HOMOMORPHISM OF AN ANTI-FUZZY HX IDEAL
OF A HX RING

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ABSTRACT. We delineated an anti-fuzzy HX left (right) ideal and anti-fuzzy HX ideal of a HX ring. We establish a notion of anti fuzzy HX ideal of a HX ring and some of its properties are explored. We present the concept of an anti-image, anti-pre-image of anti-fuzzy HX ideals of a HX ring and analyze the properties of homomorphic and anti homomorphic anti-images and pre-images of an anti-fuzzy HX ideal of a HX ring R.

1. INTRODUCTION

Fuzzy set theory is the most worthy theory for dealing with unreliability which was introduced by Lotfi Aliasker Zadeh [9] was a mathematician, computer scientist, electrical engineer, artificial intelligence researcher and professor emeritus of computer science at the University of California, Berkeley. It is an extension of a crisp set on a universal set X is defined by its characteristic function from X to \{0, 1\}, and a fuzzy set on a domain X is defined by its membership function from X to [0, 1]. Fuzzy sets drawn attention of mathematicians and grew extensively by locating applications in many areas. In ring theory or abstract algebra, a ring homomorphism is a structure preserving map between two algebraic structures of the same type. An anti homomorphism is a function defined on sets with multiplication that reverses the order of multiplication. Evaluating a function at each element of a subset X of the domain, produces a set called the image of X under the function. The inverse image or pre-image of a particular subset S of the collection of a function is the set of all elements of the domain that map to the member of S. The concept of fuzzy subring and fuzzy ideal was introduced by Wang-jin Liu [6]. Professor Li Hong

1991 Mathematics Subject Classification. 20N25, 03E72, 03F05, 06F35, 03G25.

Key words and phrases. HX ring, anti-fuzzy HX ideal, homomorphism.

2. Anti fuzzy HX ideal

**Definition 1.** A fuzzy subset $\lambda_\mu$ of $R$ is called an anti-fuzzy HX right ideal on or an anti-fuzzy right ideal induced by $\mu$ if for all $M, N \in R$,

i. $\lambda_\mu(M - N) \leq \max\{\lambda_\mu(M), \lambda_\mu(N)\}$,

ii. $\lambda_\mu(MN) \leq \lambda_\mu(M)$

where $\lambda_\mu(M) = \min\{\lambda_\mu(x) \mid x \in M \subseteq R\}$.

**Definition 2.** A fuzzy subset $\lambda_\mu$ of $R$ is called an anti-fuzzy HX left ideal on or an anti-fuzzy right ideal induced by $\mu$ if for all $M, N \in R$,

i. $\lambda_\mu(M - N) \leq \max\{\lambda_\mu(M), \lambda_\mu(N)\}$,

ii. $\lambda_\mu(MN) \leq \lambda_\mu(M)$

where $\lambda_\mu(M) = \min\{\lambda_\mu(x) \mid x \in M \subseteq R\}$.

**Definition 3.** A fuzzy subset $\lambda_\mu$ of $R$ is called an anti-fuzzy HX ideal on $R$ or an anti-fuzzy ideal induced by $\mu$ if it is both anti-fuzzy HX right ideal and anti-fuzzy HX left ideal on $R$.

That is, For all $M, N \in R$,

i. $\lambda_\mu(M - N) \leq \max\{\lambda_\mu(M), \lambda_\mu(N)\}$,

ii. $\lambda_\mu(MN) \leq \min\{\lambda_\mu(M), \lambda_\mu(N)\}$

where $\lambda_\mu(M) = \min\{\mu(x) \mid x \in M \subseteq R\}$.

3. Homomorphism and anti homomorphism of an anti-fuzzy HX ideals of a HX ring

In this subsection, we present the idea of an anti-image, anti-pre-image of anti-fuzzy HX ideals of a HX ring and examine the characteristics of homomorphic and anti homomorphic anti-images, pre-images of an anti-fuzzy HX ideal of a HX ring $R$.

**Definition 4.** Let $R_1$ and $R_2$ be any two rings. Let $R_1 \subset 2^{R_1} - \{\phi\}$ and $R_2 \subset 2^{R_2} - \{\phi\}$ be two HX rings defined on $R_1$ and $R_2$ correspondingly. Let $\mu$ and $\alpha$ be any two fuzzy subsets in $R_1$ and $R_2$ correspondingly. Let $\lambda_\mu$ and $\eta_\alpha$ be anti-fuzzy HX ideals defined on $R_1$ and $R_2$ induced by $\mu$ and $\alpha$. Let $\phi : R_1 \rightarrow R_2$ be a mapping then the anti-image of $\lambda_\mu$ denoted as $\phi(\lambda_\mu)$ is a fuzzy subset of $R_2$ defined for each $V \in R_2$. 
(ϕ(λμ))(V) = \begin{cases}
\{\inf_{\lambda \mu}(P)\} : & X \in \phi^{-1}(V); \text{if } \phi^{-1}(V) \neq \phi \\
1 & \text{otherwise}
\end{cases}

Also the anti-pre-image of ηα denoted as f□1(ηα) under f is a fuzzy subset of R1 defined as for each P belongs to R1.

