tilde{\mu}_2}) + \tilde{\mu}_2)$, $a_{22} = \tilde{l}_1(\frac{\tilde{\Lambda}_2}{\tilde{l}_h + \tilde{l}_i + \tilde{l}_j + \tilde{\mu}_2}) - (\tilde{j}_1 + \tilde{j}_3 + \tilde{\mu}_2)$, $a_{33} = \tilde{l}_2(\frac{\tilde{\Lambda}_2}{\tilde{l}_h + \tilde{l}_i + \tilde{l}_j + \tilde{\mu}_2}) - (\tilde{j}_2 + \tilde{j}_4 + \tilde{\mu}_2)$ and $a_{44} = \tilde{l}_3(\frac{\tilde{\Lambda}_2}{\tilde{l}_h + \tilde{l}_i + \tilde{l}_j + \tilde{\mu}_2}) - (\tilde{s}_3 + \tilde{s}_4 + \tilde{\mu}_2)$.

Theorem 5.1. The equilibrium point F_0 is locally asymptotically stable if $R_f < 1$ and the equilibrium F_* is locally asymptotically stable whenever $R_f > 1$.

Proof. The theorem is proved only when the conditions outlined in equation (3.3) to (3.9) are satisfied. ■

Theorem 5.2. The equilibrium point F_0 is globally asymptotically stable (GAS) if $R_f < 1$ and the point F_* is globally asymptotically stable only at $R_f > 1$.

6. Numerical simulation

The numerical results are expressed through graphs below. Fig.1 represents the stability of M_0 equilibrium when $R_m < 1$ with government intervention and no vocational training. The graph Fig.2 and Fig.3 shows the stability of M_* equilibrium for $R_m > 1$ with and without government intervention. Fig.4 explains the stability of F_0 equilibrium for $R_f < 1$. The stability of F_* equilibrium at $R_f > 1$ is shown graphically in Fig.5. The stability of F_* equilibrium at $R_f > 1$ for with and without vocational training is shown in Fig.6 and Fig.7. Providing education with and without government intervention in the female population is simulated and shown graphically in Fig.8 and Fig.9. The Fig.10 shows the stability of $R_0 < 1$ point for non existence of educated / uneducated / partially educated in the human population.

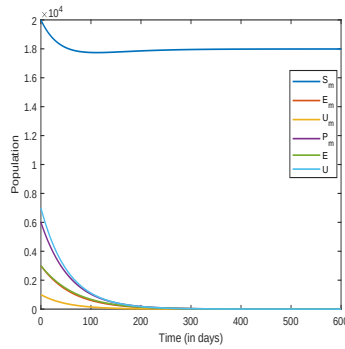


Fig. 1. Stability of M_0 equilibrium at $R_m = 0.0015$ for the values $S_m = 20000$, $E_m = 3000$, $U_m = 1000$, $P_m = 6000$, $E = 3000$, $U = 7000$

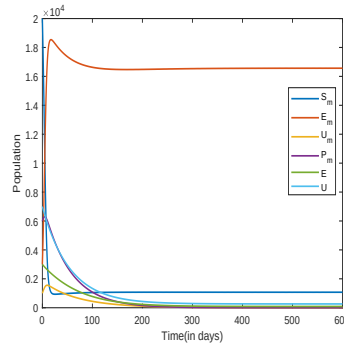


Fig. 2. Stability of M_s equilibrium With government intervention for the values $S_m = 20000$, $E_m = 3000$, $U_m = 1000$, $P_m = 6000$, $E = 3000$, $U = 7000$

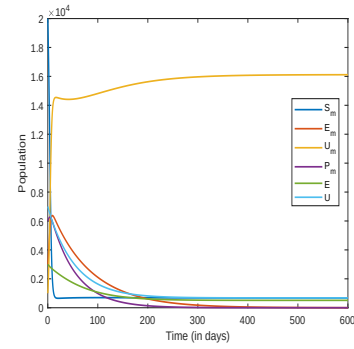


Fig. 3. Stability of M_s equilibrium Without government intervention for the values $S_m = 20000$, $E_m = 3000$, $U_m = 1000$, $P_m = 6000$, $E = 3000$, $U = 7000$

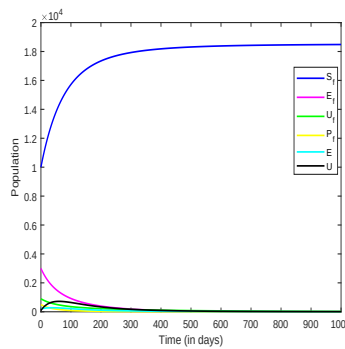


Fig. 4. Stability of F_0 equilibrium point for $R_f = 0.5653 < 1$ for the values $S_f = 10000$, $E_f = 3000$, $U_f = 900$, $P_f = 500$, $E = 200$ and $U = 90$.

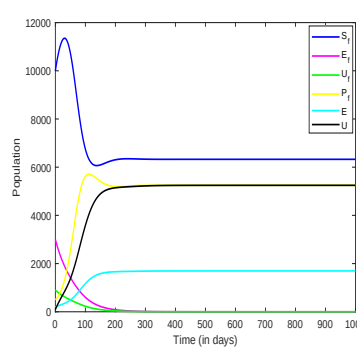


Fig. 5. Stability of F_* equilibrium point for $R_f = 1.9786 > 1$ for the values $S_f = 10000$, $E_f = 3000$, $U_f = 900$, $P_f = 500$, $E = 200$ and $U = 90$.

