MEAN AND VARIANCE OF TIME TO RECRUITMENT FOR A TWO GRADED MANPOWER SYSTEM WITH INTER-DECISION TIMES HAVING INDEPENDENT AND NON-IDENTICALLY DISTRIBUTED RANDOM VARIABLES UNDER CORRELATED WASTAGE

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Abstract

In this paper, a two graded organization subjected to random exit of personnel due to the policy decision taken by the organization is considered, there is an associated loss of manpower if a person quits the organization. As the exit of personnel is unpredictable, a new recruitment policy involving two thresholds for each grade, namely optional and mandatory is suggested to enable the organization to plan its decision on recruitment. Based on shock model approach two mathematical models are constructed using an appropriate univariate policy of recruitment. Performance measures namely mean and variance of time to recruitment is obtained when (i) the amount of wastage at each decision epoch are identically distributed constantly correlated and exchangeable exponential random variables (ii) the inter-decision times are independent and non-identically distributed random variables. (iii) The optional and mandatory thresholds are exponential random variables.

Key Words and Phrases: Manpower planning, Shock model, Univariate CUM policy, Exchangeable and constantly correlated, Hypo-exponential distribution.

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1 Introduction

Consider an organization having two grades in which depletion of manpower occurs at every decision epoch. In the Univariate policy of recruitment, based on shock model approach, recruitment is made as and when the cumulative loss of manpower crosses a threshold. In [9], [6] and [10], the authors have obtained the mean and variance of time to recruitment under the assumption that the inter-decision times are constantly correlated and exchangeable exponential random variables and the distribution of the threshold follows SCBZ property for different grades. In [2], for a single grade manpower
system author studied system characteristics when the thresholds follows geometric distribution. Recently in [8], the authors have obtained the mean and variance of time to recruitment for two grade system by assuming different distributions for thresholds with wastage having constantly correlated and exchangeable exponential random variables. In [4], the authors have obtained the mean and variance by assuming amount of wastage at each decision epoch is constantly correlated. In [7], the authors have obtained mean and variance of time to recruitment for a two graded manpower system with a univariate policy of recruitment involving (i) the loss of man hours and inter-decisions times are independent and non-identically distributed exponential random variables (ii) thresholds optional and mandatory follows exponential random variables. In [8], the authors have obtained mean and variance of time to recruitment for a two graded manpower system with a univariate policy of recruitment involving (i) the loss of man hours are independent and non-identically distributed exponential random variables (ii) the inter-decisions times are constantly correlated and exchangeable exponential random variables (iii) thresholds are optional and mandatory follows exponential random variables.

The objectives of the present paper is to study the problem of time to recruitment for a two graded manpower systems and to obtain the mean and variance of time to recruitment using CUM univariate recruitment policy for exponential thresholds with loss of man hours having constantly correlated and exchangeable exponential random variables independent and inter-decision times having non-identically distributed exponential random variables. The analytical results are numerically illustrated and the influence of nodal parameters on the mean and variance of time to recruitment is studied.

2 Notations

\( X_i \) : Loss of man hours due to the \( i^{th} \) decision epoch \( i = 1, 2, 3 \ldots \)

forming a sequence of exchangeable and constantly correlated exponential random variables with correlated \( \rho \).

\( G(.) \) : Distribution function of \( X_i \).

\( g(.) \) : Probability density function of \( X_i \) with mean \( \alpha \),

\( g(x) = \frac{1}{\alpha} e^{-x/\alpha} \).

\( S_k \) : Cumulative loss of manpower in the first \( k \)-decisions (\( k = 1, 2, \ldots \)).

\( S_k = \sum_{i=1}^{k} X_i \).

\( G_k(.) \) : Distribution function of sum of \( k \) distributed constantly correlated and exchangeable exponential random variables.

\( g_k(.) \) : Probability density function of \( S_k \).

\( g_k^*(.) \) : \( k \)-fold convolution of \( S_k \).

\( \rho \) : Correlation between \( X_i \) & \( X_j \), \( i \neq j \).