(ϕ□1(ηα))(P) = ηα(ϕ(P))

Theorem 3.1
Let λμ be an anti-fuzzy HX right ideal of R1 then φ(λμ) is an anti-fuzzy HX right ideal of R2, if (λμ) has an greatest lower bound and φ-invariant.
Proof:
Let μ be a fuzzy subset of R1 and λμ is an anti-fuzzy HX right ideal of R1. There exist P, Q ∈ R1 such that ϕ(P), ϕ(Q) ∈ R2.

i. (ϕ(λμ))(ϕ(P) − ϕ(Q)) = (ϕ(λμ))(ϕ(P) − Q))
= λμ(P − Q)
≤ max{λμ(P), λμ(Q)}
= max{ϕ(λμ))(ϕ(P)), ϕ(λμ))(ϕ(Q))}

Therefore (ϕ(λμ))(ϕ(P) − φ(Q)) ≤ max{ϕ(λμ))(ϕ(P)), ϕ(λμ))(ϕ(Q))}

ii. (ϕ(λμ))(ϕ(P)ϕ(Q)) = (ϕ(λμ))(ϕ(PQ))
= λμ(PQ)
≤ λμ(P)
= (ϕ(λμ))(ϕ(P))

Therefore (ϕ(λμ))(ϕ(P), ϕ(Q)) ≤ (ϕ(λμ))(ϕ(P))

Hence (ϕ(λμ))is an anti-fuzzy HX right ideal of R2.

Theorem 3.2
Let λμ be an anti-fuzzy HX left ideal of R1 then φ(λμ) is an anti-fuzzy HX left ideal of R2, if λμ has an greatest lower bound , φ -invariant.
Proof:
Let μ be a fuzzy subset of R1 and λμ is an anti-fuzzy HX right ideal of R1. There exist P, Q ∈ R1 such that ϕ(P), ϕ(Q) ∈ R2.
\[
i. (\phi(\lambda\mu))(\phi(P) - \phi(Q)) = (\phi(\lambda\mu))(\phi(P - Q)) = \lambda\mu(P - Q) \\
\leq \max \{\lambda\mu(P), \lambda\mu(Q)\}
\]

Therefore \((\phi(\lambda\mu))(\phi(P) - \phi(Q)) \leq \max \{\phi(\lambda\mu)(P), \phi(\lambda\mu)(Q)\}\)

\[
ii. (\phi(\lambda\mu))(\phi(P)\phi(Q)) = (\phi(\lambda\mu))(\phi(PQ)) = \lambda\mu(PQ) \\
\leq \lambda\mu(P) = (\phi(\lambda\mu))(\phi(P))
\]

Therefore \((\phi(\lambda\mu))(\phi(P)\phi(Q)) \leq (\phi(\lambda\mu))(\phi(P))\)

Hence \((\phi(\lambda\mu))\) is an anti-fuzzy HX rightleft ideal of \(R_2\).

**Theorem 3.3**

Let \(\lambda\mu\) be an anti-fuzzy HX ideal of \(R_1\) then \((\phi(\lambda\mu))\) is an anti-fuzzy HX ideal of \(R_2\) if \(\lambda\mu\) has an greatest lower bound ,\(\phi\) -invariant.

Proof:

Its clear.

**Theorem 3.4**

Let \(\eta\alpha\) be an anti-fuzzy HX right ideal of \(R_2\) then \(\phi^{-1}(\eta\alpha)\) is an anti-fuzzy HX right ideal of \(R_1\)

Proof:

\[
i. \phi^{-1}(\eta\alpha)(P - Q) = \eta\alpha(\phi(P - Q)) = \eta\alpha(\phi(P) - \phi(Q)) \\
\leq \max \{\eta\alpha(\phi(P)), \eta\alpha(\phi(Q))\} = \max \{\phi^{-1}(\eta\alpha)(P), \phi^{-1}(\eta\alpha)(Q)\}
\]

