

PULSATILE FLOW OF NEWTONIAN FLUID MODEL FOR HUMAN AIRWAYS WITH BODY ACCELERATION

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Abstract: A mathematical model has been developed for the studying of the unsteady flow of human airways through a constricted trachea with periodic body acceleration, by characterizing it as a Newtonian fluid. Perturbation method is used to solve the resulting system of non-linear partial differential equation with appropriate boundary condition to analyze the flow. Analytical expressions are obtained for axial velocity, volumetric flow rate and wall shear stress. Effects of various parameters on the flow characteristics through constricted trachea are plotted in graphs.

Keywords: Pulsatile Flow, Body Acceleration, Newtonian Fluid, Constricted Human Airways, Slip velocity.

1. Introduction

The human respiratory tract is a system where particulate matter can be inhaled via the mouth/nose and may deposit at focal points in the airways. Toxicologic or therapeutic health effects of inhaled particulate matter depend largely on the particle deposition patterns and concentrations at critical sites in the respiratory airways. Excessive retention of inhaled particles has been known to be directly related to a number of occupational diseases, such as coal worker's pneumoconiosis, silicosis and asbestosis. Deposition of inhaled allergens causes acute bronchoconstriction for asthma sufferers, and elevated depositions of some particulate pollutant cause tissue injuries and lung diseases such as bronchial carcinoma and even lung cancer.

The health hazards originating from particulate matter deposited in the human respiratory tract requires the determination of total and regional deposition of inhaled particles of different materials entering the body *via* breathing. If aerosol particles are inhaled, it will be deposit on the walls of the human respiratory tract. Every day billions of particles are inhaled with the ambient air by every human being, many of these particles are deposited in the respiratory tract depending on the size, density, shape, charge, and surface properties of the particles and the breathing pattern of the individual.

It is important to analyze the effect of periodic body acceleration on different part of body. Prolonged exposure of a healthy human body to external acceleration may cause serious health diseases like chronic obstructive pulmonary disease (COPD), asthma, strokes, lung infections and lung cancer. Due to physical importance of body acceleration, many researchers have been proposed for human airways with body acceleration.

Lippmann M, Albert R.E (1969) studied the effect of particle size on the regional deposition of inhaled aerosols in the human respiratory tract. Joachim Heyder (2004) studied on deposition of inhaled particles in the human respiratory tract and consequences for regional targeting in respiratory drug delivery. P.K.Hopke, Z.Wang (2012) has been investigated particle deposition in the human respiratory tract. Varun Kumar Tanwar and N.K.Varshney (2014) has studied pulsatile flow of blood through a stenosed artery with body acceleration. B.Asgharian, W.Hofmann & R. Bergmann (2001) has discussed the particle deposition in a multiple-path model of the human lung. M.Chitra, D.Karthikeyan (2017) has been studied oscillating flow of blood in porous vessel of a stenosed artery with variable viscosity effects of magnetic field.

In the present analysis an effect has been made to the unsteady oscillatory flow of Newtonian fluid for human airways with body acceleration subject to a slip velocity condition at the constricted wall of the trachea. Analytical expressions for axial velocity, volumetric flow rate and wall shear stress have been derived and the effects of various parameter on these flow characteristics have been discussed through graphs.

2. Mathematical Formulation

We assumed a constriction in the trachea, in an axially-symmetric manner. The airflow is assumed to be characterized by the Newtonian model whose consistency varies symmetrical along the radial direction.

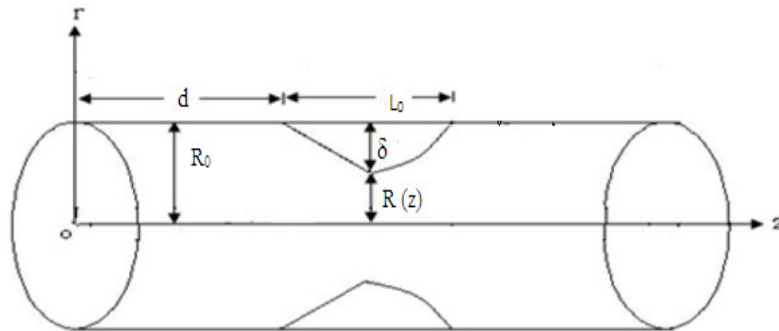


Figure1: Geometry of trachea with constricted region

The radius of the constricted trachea $R(z)$ can be taken as

$$R(z) = \begin{cases} R_0 - \frac{\delta}{2} \left(1 + \cos \frac{2\pi}{L_0} \left(z - d - \frac{L_0}{2} \right) \right) & ; d \leq z \leq d + L_0 \\ R_0 & ; \text{otherwise} \end{cases} \quad (1)$$

Where R_0 and $R(z)$ represent the radii of the uniform and constricted region of trachea.