\( \alpha \) : Mean of inter-decision times, \( \alpha = \frac{b}{1-\rho} \).
\[ \Psi(n, x) : \int_{0}^{x} e^{-\varepsilon} \varepsilon^{n-1} d\varepsilon. \]

\[ U_k : \text{Inter-decision times are independent and non-identically distributed exponential random variables between } (k-1)^{th} \text{ and } k^{th} \text{ decisions with parameters } \beta_k (\beta_k > 0). \]

\[ F_k(.) : \text{Distribution function of } U_k. \]

\[ f_k(.) : \text{Probability density function of } U_k \text{ with mean } \frac{1}{\beta_k} (\beta_k > 0). \]

\[ Y_1, Y_2 : \text{Continuous random variables denoting the optional threshold levels for the grade 1 & grade 2 follows exponential distribution with parameters } \lambda_1, \lambda_2 \text{ respectively.} \]

\[ Z_1, Z_2 : \text{Continuous random variables denoting the mandatory threshold levels for the grade 1 & grade 2 follows exponential distribution with parameters } \mu_1, \mu_2 \text{ respectively.} \]

\[ W : \text{Continuous random variable denoting the time to recruitment in the organization.} \]

\[ p : \text{Probability that the organization is not going for recruitment whenever the total loss of manpower crosses the optional threshold.} \]

\[ V_k(t) : \text{Probability that exactly } k\text{-decisions are taken in } [0, t). \]

\[ L(.) : \text{Distribution function of } W. \]

\[ l(.) : \text{Probability density function of } W. \]

\[ l^*(.) : \text{Laplace transform of } l(.). \]

\[ E(W) : \text{Expected time to recruitment.} \]

\[ V(W) : \text{Variance of the time to recruitment.} \]

**CUM policy** : Recruitment is done whenever the cumulative loss of manpower crosses the mandatory threshold. The organization may or may not go for recruitment if the cumulative loss of manpower crosses the optional threshold.

**Main Results**

Analytical results for the above cited measures related to time to recruitment \( W \) are derived for the present model. The tail distribution of time to recruitment is given by

\[ P(W > t) = \sum_{k=0}^{\infty} V_k(t)P(S_k < Y) + \sum_{k=0}^{\infty} V_k(t)P(S_k \geq Y)(S_k < Z)p \quad (1) \]

Evaluate \( P(S_k < Y) \), Using law of total probability and conditioning upon \( Y \), we get

\[ P(S_k < Y) = \int_{0}^{\infty} G_k(y)h(y)dy \quad (2) \]

Since \( X_i \)s are assumed to be identical, exchangeable and constantly correlated random variables each following the exponential distribution with probability density function
\( g(x) = \frac{1}{\alpha} e^{-\left(\frac{x}{\alpha}\right)} \), \((a > 0)\), the cumulative distribution function of the partial sum \( S_k = (X_1 + X_2 + \cdots + X_k) \) is given by Gurdan (1995) as

\[
G_k(y) = (1 - \rho) \sum_{k=0}^{\infty} \frac{(k\rho)^i \varphi(k + i, y/b)}{(1 - \rho + k\rho)^{i+1}(k + i - 1)!}
\]

where \( \rho \) is the constant correlation between \( X_i \) & \( X_j \), \( i \neq j \)

\[
\varphi(k + i, y/b) = \int_0^{y/b} e^{-z} z^{k+i-1} dz \quad \text{and} \quad \alpha = \frac{b}{1 - \rho}
\]

**Model: I Maximum Model** \( Y = \max(Y_1, Y_2) \)

\[
H(y) = 1 - e^{-\lambda_2 y} - e^{-\lambda_1 y} + e^{-(\lambda_1 + \lambda_2)y}
\]

\[
h(y) = \lambda_1 e^{-\lambda_1 y} + \lambda_2 e^{-\lambda_2 y} - (\lambda_1 + \lambda_2)e^{-(\lambda_1 + \lambda_2)y}
\]

Using equations (3) and (5) in (2), we get

\[
P(S_k < Y) = \int_0^\infty (1 - \rho) \sum_{k=0}^{\infty} \frac{(k\rho)^i \varphi(k + i, y/b)}{(1 - \rho + k\rho)^{i+1}(k + i - 1)!} \\
\left[\lambda_1 e^{-\lambda_1 y} + \lambda_2 e^{-\lambda_2 y} - (\lambda_1 + \lambda_2)e^{-(\lambda_1 + \lambda_2)y}\right] dy
\]

Using Gamma integrals and on further simplification, we get

\[
\int_0^\infty \varphi(k + i, y/b)e^{-\lambda_1 y} dy = \frac{1}{\lambda_1} \frac{(k + i - 1)!}{(1 + b\lambda_1)^{k+i}}
\]

