UNSTEADY TWO-LAYER FLOW OF A VISCOUS FLUID THROUGH A CIRCULAR PIPE: EFFECT OF MAGNETIC FIELD

M. Chitra¹, V.Kavitha²

¹Associate Professor, ²Research Scholar, Department of Mathematics, Thiruvalluvar University, Vellore-632 115, Tamilnadu, India
Email: ¹chitratvu@gmail.com, ²kavithavijayakumar94@gmail.com

Abstract:

The unsteady MHD two–layer flow of a viscous fluid through a circular pipe is bounded by a porous medium with the effects of magnetic field are investigated. The governing equation is solved by using Bessel function. The effects of magnetic field on the velocity profile, shear stress and flow rate for both the layers are depicted in graphs.

Key words:

Viscous fluid, MHD flows, Porous medium, Bessel function, Slip velocity.

Nomenclature:

µ = Viscosity of the fluid in the first layer
µₑ = Viscosity of the fluid in the second layer
u₁ = Velocity of the fluid in the z direction in the first layer
u₂ = Velocity of the fluid in the z direction in the second layer
k = Permeability of the porous medium
j = Magnetic flux
E = Electrical conductivity
B = Magnetic field
σ = Electrical conductivity
ρ = Density of the fluid
µₑ = Permeability of the medium
H₀ = Magnetic field intensity
β = Constant (both positive and negative)
A₁, A₂ = Constants
1. Introduction:

The unsteady two-layer flow of a viscous fluid through a circular pipe in a porous medium with the effects of magnetic field has received much attention for a long time. Because of their wide usage area in the industry such as petroleum industry, agriculture, chemical spills and polymer technology applications and in many devices such as MHD power generation, MHD pumps, purification of crude oil etc. Ishimoto and Kamiyama (1997) studied effect of non-uniform magnetic field on the linear and nonlinear wave propagation phenomena in two-phase pipe flow of magnetic fluid. The governing equations of two-phase flow are numerically analyzed by using the finite volume method. MHD flow of a non-Newtonian fluid in a channel of slowly varying cross section in the presence of a uniform transverse magnetic field was studied by Misra et al. (1998). Several authors have studied problems of flow of fluid in two regions: in region I fluid is free to flow and in region II the fluid flows through porous media. Such types of coupled flow with different geometries, and several kinds of matching conditions at the interface have been examined Srinivastava (1999). The study of magneto hydrodynamic flows through porous medium has been studied by Yamamoto (2004). Zhang et al. (2004) have solved the model which they have built for liquid-solid dimensional flow and compared it with experimental results. Srivastavaet.al. (2005) have developed a model for flow of viscous fluid through a circular pipe and its surrounding porous medium bounded by a rigid cylinder and the unbounded regions. Attia and Ahmed (2005) investigated numerically the unsteady flow of a dusty viscous incompressible electrically conducting Bingham fluid through a circular pipe at constant pressure gradient. The effect of slip condition on MHD steady flow in a channel with permeable boundaries has been discussed by Makinde and Osalusi (2006). Attia (2006) studied the transient flow of a dusty viscous incompressible electrically conducting non-Newtonian Bingham fluid through a circular pipe by taking the Hall Effect into consideration. Misra ans Shit (2007) studied the effect of magnetic field on blood flow through an artery in unsteady situation and observed the effect of magnetic parameter, unsteady parameter and the radius phase angle on the flow characteristics.

Sezgin and Bozkaya (2008) have been discussed boundary element method solution of magnetohydrodynamic flow in rectangle duct with conducting walls parallel to applied magnetic field. Biswas and Laskar (2011) studied the influence of magnetic field and slip velocity on pulsatile blood flow through a constricted artery. R. Ponalagusamy, R. Tamil Selvi(2011) have considered the flow of blood represented by a two-layered model. Chauhan (2015) discussed the flow of a viscous fluid through a porous circular pipe in presence of magnetic field. The main objective of the present paper is to study the unsteady MHD two-layer flow through a circular pipe which is bounded by a porous medium fully saturated with the viscous fluid in the effects of magnetic field.