L_0 is the length of the constriction. δ is the maximum thickness of the constriction.

r and z are the radial and axial co-ordinates.

The pressure gradient and body acceleration are given by

$$-\frac{\partial p}{\partial z} = A_0 + A_1 \cos(\omega_p t) \quad (2)$$

$$G(t) = a_0 \cos(\omega_b t + \phi) \quad (3)$$

Where A_0 and A_1 are pressure gradient of unsteady flow and amplitude of oscillatory part respectively, a_0 is the amplitude of body acceleration, $\omega_p = 2\pi f_p$, $\omega_b = 2\pi f_b$ with f_p is the pulse frequency and f_b is the body acceleration frequency, ' ϕ ' is the phase angle of body acceleration ' p ' is the pressure gradient and ' t ' is time.

$$\rho \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial z} + \frac{1}{r} \frac{\partial}{\partial r} (r\tau) + G(t) \quad (4)$$

$$\rho \frac{\partial u}{\partial t} = A_0 + A_1 \cos(\omega_p t) + \frac{1}{r} \frac{\partial}{\partial r} (r\tau) + a_0 \cos(\omega_b t + \phi) \quad (5)$$

$$\frac{\partial p}{\partial r} = 0 \quad (6)$$

Where r and z denotes the radial and axial co-ordinates respectively and ρ denotes density, u axial velocity of trachea and τ is the share stress.

The constitutive equation for Herschel-Bulkley Newtonian fluid may be written as

$$-\frac{\partial u}{\partial r} = \begin{cases} \frac{\tau}{\mu_H} & \text{if } \tau \geq \tau_H \\ 0 & \text{if } \tau < \tau_H \end{cases} \quad (7)$$

Where u stands for the axial velocity of trachea, $\tau_H (\geq 0)$ is the yield shear stress, μ_H is the coefficient of viscosity for Herschel-Bulkley Newtonian fluid.

The boundary conditions are

$$u = u_s \quad \text{at } r = R(z) \quad (8)$$

$$\frac{\partial u}{\partial r} = 0 \quad \text{at } r = 0 \quad (9)$$

Where u_s is the axial slip velocity.

Introducing the non-dimensional variables:

$$\bar{u} = \frac{4\mu_0 u}{A_0 R_0^2}, \bar{z} = \frac{z}{R_0}, \bar{t} = \omega_p t, \bar{\delta} = \frac{\delta}{R_0}, \bar{\tau} = \frac{2\tau}{A_0 R_0}, \bar{\tau}_H = \frac{2\tau_H}{A_0 R_0},$$

$$\bar{R}(\bar{z}) = \frac{R(z)}{R_0}, \bar{r} = \frac{r}{R_0}, e = \frac{A_1}{A_0}, B = \frac{a_0}{A_0},$$

$$\omega = \frac{\omega_b}{\omega_p}, Z_0 = \frac{z_0}{R_0}, \mu_0 = \bar{\mu}_H \left(\frac{2}{R_0 A_0} \right)^{\left(\frac{1}{n} - 1 \right)} \quad (10)$$

The non-dimensional momentum equation (4) becomes after dropping bars as

$$\alpha^2 \frac{\partial u}{\partial t} = 4(1 + e \cos t) + 4B \cos(\omega_p t + \varphi) + \frac{2}{r} \frac{\partial}{\partial r}(r\tau) \quad (11)$$

Where $\alpha^2 = \frac{\rho \omega_p R_0^2}{\mu}$, α is Womersley frequency parameter.

Equation (7) can be written as

$$\tau = -\frac{1}{2} \frac{\partial u}{\partial r} \quad \text{if } \tau \geq \tau_H \quad (12)$$

$$\frac{\partial u}{\partial r} = 0 \quad \text{if } \tau < \tau_H \quad (13)$$

The boundary conditions (8) and (9) reduces to

$$\text{i) } u = u_s \quad \text{at } r = R(z) \quad (14)$$

$$\text{ii) } \frac{\partial u}{\partial r} = 0 \quad \text{at } r = 0 \quad (15)$$

The geometry of the constriction is given by

$$R'(z) = \begin{cases} 1 - \frac{\delta}{2} \left(1 + \cos \frac{2\pi}{L_0} \left(z - d - \frac{L_0}{2} \right) \right) & ; d \leq z \leq d + L_0 \\ 1 & ; \text{otherwise} \end{cases} \quad (16)$$