Therefore \(\phi^{-1}(\eta\alpha)(P - Q) \leq \max \{\phi^{-1}(\eta\alpha)(P), \phi^{-1}(\eta\alpha)(Q)\}\)

\[
ii. \phi^{-1}(\eta\alpha)(PQ) = \eta\alpha(\phi(PQ)) = \eta\alpha(\phi(P)\phi(Q)) \\
\leq \eta\alpha(\phi(P)) = \phi^{-1}(\eta\alpha)(P)
\]

Therefore \(\phi^{-1}(\eta\alpha)(PQ) \leq \phi^{-1}(\eta\alpha)(P)\)

Hence, \(\phi^{-1}(\eta\alpha)\) is an anti-fuzzy HX right ideal of \(R_1\).
Theorem 3.5
Let $\eta_\alpha$ be an anti-fuzzy HX left ideal of $R_2$ then $\phi^{-1}(\eta_\alpha)$ is an anti-fuzzy HX left ideal of $R_1$.

Proof:

$$i. \phi^{-1}(\eta_\alpha)(P - Q) = \eta_\alpha(\phi(P) - \phi(Q)) \leq \max\{\eta_\alpha(\phi(P)), \eta_\alpha(\phi(Q))\}$$

Therefore $\phi^{-1}(\eta_\alpha)(P - Q) \leq \max\{\phi^{-1}(\eta_\alpha)(P), \phi^{-1}(\eta_\alpha)(Q)\}$

$$ii. \phi^{-1}(\eta_\alpha)(PQ) = \eta_\alpha(\phi(P)\phi(Q)) \leq \eta_\alpha(\phi(Q)) = \phi^{-1}(\eta_\alpha)(Q)$$

Therefore $\phi^{-1}(\eta_\alpha)(PQ) \leq \phi^{-1}(\eta_\alpha)(Q)$

Hence, $\phi^{-1}(\eta_\alpha)$ is an anti-fuzzy HX left ideal of $R_1$.

Theorem 3.6
Let $\eta_\alpha$ be an anti-fuzzy HX ideal of $R_2$ then $\phi^{-1}(\eta_\alpha)$ is an anti-fuzzy HX ideal of $R_1$.

Proof:

It is clear

Remark:
For the following theorems $f$ is an antihomomorphism.

Theorem 3.7
Let $\lambda_\mu$ be an anti-fuzzy HX right ideal of $R_1$ then $\phi(\lambda_\mu)$ is an anti-fuzzy HX left ideal of $R_2$, if $(\lambda_\mu)$ has an greatest lower bound and $\phi$-invariant.

Proof:

Let $\mu$ be a fuzzy subset of $R_1$ and $\lambda_\mu$ is an anti-fuzzy HX right ideal of $R_1$. There exist $P, Q \in R_1$ such that $\phi(P), \phi(Q) \in R_2$. 

\[(\phi(\lambda \mu))(\phi(P) - \phi(Q)) = (\phi(\lambda \mu))(\phi(Q - P)) = \lambda \mu(Q - P) \leq \max\{\lambda \mu(Q), \lambda \mu(P)\} = \max\{\phi(\lambda \mu)(\phi(Q)), \phi(\lambda \mu)(\phi(P))\}\]

Therefore \(\phi(\lambda \mu)(\phi(P) - \phi(Q)) \leq \max\{\phi(\lambda \mu)(\phi(Q)), \phi(\lambda \mu)(\phi(P))\}\)

\[\text{ii. } (\phi(\lambda \mu))(\phi(P)\phi(Q)) = (\phi(\lambda \mu))(\phi(QP)) = \lambda \mu(QP) \leq \lambda \mu(P) = (\phi(\lambda \mu))(\phi(P))\]

Therefore \(\phi(\lambda \mu)(\phi(P)\phi(Q)) \leq (\phi(\lambda \mu))(\phi(P))\)

Hence \((\phi(\lambda \mu))\) is an anti-fuzzy HX right ideal of \(R_2\).

Theorem 3.9
Let \(\mu\) be a fuzzy subset of \(R_1\) and \(\lambda \mu\) is an anti-fuzzy HX right ideal of \(R_2\), if \((\lambda \mu)\) has an greatest lower bound and \(\phi\)-invariant.

Proof:
Let \(\mu\) be a fuzzy subset of \(R_1\) and \(\lambda \mu\) is an anti-fuzzy HX right ideal of \(R_1\). There exist \(P, Q \in R_1\) such that \(\phi(P) \in R_2\).

\[
i.(\phi(\lambda \mu))(\phi(P) - \phi(Q)) = (\phi(\lambda \mu))(\phi(Q - P)) = \lambda \mu(Q - P) \leq \max\{\lambda \mu(Q), \lambda \mu(P)\} = \max\{\phi(\lambda \mu)(\phi(Q)), \phi(\lambda \mu)(\phi(P))\}\]

Therefore \(\phi(\lambda \mu)(\phi(P) - \phi(Q)) \leq \max\{\phi(\lambda \mu)(\phi(Q)), \phi(\lambda \mu)(\phi(P))\}\)

\[\text{ii. } (\phi(\lambda \mu))(\phi(P)\phi(Q)) = (\phi(\lambda \mu))(\phi(QP)) = \lambda \mu(QP) \leq \lambda \mu(P) = (\phi(\lambda \mu))(\phi(P))\]

Therefore \(\phi(\lambda \mu)(\phi(P)\phi(Q)) \leq (\phi(\lambda \mu))(\phi(P))\)

Hence \((\phi(\lambda \mu))\) is an anti-fuzzy HX right ideal of \(2\).

Theorem 3.9
Let \( \lambda_\mu \) be an anti-fuzzy HX ideal of \( R_1 \) then \( (\phi(\lambda_\mu)) \) is an anti-fuzzy HX ideal of \( R_2 \), if \( \lambda_\mu \) has a greatest lower bound, \( \phi \)-invariant.

Proof:
It's clear.

Theorem 3.10
Let \( \eta_\alpha \) be an anti-fuzzy HX right ideal of \( R_2 \) then \( \phi^{-1}(\eta_\alpha) \) is an anti-fuzzy HX left ideal of \( R_1 \).

Proof:

\[
i. \phi^{-1}(\eta_\alpha)(P - Q) = \eta_\alpha(\phi(P - Q)) \\
= \eta_\alpha(\phi(Q) - \phi(P)) \\
\leq \max\{\eta_\alpha(\phi(Q)), \phi(P)\} \\
= \max\{\phi^{-1}(\eta_\alpha)(P), \phi^{-1}(\eta_\alpha)(Q)\}
\]

Therefore \( \phi^{-1}(\eta_\alpha)(P - Q) \leq \max\{\phi^{-1}(\eta_\alpha)(P), \phi^{-1}(\eta_\alpha)(Q)\} \)

\[
ii. \phi^{-1}(\eta_\alpha)(PQ) = \eta_\alpha(\phi(PQ)) \\
= \eta_\alpha(\phi(Q), \phi(P)) \\
\leq \eta_\alpha(\phi(Q)) \\
= \phi^{-1}(\eta_\alpha)(Q)
\]

Therefore \( \phi^{-1}(\eta_\alpha)(PQ) \leq \phi^{-1}(\eta_\alpha)(Q) \)

Hence, \( \phi^{-1}(\eta_\alpha) \) is an anti-fuzzy HX right ideal of \( R_1 \).

Theorem 3.11
Let \( \eta_\alpha \) be an anti-fuzzy HX left ideal of \( R_2 \) then \( \phi^{-1}(\eta_\alpha) \) is an anti-fuzzy HX right ideal of \( R_1 \).

Proof:

\[
i. \phi^{-1}(\eta_\alpha)(P - Q) = \eta_\alpha(\phi(P - Q)) \\
= \eta_\alpha(\phi(Q) - \phi(P)) \\
\leq \max\{\eta_\alpha(\phi(P)), \eta_\alpha(\phi(Q))\} \\
= \max\{\phi^{-1}(\eta_\alpha)(Q), \phi^{-1}(\eta_\alpha)(P)\}
\]

Therefore \( \phi^{-1}(\eta_\alpha)(P - Q) \leq \max\{\phi^{-1}(\eta_\alpha)(P), \phi^{-1}(\eta_\alpha)(Q)\} \)

\[
ii. \phi^{-1}(\eta_\alpha)(PQ) = \eta_\alpha(\phi(PQ)) \\
= \eta_\alpha(\phi(Q), \phi(P)) \\
\leq \eta_\alpha(\phi(P)) \\
= \phi^{-1}(\eta_\alpha)(P)
\]

Therefore \( \phi^{-1}(\eta_\alpha)(PQ) \leq \phi^{-1}(\eta_\alpha)(P) \)
Hence, $\phi^{-1}(\eta_{\alpha})$ is an anti-fuzzy HX right ideal of $R_1$.

Theorem 3.12
Let $\eta_{\alpha}$ be an anti-fuzzy HX ideal of $R_2$ then $\phi^{-1}(\eta_{\alpha})$ is an anti-fuzzy HX ideal of $R_1$.

Proof:
It is clear.

REFERENCES


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