2. Mathematical Formulation of the Problem:

Consider the unsteady two-layer flow of a viscous fluid through a circular pipe in a porous medium with the effects of magnetic field. The layer - I inside the pipe in which clear fluid is flowing and the layer – II outside the pipe $r \geq \eta$ which is occupied by the porous medium fully saturated with the viscous fluid is flowing and both the layers are subjected to a constant applied magnetic field in the direction normal to the common axis (fig.1). The velocity component in the direction of $z$ is denoted by $u_1(r)$ and $u_2(r)$ in the layer- I and layer - II respectively. Both $u_1$ and $u_2$ are functions of $r$ only.
Figure 1. Schematic diagram for flow geometry.

We further assume that the flows in layer - I and II are respectively governed by Navier-Stokes equation and Brinkman equation [1947]. Pressures in both the layers are functions of $z$ alone and in both the layers are same.

Equation of motion in $z$ direction in layer - I is given by:

$$
\rho \frac{du_1}{dt} = -\frac{dp}{dz} + \mu \left[ \frac{d^2u_1}{dr^2} + \frac{1}{r} \frac{du_1}{dr} \right] + J \times B \\
0 \leq r < \eta 
$$

Equation of motion in $z$ direction in layer - II is given by:

$$
\rho \frac{du_2}{dt} = -\frac{dp}{dz} + \mu_e \left[ \frac{d^2u_2}{dr^2} + \frac{1}{r} \frac{du_2}{dr} \right] + J \times B - \frac{\mu}{k} u_2 \\
0 > \eta 
$$

Ohm’s law

$$
J = \sigma (E + u_1 \times B) 
$$

$$
J = \sigma (E + u_2 \times B) 
$$

The value of

$$
J \times B = -\sigma \mu_e \beta H^2 U_0 u_1 \quad \text{(Layer – I)} 
$$

$$
J \times B = -\sigma \mu_e \beta H^2 U_0 u_2 \quad \text{(Layer – II)} 
$$

Boundary and Matching Conditions

$$
u_i(r) = u_j(r) \quad \text{at} \quad r = \eta 
$$

$$
\mu_e \frac{du_1}{dr} - \mu \frac{du_1}{dr} = \frac{\mu \beta}{\sqrt{k}} u_2 \quad \text{at} \quad r = \eta 
$$
\[
\frac{du_1}{dr} = 0 \text{ at } r = 0 \\
\frac{du_2}{dr} = 0 \text{ at } r = 0
\]

(9)  
(10)

The pressure gradient is given by
\[
\frac{dp}{dz} = -\lambda e^{i\omega t}
\]

(11)

Let us introduce the following dimensionless quantities:
\[
u_i = \eta u_i \omega, \quad u_2 = \eta u_2 \omega, \quad r = \eta r \omega, \quad t = \frac{t}{\omega}, \quad \lambda = \frac{\mu \lambda}{\eta} \omega
\]

(12)

The Womersley parameter is defined by:
\[
\alpha = \eta \sqrt{\frac{\rho \omega}{\mu}}
\]

(13)

In terms of these variables, equations (1) and (2) [after dropping stars] becomes
\[
\frac{df}{dt} = \lambda e^{i\omega t} + \frac{d^2 u_1}{dr^2} + \frac{1}{r} \frac{du_1}{dr} - M^2 u_1 \quad 0 < r < 1
\]

(14)
\[
\frac{df}{dt} = \frac{\lambda}{\gamma^2} e^{i\omega t} + \frac{d^2 u_2}{dr^2} + \frac{1}{r} \frac{du_2}{dr} - Q^2 u_2 \quad r > 1
\]

(15)

Where
\[
M^2 = \frac{\sigma \mu_2^{H_2^2} r_0^2}{\mu}, \quad N^2 = \frac{\sigma \mu_1^{H_1^2} r_0^2}{\mu}, \quad Q^2 = \delta^2 + N^2, \quad \gamma^2 = \frac{\mu_1}{\mu}, \quad \sigma = \frac{r_0}{\sqrt{k}} \quad \text{and} \quad \delta = \frac{\sigma}{\gamma}
\]

With boundary and matching conditions
\[
u_i = u_2 \text{ at } r = 1
\]