3. Method of Solution

On using perturbation method, the velocity u and τ , plug core velocity u_p plug core radius R_p are expanded as follows in terms of α^2 (where $\alpha^2 \ll 1$)

$$u(z, r, t) = u_0(z, r, t) + \alpha^2 u_1(z, r, t) + \dots \quad (17)$$

$$\tau(z, r, t) = \tau_0(z, r, t) + \alpha^2 \tau_1(z, r, t) + \dots \quad (18)$$

$$u_p(z, r, t) = u_{0p}(z, r, t) + \alpha^2 u_{1p}(z, r, t) + \dots \quad (19)$$

$$R_p(z, r, t) = R_{0p}(z, r, t) + \alpha^2 R_{1p}(z, r, t) + \dots \quad (20)$$

Substituting (17) and (18) in (11) and equating the constant term and α^2 term we get,

$$\frac{\partial}{\partial r}(r\tau_0) = -2r\{(1 + e \cos t) + B \cos(\omega t + \varphi)\} \quad (21)$$

$$\frac{\partial u_0}{\partial t} = \frac{2}{r} \frac{\partial}{\partial r}(r\tau_1) \quad (22)$$

Integrate equation (21) and using boundary condition (15), we get

$$\tau_0 = -f(t)r \quad (23)$$

Where $f(t) = (1 + e \cos t) + B \cos(\omega t + \varphi)$

Substituting equation (17) and (18) in (12)

$$-\frac{\partial u_0}{\partial r} = 2rf(t) \quad (24)$$

$$-\frac{\partial u_1}{\partial r} = 2\tau_1 \quad (25)$$

Integrating equation (24) using the relation (23) and the boundary condition (15) we obtain

$$u_0 = u_s + f(t) \left[(R(z))^2 - r^2 \right] \quad (26)$$

The plug core velocity u_{0p} can be obtained from (26) as

$$u_{0p} = u_s + f(t) \left[(R(z))^2 - R_{0p}^2 \right] \quad (27)$$

Neglecting the terms of $o(\alpha^2)$ and higher powers of α in (20) R_{0p} can be obtained from (23) as

$$R_{0p} = \frac{\tau_0}{f(t)} \quad (28)$$

Using equation (22) we get the solution for τ_1 as

$$\tau_1 = \frac{f'(t)}{8} \left[2(R(z))^2 r - r^3 \right] \quad (29)$$

Similarly using (25) and (29) we can obtain the solution for u_1 as

$$u_1 = u_s + 2\tau_1 [R(z) - r] \quad (30)$$

$$u_{1p} = u_s + 2\tau_1 [R(z) - R_{0p}] \quad (31)$$

Using (17) and (18) the total velocity distribution and shear stress can be written as

$$u = u_s + f(t) \left[(R(z))^2 - r^2 \right] + \alpha^2 \frac{f'(t)}{4} \left[2(R(z))^2 r - r^3 \right] \quad (32)$$

$$u_p = u_s + f(t) \left[(R(z))^2 - R_{0p}^2 \right] + \alpha^2 \frac{f'(t)}{4} \left[2(R(z))^2 R_{0p} - R_{0p}^3 \right] \quad (33)$$

$$\tau = (\tau_0 + \alpha^2 \tau_1)_{r=R}$$

$$\tau = -Rf(t) + \alpha^2 \frac{f'(t)}{8} \left[2(R(z))^2 R - R^3 \right] \quad (34)$$

The second approximation plug core radius R_{1p} can be obtained by neglecting terms of $o(\alpha^4)$ and higher powers of α in (20) as

$$R_{1p} = \frac{\tau_1(R_{0p})}{f(t)} \quad (35)$$

While the help of equations (20), (28) and (34), R_p can be given by

$$R_p = \frac{\tau_0}{f(t)} + \frac{\alpha^2 f'(t)}{f(t)} \left[2(R(z))^2 R_{0p} - R_{0p}^3 \right] \quad (36)$$

The volumetric flow rate Q of the constricted trachea is

$$Q = \int_0^{R(z)} 2\pi r u(z, r, t) dr$$

$$= 2\pi r \int_0^{R(z)} \left(u_s + f(t) \left[(R(z))^2 - r^2 \right] + \alpha^2 \frac{f'(t)}{4} \left[2(R(z))^2 r - r^3 \right] \right) dr$$

$$= \pi [R(z)]^2 \left[u_s + \frac{f(t) R(z)^2}{2} + \frac{7\alpha^2 f'(t) R(z)^3}{30} \right]$$

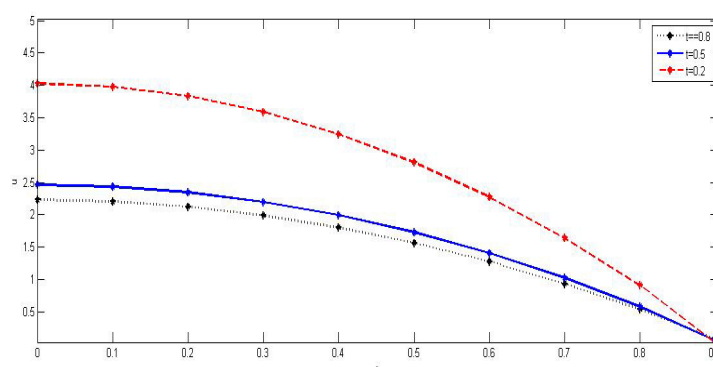


Figure 2. Variation of axial velocity u with radial position r for varying time t

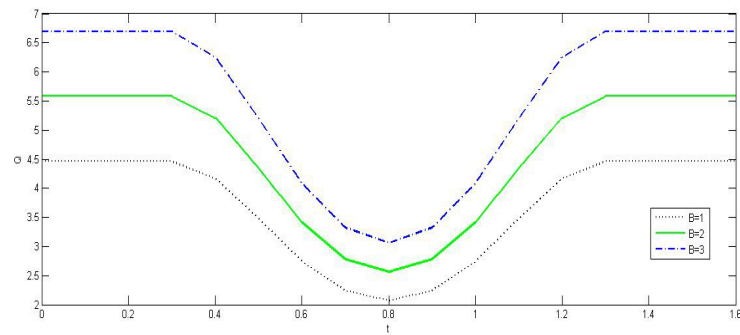


Figure 3. Variation of volumetric flow rate Q with time for t for varying body acceleration B with fixed $e = 1, \varphi = 0.2, \omega = 1, \alpha = 0.2, u_s = 0.09$

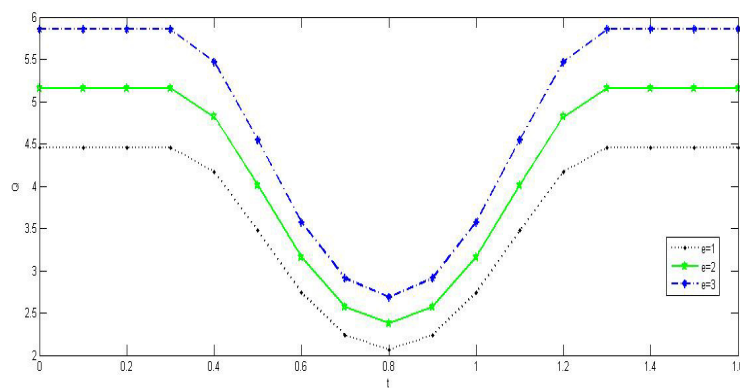


Figure 4. Variation of volumetric flow rate Q with time for t for varying parameter e with fixed $B = 1, \varphi = 0.2, \omega = 1, \alpha = 0.2, u_s = 0.09$

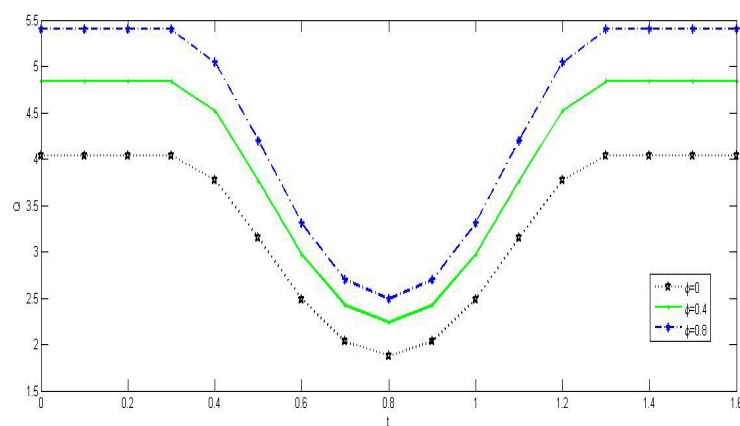


Figure 5. Variation of volumetric flow rate Q with time for t for varying phase angle φ with fixed $e = 1, B = 1, \omega = 1, \alpha = 0.2, u_s = 0.09$

4. Conclusion

The figure 2.shows that the variation of the velocity profile depends on the time. The velocity u decreases as the increase of time t . figure 3.shows that the velocity u decreases as the increase of body acceleration. Figure 4 and figure 5, Shows that the phase angel of body acceleration ϕ increases, as the increase of velocity profile for different values of parameter e .

From these results it is clear that the body acceleration affects largely on the axial velocity of the constricted trachea. The values of the parameters of body acceleration, yield shear stress will affect the flow characteristic and hence can be utilized for medical.

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