Consider the first term of equation (6) and using (7), we get

\[
\int_0^\infty (1 - \rho) \sum_{i=0}^{\infty} \frac{(k\rho)^i \varphi(k + i, y/b)}{(1 - \rho + k\rho)^{i+1}(k + i - 1)!} \lambda_1 e^{-\lambda_1 y} dy \\
= \frac{(1 - \rho)}{(1 + b\lambda_1)^{k+i}} \left[ \frac{1}{(1 + b\lambda_1)(1 - \rho + k\rho) - k\rho} \right]
\]

\[
\int_0^\infty (1 - \rho) \sum_{i=0}^{\infty} \frac{(k\rho)^i \varphi(k + i, y/b)}{(1 - \rho + k\rho)^{i+1}(k + i - 1)!} \lambda_2 e^{-\lambda_2 y} dy \\
= \frac{(1 - \rho)}{(1 + b\lambda_2)^{k+i}} \left[ \frac{1}{(1 + b\lambda_2)(1 - \rho + k\rho) - k\rho} \right]
\]
\[
\int_0^\infty (1 - \rho) \sum_{i=0}^{\infty} \frac{(k\rho)^i}{(1 - \rho + k\rho)^{i+1}} \frac{\varphi(k + i, y/b)}{(k + i - 1)!} (\lambda_1 + \lambda_2) e^{-(\lambda_1 + \lambda_2)y} dy
\]

\[
= \frac{(1 - \rho)}{(1 + b(\lambda_1 + \lambda_2))^{k-1}} \left[ \frac{1}{(1 + b(\lambda_1 + \lambda_2))(1 - \rho + k\rho) - k\rho} \right]
\]

Substituting there in (6), we get

\[
(S_k < Y) = (1 - \rho) [B_{1k} + B_{2k} - B_{3k}]
\]

(9)

where

\[
B_{1k} = \frac{1}{(1 + b\lambda_1)^{k-1}} \left[ \frac{1}{(1 + b\lambda_1)(1 - \rho + k\rho) - k\rho} \right]
\]

\[
B_{2k} = \frac{1}{(1 + b\lambda_2)^{k-1}} \left[ \frac{1}{(1 + b\lambda_2)(1 - \rho + k\rho) - k\rho} \right]
\]

\[
B_{3k} = \frac{1}{(1 + b(\lambda_1 + \lambda_2))^{k-1}} \left[ \frac{1}{(1 + b(\lambda_1 + \lambda_2))(1 - \rho + k\rho) - k\rho} \right]
\]

Evaluate \(P(S_k < Z)\), using law of total probability and conditioning upon \(Z\), we get

\[
P(S_k < Z) = \int_0^\infty (1 - \rho) \sum_{i=0}^{\infty} \frac{(k\rho)^i}{(1 - \rho + k\rho)^{i+1}} \frac{\varphi(k + i, z/b)}{(k + i - 1)!} \left[ \mu_1 e^{-\mu_1 z} + \mu_2 e^{-\mu_2 z} - (\mu_1 + \mu_2) e^{-(\mu_1 + \mu_2)z} \right] dz
\]

(10)

From (7), we get

\[
\int_0^\infty \varphi(k + i, \frac{z}{b}) \mu_1 e^{-\mu_1 z} dz = \frac{1}{\mu_1} \frac{(k + i - 1)!}{(1 + b\mu_1)^{k+i}}
\]

\[
\int_0^\infty \varphi(k + i, \frac{z}{b}) \mu_2 e^{-\mu_2 z} dz = \frac{1}{\mu_2} \frac{(k + i - 1)!}{(1 + b\mu_2)^{k+i}}
\]

\[
\int_0^\infty \varphi(k + i, \frac{z}{b}) (\mu_1 + \mu_2) e^{-(\mu_1 + \mu_2)z} dz = \frac{1}{(\mu_1 + \mu_2)} \frac{(k + i - 1)!}{(1 + b(\mu_1 + \mu_2))^{k+i}}
\]

Substituting these equations in (10), we get

\[
P(S_k < Z) = (1 - \rho) [B_{4k} + B_{5k} - B_{6k}]
\]

Using the equations (9) and (10) in (1), we get

\[
P(W > t) = (1 - \rho) \left\{ \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)](B_{1k} + B_{2k} - B_{3k}) + p \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)](B_{4k} + B_{5k} - B_{6k}) \right\}
\]

\[-(1 - \rho)p \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)](B_{4k} + B_{5k} - B_{6k})(B_{4k} + B_{5k} - B_{6k}) \}
\]
\[ L(t) = 1 - P(W > t) \] and using \( \frac{d}{dt} L(t) = l(t) \), we get

\[
l(t) = -(1 - \rho) \left\{ \sum_{k=0}^{\infty} [f_k(t) - f_{k+1}(t)] \left( (B_{1k} + B_{2k} - B_{3k}) + p(B_{4k} + B_{5k} - B_{6k}) \right) - (1 - \rho) p(B_{1k} + B_{2k} - B_{3k}) (B_{4k} + B_{5k} - B_{6k}) \right\} \tag{11}
\]