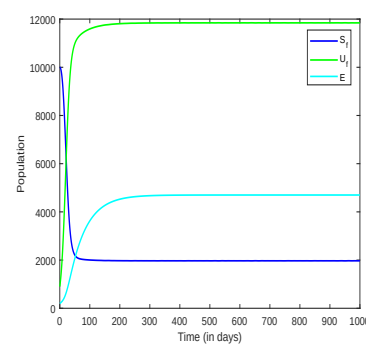


Fig. 6. Stability of F_* equilibrium point without vocational training for the values $S_f = 10000$, $E_f = 3000$, $U_f = 900$, $P_f = 500$, $E = 200$ and $U = 90$

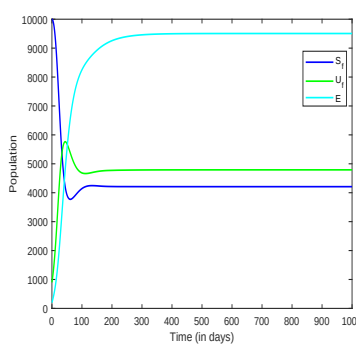


Fig. 7. Stability of F_* equilibrium point with vocational training for the values $S_f = 10000$, $E_f = 3000$, $U_f = 900$, $P_f = 500$, $E = 200$ and $U = 90$

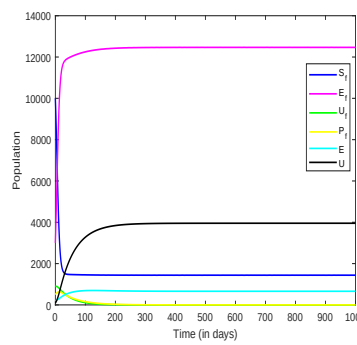


Fig. 8. Graph showing the stability of female population with government intervention for the values $S_f = 10000$, $E_f = 3000$, $U_f = 900$, $P_f = 500$, $E = 200$ and $U = 90$

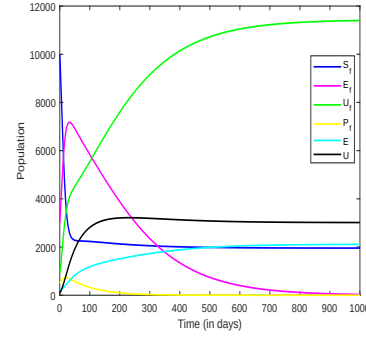


Fig. 9. Graph showing the stability of female population without government intervention for the values $S_f = 10000$, $E_f = 3000$, $U_f = 900$, $P_f = 500$, $E = 200$ and $U = 90$

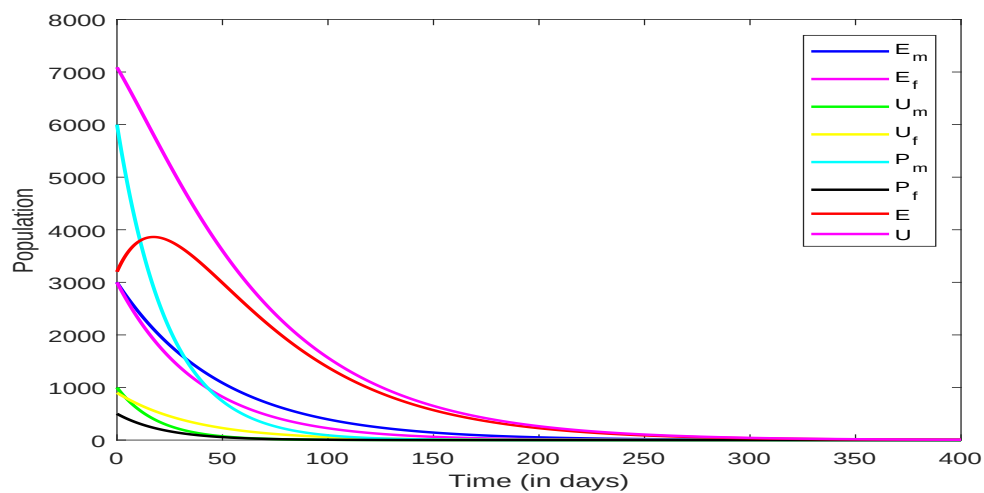


Fig. 10. Stability of graph showing no educated / no uneducated / no partially educated in the total population

7. Conclusion

The government intervention in education already exists but it is not that much efficient in increasing the literacy. Literacy rate will not be increased only by providing free education. Even though, the government provide free education, the students still skip their classes and go to work. Most of the girl children stops their schooling up to eighth / ninth standard. According to our model, strong government intervention (flexible schedules in schools, create awareness in underprivileged societies, change in educational structure according to students psychology etc.) is provided, then literacy rate will be automatically increased and providing vocational training to individuals brings out less unemployed.

References

- [1] Driessche, P.V., Watmough, J., "Reproduction numbers and sub-threshold endemic equilibria for compartmental models of disease transmission" *Mathematical Biosciences* 180 (2002), 29-48.
- [2] Fred Brauer, Pauline van den Driessche Jianhong Wu (Eds.) "Mathematical Epidemiology" *Mathematical Biosciences Subseries* (1945).
- [3] <https://www.worldpulse.com/fr/node/15428>
- [4] <https://www.mapsofindia.com/my-india/society/low-female-literacy-rate-and-its-impact-on-our-society>