(16)
\[
\gamma^2 \frac{du_2}{dr} - \frac{du_i}{dr} = \beta \sigma u_2 \text{ at } r = 1
\]

(17)
\[
\frac{du_i}{dr} = 0 \text{ at } r = 0
\]

(18)
\[
\frac{du_2}{dr} = 0 \text{ at } r = 0
\]

(19)

3. Solution of the Problems:

To solve the equation (14) and (15) under the boundary condition (16-19) following Schlichting, we assume the solution of the form
\[ u_1(r,t) = f_1(r, t)e^{i\omega t} \]  
\[ u_2(r,t) = f_2(r, t)e^{i\omega t} \]

We have

\[ f_1'(r) + \frac{1}{r} f_1'(r) + \left(\sqrt{-C}\right)^2 f_1(r) = -\lambda \]  
\[ f_2'(r) + \frac{1}{r} f_2'(r) + \left(\sqrt{-D}\right)^2 f_2(r) = -\frac{\lambda}{\gamma^2} \]

Where \( C = (M^2 + \alpha^2 i\omega) \) and \( D = (Q^2 + \alpha^2 i\omega) \)

Equations (22) and (23) are Bessel equations of order zero

We get,

\[ u_1(r,t) = \left[ \frac{\lambda}{C} + A_1 J_0(\sqrt{-C}r) \right] e^{i\omega t} \]  
\[ u_2(r,t) = \left[ \frac{\lambda}{D\gamma^2} + A_2 J_0(\sqrt{-D}r) \right] e^{i\omega t} \]

Where

\[ A_1 = \frac{(C-D\gamma^2)\lambda J_0(\sqrt{-D})(\sqrt{-C}) + D\beta\lambda J_0(\sqrt{-D})}{CD \left[ \gamma^2 J_0(\sqrt{-D})(\sqrt{-D}) - \beta\lambda J_0(\sqrt{-D}) \right] J_1(\sqrt{-C}) - J_0(\sqrt{-C})(\sqrt{-D}) J_1(\sqrt{-D})} \]

\[ A_2 = \frac{(C-D\gamma^2)\lambda J_0(\sqrt{-D})(\sqrt{-C}) + C\beta\lambda J_0(\sqrt{-D})}{CD\gamma^2 \left[ \gamma^2 J_0(\sqrt{-D})(\sqrt{-D}) - \beta\lambda J_0(\sqrt{-D}) \right] J_1(\sqrt{-C}) - J_0(\sqrt{-C})(\sqrt{-D}) J_1(\sqrt{-D})} \]

The expression for the shear stress \( \tau_1 \) (layer-I) can be obtained from

\[ \tau_1 = -\mu \left( \frac{du_1}{dr} \right)_{r=1} \]  
\[ \tau_1 = \frac{\left[ (C-D\gamma^2)\lambda J_0(\sqrt{-D})(\sqrt{-C}) + D\beta\lambda J_0(\sqrt{-D}) \right] J_1(\sqrt{-C})(\sqrt{-C})e^{i\omega t}}{CD \left[ \gamma^2 J_0(\sqrt{-D})(\sqrt{-D}) - \beta\lambda J_0(\sqrt{-D}) \right] J_1(\sqrt{-C}) - J_0(\sqrt{-C})(\sqrt{-D}) J_1(\sqrt{-D})} \]

Similarly the expression for the shear stress \( \tau_2 \) (layer-II) can be obtained from

\[ \tau_2 = -\mu \left( \frac{du_2}{dr} \right)_{r=1} \]  
\[ \tau_2 = \frac{\left[ (C-D\gamma^2)\lambda J_0(\sqrt{-D})(\sqrt{-C}) + C\beta\lambda J_0(\sqrt{-D}) \right] J_1(\sqrt{-D})(\sqrt{-C})e^{i\omega t}}{CD\gamma^2 \left[ \gamma^2 J_0(\sqrt{-D})(\sqrt{-D}) - \beta\lambda J_0(\sqrt{-D}) \right] J_1(\sqrt{-C}) - J_0(\sqrt{-C})(\sqrt{-D}) J_1(\sqrt{-D})} \]
The expression for the flow rate $Q_1$ (layer-I) can be written as

$$Q_1 = 2\pi \int_0^1 r u_1 dr$$

(30)