Taking Laplace transform on both sides of (11), we get

\[
l^*(s) = -(1 - \rho) \left\{ \sum_{k=0}^{\infty} [f^*_k(s) - f^*_{k+1}(s)] \left( (B_{1k} + B_{2k} - B_{3k}) + p(B_{4k} + B_{5k} - B_{6k}) \right) - (1 - \rho) p(B_{1k} + B_{2k} - B_{3k}) (B_{4k} + B_{5k} - B_{6k}) \right\} \tag{12}
\]

\[
E(W) = - \left[ \frac{d}{ds} l^*(s) \right]_{s=0} \tag{13}
\]

\[
E(W^2) = - \left[ \frac{d^2}{ds^2} l^*(s) \right]_{s=0} \tag{14}
\]

\[
Var(W) = E(W^2) - (E(W))^2 \tag{15}
\]

The inter-decision times are independent and non-identically distributed random variables so it follows Hypo-exponential distribution. We note that

\[
F_k(t) = \sum_{i=1}^{k} b_i (1 - e^{-\beta_i t}), \quad f_k(t) = \sum_{i=1}^{k} b_i \beta_i e^{-\beta_i t}
\]

\[
f^*_k(s) = \sum_{i=1}^{k} b_i \frac{\alpha_i}{\alpha_i + s}, \quad \text{where} \quad b_i = \prod_{j=1, j \neq i}^{k} \frac{\beta_j}{\beta_j - \beta_i}, \quad i = 1, 2, 3, \ldots, k.
\]

Consider

\[
\sum_{k=0}^{\infty} [f_{k+1}(t) - f_k(t)] = \sum_{k=0}^{k+1} \sum_{i=1}^{k} \frac{b_i \beta_i}{(\beta_i + s)} - \sum_{k=0}^{k} \sum_{i=1}^{k} \frac{b_i \beta_i}{(\beta_i + s)}
\]

\[
\frac{d}{ds} \left[ (f^*_{k+1}(s) - f^*_k(s)) \right]_{s=0} = - \sum_{k=0}^{\infty} \sum_{i=1}^{k+1} \frac{1}{\beta_i} = - \sum_{k=0}^{\infty} \frac{1}{\beta_{k+1}} \tag{16}
\]
\[ E(w) = \left[ -\frac{d^2}{ds^2} l^s(s) \right]_{s=0} = (1 - \rho) \left\{ \sum_{k=0}^{\infty} \frac{1}{\beta_{k+1}} \left\{ \left( B_{1k} + B_{2k} - B_{3k} \right) + p\left( B_{4k} + B_{5k} - B_{6k} \right) \right\} \right\} \]

Equation (17) gives mean time to recruitment of maximum model

\[ \frac{d^2}{ds^2} \left[ \left( f_{k+1}^* - f_k^* \right) \right]_{s=0} = 2 \sum_{k=0}^{\infty} \frac{1}{(\beta_{k+1})^2} \]  

(18)

\[ E(w^2) = \left[ -\frac{d^2}{ds^2} l^s(s) \right]_{s=0} = 2(1 - \rho) \left\{ \sum_{k=0}^{\infty} \frac{1}{(\beta_{k+1})^2} \left\{ \left( B_{1k} + B_{2k} - B_{3k} \right) + p\left( B_{4k} + B_{5k} - B_{6k} \right) \right\} \right\} \]

Substitute equations (17) and (19) in (15), we get the variance of maximum model.

**Numerical Illustrations**

The parameters of inter-decision times \((\beta_2, \beta_3, \beta_4, \beta_5, \beta_6)\) are fixed. The value of \(\rho\) and \(p\) vary. \(\beta_2 = 0.6, \beta_3 = 0.7, \beta_4 = 0.8, \beta_5 = 0.9, \beta_6 = 1.2, \lambda_1 = 0.01, \lambda_2 = 0.12, \mu_1 = 0.02, \mu_2 = 0.025.\)

<table>
<thead>
<tr>
<th>(\rho/p)</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.55</th>
<th>0.6</th>
</tr>
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<tbody>
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<td>(E(w))</td>
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<td>0.9750</td>
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</tr>
<tr>
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<td>0.6187</td>
<td>0.6275</td>
</tr>
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</table>

**Findings**

7
From the table, we observe the following:
1. The mean and variance of the time to recruitment decreases as the value of $\rho$ and $p$ are increases simultaneously.
2. If $\rho$, the correlation alone increases, while the mean and variance of the time to recruitment decreases.
3. As $p$, the probability value alone increase, the mean and variance of the time to recruitment increase.

Model: II Minimum Model

$Y = \min(Y_1, Y_2)$ Consider $P(S_k < Y)$, $Y_1, Y_2$ are independent exponential random variables with mean $\frac{1}{\lambda_1}$ and $\frac{1}{\lambda_2}$.

The distribution $H(.)$ of $Y$ is given by

$$H(y) = 1 - e^{-\lambda_1 y}e^{-\lambda_2 y} = 1 - e^{(\lambda_1 + \lambda_2)y} \quad (20)$$

The probability density function is

$$h(y) = (\lambda_1 + \lambda_2)e^{(\lambda_1 + \lambda_2)y} \quad (21)$$

Using (3) and (21) in (2)

$$P(S_k < Y) = \int_0^\infty (1 - \rho) \left[ \sum_{i=0}^{\infty} \frac{(kp)^i \varphi(k + i, g/y)}{(1 - \rho + kp)^{i+1}(k + i - 1)!} \right] \left[ (\lambda_1 + \lambda_2)e^{-(\lambda_1 + \lambda_2)y} \right] dy$$

$$\int_0^\infty \varphi(k + i, g/y)e^{-(\lambda_1 + \lambda_2)y} dy = \frac{1}{(\lambda_1 + \lambda_2)(1 + b(\lambda_1 + \lambda_2))^{k+1}} \quad (22)$$

$$P(S_k < Y) = (1 - \rho)B_{3k} \quad (23)$$

Similarly,

$$P(S_k < Z) = (1 - \rho)B_{6k} \quad (24)$$

Substitute equations (23) and (24) in (1), we get

$$P(W > t) = (1 - \rho) \sum_{k=0}^{\infty} V_k(t)[B_{3k} + pB_{6k} - p(1 - \rho)B_{3k}B_{6k}]$$

It known that $\frac{d}{ds}L(t) = l(t)$ and Laplace transorm $l(t) = l^*(s)$, we get

$$l^*(s) = 1 - (1 - \rho) \sum_{k=0}^{\infty} \left[ f_k^*(t) - f_{k+1}^*(t) \right][B_{3k} + pB_{6k} - p(1 - \rho)B_{3k}B_{6k}] \quad (25)$$

From equations (13) and (14), we get

$$E(w) = \left[ \frac{d}{ds} l^*(s) \right]_{s=0} = (1 - \rho) \sum_{k=0}^{\infty} \frac{1}{\beta_{k+1}} [B_{3k} + pB_{6k} - p(1 - \rho)B_{3k}B_{6k}] \quad (26)$$
Equation (26) gives mean time to recruitment for minimum model
\[
E(w^2) = 2(1 - \rho) \sum_{k=0}^{\infty} \frac{1}{\beta_k^2} \left[ B_{3k} + pB_{6k} - p(1 - \rho)B_{3k}B_{6k} \right] \tag{27}
\]
Using equations (26) and (27) in (15), we get the variance of time to recruitment for minimum model.

**Numerical Illustrations**

The parameters of inter-decision times \((\beta_2, \beta_3, \beta_4, \beta_5, \beta_6)\) are fixed. The value of \(\rho\) and \(p\) vary. \(\beta_2 = 0.6, \beta_3 = 0.7, \beta_4 = 0.8, \beta_5 = 0.9, \beta_6 = 1.2, \lambda_1 = 1.7, \lambda_2 = 1.92, \mu_1 = 2.85, \mu_2 = 3.93, b = 1, v = 1.06\).

<table>
<thead>
<tr>
<th>(\rho/p)</th>
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<td>0.2463</td>
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</table>

**Findings**

From the table, we observe the following:

1. The mean and variance of the time to recruitment decreases as the value of \(\rho\) and \(p\) are increases simultaneously.
2. If \(\rho\), the correlation alone increases, while the mean and variance of the time to recruitment decreases .
3. As \(p\), the probability value alone increase, the mean and variance of the time to recruitment increase.

**References**