Similarly, the expression for the flow rate $Q_2$ (layer-II) can be written as

$$Q_2 = 2\pi \int_0^1 r u_2 dr$$

(32)

$$Q_1 = 2\pi \left[ \frac{\lambda}{2C} + \frac{\gamma J_r(\sqrt{D}) J_\beta J_\sigma(\sqrt{D})}{CD} \int_0^1 \left[ \frac{(C-D)r^2}{(C-D)^2} J_\alpha(\sqrt{D}-\beta) J_\gamma(\sqrt{D}) + D\beta\sigma J_\beta(\sqrt{D}) J_\gamma(\sqrt{D}) \right] \right] e^{\omega t}$$

(31)

$$Q_2 = 2\pi \left[ \frac{\lambda}{2D\gamma} \int_0^1 \left[ \frac{(C-D)r^2}{(C-D)^2} J_\alpha(\sqrt{D}-\beta) J_\gamma(\sqrt{D}) + D\beta\sigma J_\beta(\sqrt{D}) J_\gamma(\sqrt{D}) \right] \right] e^{\omega t}$$

(33)

4. Graphical results and Discussions:

The present study deals with our mathematical analysis and graphical investigation of unsteady two-layer flow of a viscous fluid through a circular pipe with effects of magnetic field. The problem is solved by Bessel function: it gives the solution of velocity distribution, shear stress and flow rate. The velocity profile, shear stress and flow rate for both the layers are computed for different values of magnetic field and results are shown graphically.

Fig. 2 Variation of velocity distribution $u_i$ with radial position $r$ for different values of magnetic field $M$ ($\gamma = 1, \beta = 1, \sigma = 1, \lambda = 1, \alpha = 1, \omega = 1, t = 1$)
Fig. 3: Variation of velocity distribution $u_r$ with radial position $r$ for different values of magnetic field $M$ ($\gamma = 1, \beta = 1, \sigma = 1, \lambda = 1, k = 0.5, \alpha = 1, \omega = 1, t = 1$)

Fig. 4: Variation of shear stress $u_t$ with time $t$ for different values of magnetic field $M$ ($\gamma = 1, \beta = 1, \sigma = 1, \lambda = 1, \alpha = 1, \omega = 1$)
Fig. 5 Variation of shear stress $u_2$ with time $t$ for different values of magnetic field $M$
($\gamma = 1, \beta = 1, \sigma = 1, \lambda = 1, k = 0.5, \alpha = 1, \omega = 1$)

Fig. 6 Variation of flow rate $u_1$ with time $t$ for different values of magnetic field $M$
($\gamma = 1, \beta = 1, \sigma = 1, \lambda = 1, \alpha = 1, \omega = 1$)
The figure 2 and 3 displays the velocity profiles \( (u_1, u_2) \) decreases with increases of magnetic field.

The figure 4 indicate the shear stress profile \( (u_1) \) decreases with increases of magnetic field and the figure 5 illustrates the shear stress profile \( (u_2) \) increases with increases of magnetic field.

The figure 6 shows the flow rate \( (u_1) \) increases with increases of magnetic field and the figure 7 present the flow rate \( (u_2) \) decreases with increasing of magnetic field.

5. Conclusions:

In this analysis, the impact of magnetic field on unsteady two-layer flow of viscous fluid in a circular pipe bounded by a porous medium is studied. The fluid and changing expressions of velocities, shear stress and flow rate of the flow depending on physical parameter of the pipe have been obtained. The fluid velocity profiles, shear stress and flow rate for the effects of magnetic field are examined graphically. The fluid velocity decreases as the increasing of magnetic field in both the layers. It is clear that the magnetic field is unaffected by the porous medium in velocity profiles. The shear stress profiles layer-I decreases and layer-II increases for the increase of magnetic field. The flow rate of layer-I increases and layer-II decreases for the increase of magnetic field. Thus, the mathematical expressions give a solution for more complex problems involving the various physical effects.

References:
