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Unsteady Heat and Mass Transfer Flow of Cu-Water Nano-Fluid over an Inclined Plate with Free Convection and Chemical Reactions

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Abstract

Convective heat and mass transfer using nano fluid has got its potential in heating and cooling systems for the past 2 decades. Presentably flow is considered over inclined plate under the free convection with chemical reactions. Graham and Choi models for viscosity and thermal conductivity are considered because of presence of particle size. The particle size during the flow is not constant due to reaction with solvent and other factors. The change in the particle size certainly affects the viscosity and thermal conductivity of the nano-fluid. So the study of convective heat and mass transfer through nano-fluid attracted the researchers. The

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Cu water nano fluid is considered due to the high thermal conductivity and of industrial importance. Motion of the fluid is analyzed with the radiation effects.

AMS Subject Classification: 80A20, 76D10

Key Words and Phrases: Nano - fluid, Viscosity, Heat Source, Radiation, Vertical Plate, R-K 6th Order

1 Introduction

The heat transfer has gained great demand due to its applications in many areas like, cooling of electronic devices, reactor cores, high voltage power transformers, energy storages, petroleum industries etc. In the heat transfer all the above areas study can be done with a typical geometry, like a flat plate. The inclined flat plate may consider for various application. If the system is like then the oscillation of the plate shall be considered. In view of this researchers studied the convective heat transfer with this geometry. Among those studies, [1] studied the convective heat transfer with a vertical permeable, rotating flat plate and reported that heat transfer enhances with nano particles presence.[2] studied the fluid rate in a wedge moving of a nano fluid and reported that the Brownian motion of nano particle influences the flow and heat transfer significantly. [3] also stressed the importance of next generation coolants like nano fluids. [4] reported that the increase in Brownian motion of the particles enhances the momentum and heat transfer. [5] reported that the performance of solar collector increases with increase in inclination of the solar cell and concentration of the nano material.[6] reported thermophoretic effect of convectional heat transfer over a permeable inclined plate. Many researchers [7] , [8] studied the significance of nano fluid in convective heat transfer by performing the experiments. [9] described the effect of suction/injection on convective boundary layer flow along with some more physical phenomenon. The reported that range of solution for the injection case is smallest for cu-water nano fluid. Few researchers [10], [11], [12], [13] stressed the importance of the moving flat plate geometry in convective heat transfer with nano fluids. Recently [14] studied the effect of nano particle diameter on flow and heat transfer along a vertical flat plate. To the best of available literature, the study of convective heat and mass transfer through the particle size (diameter) and the inter particle spacing in live cases like moving/oscillating plates have not noted. So we made here an attempt to study the effects of size of nano particles and the inter particles spacing in convective heat and mass transfer past a permeable, oscillating, inclined flat plate. We also considered the magnetic field, radiation, heat source, suction to study.

2 Mathematical Formulation

Cu water nano fluid flow is assumed along inclined flat plate. It is taken the assumption that the flow is in horizontal direction as per the 3D Cartesian coordinate system. The plate is assumed to be permeable, electrically non-conducting and oscillates, so $u(0,t) = U_0(1 + \cos nt)$.
The following assumptions are made to generate the governing equations:

- The plate and the flow are taken to be at rest initially
- Magnetic field is applied in the z direction
- The surface temperature is assumed to be more than the ambient temperature
- The base fluid and nano particles are at thermal equilibrium and having no slip
- Boussinesq and boundary layer approximations are considered

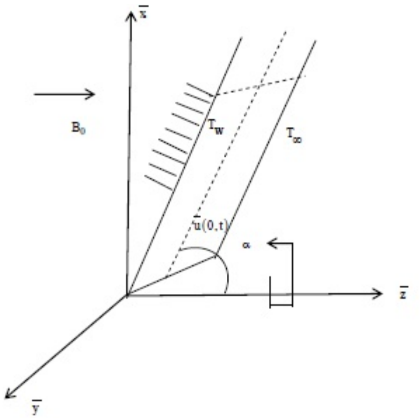

Figure 1: Schematic diagram.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial w}{\partial z} = 0 \tag{2}$$

$$\frac{\partial u}{\partial t} + w\frac{\partial u}{\partial z} = \frac{1}{\rho_{nf}}\left[\mu_{nf}\frac{\partial^2 u}{\partial z^2} + (\rho\beta_T)_{nf} g(T - T_\infty)\cos\gamma + (\rho\beta_C)_{nf} g(c - c_\infty)\cos\gamma - \sigma B_0^2 u\right] \tag{3}$$

$$\frac{\partial T}{\partial t} + w\frac{\partial T}{\partial z} = \alpha_{nf}\frac{\partial^2 T}{\partial z^2} - \frac{Q}{(\rho c_p)_{nf}}(T - T_\infty) - \frac{1}{(\rho c_p)_{nf}}\frac{\partial q_r}{\partial z} \tag{4}$$

$$\frac{\partial c}{\partial t} + w\frac{\partial c}{\partial z} = D_{nf}\frac{\partial^2 c}{\partial z^2} + k_l(c - c_\infty) \tag{5}$$

The following initial and boundary conditions are considered to solve the system of governing equations:

$u(z,t) = 0, T = T_\infty, c = c_\infty, t < 0, \forall z$
$u(0,t) = U_0[1 + \frac{\epsilon}{2}(e^{int} + e^{-int})], T(0,t) = T_w, c(0,t) = c_w, t \geq 0$
$u(\infty,t) \to 0, T(\infty,t) \to T_\infty, \epsilon < 0$ (6)

The Thermo-Physical properties are considered as:
$\rho_{nf} = (1-\phi)\rho_{nf} + \phi\rho_s, \alpha_{nf} = \frac{k_{nf}}{(\rho c_p)_{nf}},$

$(\rho c_p)_{nf} = (1-\phi)((\rho c_p)_{nf}) + \rho((\rho c_p)_s)$
$(\rho\beta)_{nf} = (1-\phi)(\rho\beta)_s, \frac{\mu_{nf}}{\mu_f} = 1 + 1.25\phi + 4.5[\frac{h}{d_p}(2+\frac{h}{d_p})(2+\frac{h}{d_p})^2]^{-1},$
$\frac{k_{nf}}{k_f} = 1 + 64.7\phi^{0.7460}(\frac{d_f}{d_p})^{0.3690}(\frac{k_p}{k_f})^{0.7476}(Pr)^{0.9955})(Re)^{0.9955}$ (7)

The solution of Equation (2) leads to

$$w = -w_0 \tag{8}$$

here w_0 represents the normal velocity at the plate, it is positive for suction $w_0 > 0$.
The following non dimensional variables are introduced to solve the generated system of equations:
$z = (\frac{v_f}{U_0})Z, t = (\frac{v_f}{U_0^2})\tau, n = (\frac{U_0^2}{v_f})\eta, u = UU_0, \theta = \frac{T-T_\infty}{T_w-T_\infty}, C = \frac{c-c_\infty}{c_w-c_\infty}$
$q_r = -\frac{4\sigma_1}{3\delta}\frac{\partial T^4}{\partial y}, Pr = \frac{v_f}{\alpha_f}, S = \frac{w_0}{U_0}, M = \frac{\sigma B_0^2 V_f}{\rho_f U_0^2}, Ra = \frac{4\alpha\sigma_1 T_\infty^3}{\delta k_{nf}}$
$Q_H = \frac{Qv_f^2}{k_f U_0^2}, K = \frac{K_l v_f}{U_0^2}, \frac{Gr}{Gc} = \frac{g\beta_{cf}(c_w-c_\infty)v_f}{g\beta_{Tf}(T_w-T_\infty)v_f},$
$U_0 = [g\beta(T_w-T_\infty)v_f]^{\frac{1}{3}}$ (9)
The temperature differences in the flow are assumed to be sufficiently small hence T^4can be expressed as a linear function. The following approximation is the expansion by using Taylors series about T_∞ and neglecting higher-order terms: $T^4 \cong 4T_\infty^3 T - 3T_\infty^4$
also,$\frac{\partial q_r}{\partial z} = -\frac{16\sigma_1}{3\delta}\frac{\partial^2 z T^4 T_\infty^3}{\partial Z^2}$ (10)
The dimensionless system of equations thus formed is:

$$\Big[1-\phi+\phi\Big(\frac{\rho_s}{\rho_f}\Big)\Big]\Big(\frac{\partial U}{\partial \tau} - S\frac{\partial U}{\partial Z}\Big) = 1 + 1.25\phi + 4.5\Big[\frac{h}{d_p}(2+\frac{h}{d_p})(1+\frac{h}{d_p})^2\Big]^{-1}$$
$$\frac{\partial^2 U}{\partial Z^2} + \Big[1-\phi+\phi\frac{(\rho\beta_T)_s}{(\rho\beta_T)_f}]Gr\theta\cos\gamma + \Big[1-\phi+\phi\frac{(\rho\beta_c)_s}{(\rho\beta_c)_f}]\frac{Gr}{Gc}C\theta\cos\gamma] - MU \tag{11}$$

$$\Big[1-\phi+\phi\Big(\frac{(\rho c_p)_s}{(\rho c_p)_f}\Big)\Big]\Big(\frac{\partial\theta}{\partial\tau} - S\frac{\partial\theta}{\partial Z}\Big) = \frac{1}{Pr}\Big[(1 + 64.7\phi^{0.7460}(\frac{d_r}{d_p})^{0.3690} +$$
$$(\frac{k_p}{k_r})^{0.7476}(Pr)^{0.9955}(\frac{1.381\times10^{-3}\times300}{3\times3.14\times0.738\times8.94\times10^{-4}})^{1.2321})\frac{\partial^2\theta}{\partial Z^2}] -$$
$$\frac{1}{Pr}Q_H\theta + \frac{1}{Pr}\frac{4}{3}\frac{1}{Ra}\frac{\partial^2\theta}{\partial Z^2} \tag{12}$$

$$(\frac{\partial C}{\partial\tau} - S\frac{\partial C}{\partial Z}) = \frac{1}{Sc}\frac{\partial^2 C}{\partial C^2} + KC \tag{13}$$

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and the resulted dimensionless boundary conditions are:
$U(z,t) = 0, \theta(z,t) = 0, c(z,t) = 0 for t < 0 \forall z$
$U(0,t) = U_0[1 + \frac{\epsilon}{2}(e^{int} + e^{-int})], \theta(0,t) = 1, c(0,t) = 1,$
$U(\infty,t) \to 0, \theta(\infty,t) \to 0, c(\infty,t) \to 0 \forall t \geq 0$
The local Nusselt number and local Sherwood number are evaluated using the following expressions:
$Nu = -\frac{k_{nf}}{k_f}\theta'(0), Sh = -\frac{v_f}{D_{nf}}C'(0)$

3 Solution of the Problem

The semi-infinite plate length is limited to 6 for computationsbecause the Diffusion reaches boundary at 6 for variation with Sc. We have generalized the semi-infinite plate length as 6by using Runge-Kuta 6th order Method. The convergences of the method are guaranteed by satisfaction of the boundary conditions.The standard values throughout the computations are: $\phi = 0.02, s = 1, h = 4, d_p = 40, Gc = 5, Gr = 5, M = 5, \gamma = \frac{\pi}{3}, QH = 5, Ra = 0.4, Sc = 0.6, K = 0.5$.

4 Results and Discussions

The effect of various parameters viz. solid volume fraction ϕ, thickness of liquid like layer around the solid particle (h), diameter of the solid particle d_p, magnetic parameter (M), inclination angle of the plate γ, heat source parameter Q_H, Schmidt number (Sc), suction parameter (s) and chemical reaction parameter (K) on velocity (U), temperature θ and diffusion (c) are exhibited in graphs from Figures 2 to 21. The other parameters were assumed to be constant. The Prandtl Number (Pr) kept constant as 7 (for water), $\epsilon = 0.02$ and $nt = \frac{\pi}{2}$. The velocity is found maximum near the base of the plate for all parameters and it is found that it drastically decreases as we move along the plate (Figures 2 to 12). The significance of the nano fluid has clearly observed from Figure 2. It is evident that the dissolved Cu nano-particles increase the momentum along the plate. The momentum further enhances with the increase in the solid portion of the nano-fluid because the flow enhances for 0 to 5 solid portion in the fluid. The increase in inter particle spacing (h) opposes the momentum and it is found from

Figure 3 that the increase in spacing from 2 nm to 10 nm reduces the momentum. The momentum increases with size of the particle (Figure 4). This may be due to the slip along the surface of the particle. Unlike micro particles the nano-particles are dynamic in nature and this nature increase with diameter of the particle (from 20nm to 100nm). The effect of magnetic field (M) reduces the momentum of the nano-fluid (Figure 4). The magnetization of the Cu nano-particle further opposes the motion of the fluid because the absence of the magnetic field rapidly enhancing the motion of the fluid. The momentum boundary layer is reducing with the increase in the inclination angle (Figure 6). The nano-fluid flow is more when the plate is tending to be horizontal. The heat source (QH) opposes the nano-fluid flow (Figure 6). This is due to various factors like evaporation of liquid, increase in size of nano-particle and Brownian motion of particles etc. The fluid flow reduces with decrease in the diffusivity (Sc) (Figure 8). The destructive chemical reaction $K > 0$ enhances the momentum and the generative chemical reaction $K < 0$ reduces the momentum when compared with no chemical reaction (Figure 9). The increase in suction reduces the momentum (Figure 9) as the plate is permeable. The increase in molecular Grashof number (Gc) reduces the momentum (Figure 11). The increase in thermal Grashof number (Gr) reduces the momentum (Figure 12). The temperature profiles (Figures 13-18) shows that variation of temperature is more with inter particle spacing of nano-particles (h), diameter of the particle d_p, heat source(QH), Radiation(Ra) and suction(s). The temperature is high near the base of the plate and decreases rapidly as we move along the plate. Temperature enhances with the dispersion of Cu particles (Figure 13). It is observed that the temperature is more for $0\% to 5\%$ of particle dispersion in nano-fluid. The inter particle spacing enhances the temperature (Figure 14). The spacing of 2nm to 10nm is studied; transfer of temperature distribution is increased. The variation of temperature for various Cu nano-particle sizes (dp = 20nm - 100nm) is depicted in Figure 15. The temperature enhances with size of the particle, which is basic advantage of nano-fluid. The temperature decreases with increase in the heat source (QH) from Figure 16. The variation of temperature is more for radiation (Ra) is around 0.1 but the variation is less when the radiation is above 0.4 (Figure 17). The temperature decreases with increase in radiation. The temperature is more for no suction, and decreases with increase in suction parameter (s) from Figure 18. The variation of diffusion for various parameters is shown in Figures 19 21. The variation of diffusion is found more for chemical reaction (K), Schmidt (Sc) and suction (S). The diffusion decreases with increase in Sc and diffusion is very less when Sc is around 1.3 (Figure 19). Diffusion is less for generative chemical reaction and more for destructive chemical reaction (K) and moderate for no reaction (Figure 20). Diffusion is more for no suction, less for suction, as s is more than or equal to 1 (Figure 21).

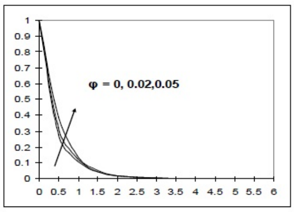

Figure 2: Profiles of U with ϕ

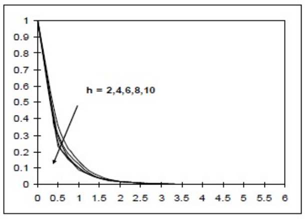

Figure 3: Profiles of U with h

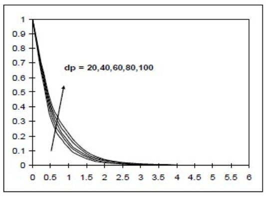

Figure 4: Profiles of U with d_p

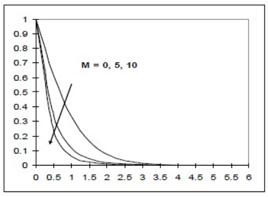

Figure 5: Profiles of U with M

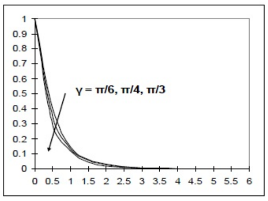

Figure 6: Profiles of U with γ

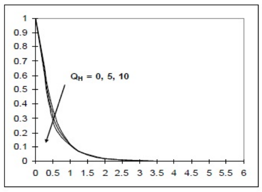

Figure 7: Profiles of U with QH

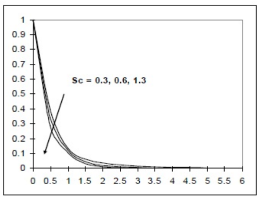

Figure 8: Profiles of U with Sc

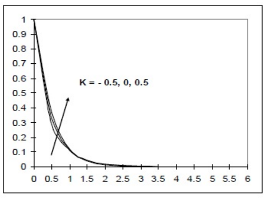

Figure 9: Profiles of U with K

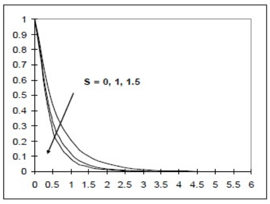

Figure 10: Profiles of U with S

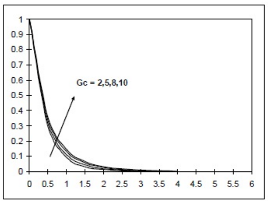

Figure 11: Profiles of U with Gc

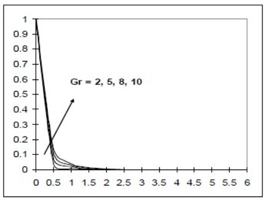

Figure 12: Profiles of U with Gr

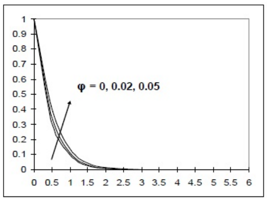

Figure 13: Profiles of θ with ϕ

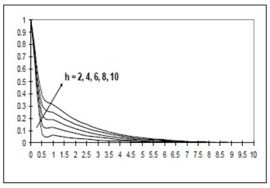

Figure 14: Profiles of θ with h

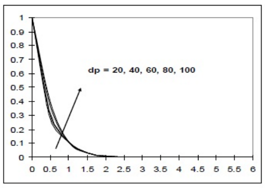

Figure 15: Profiles of θ with dp

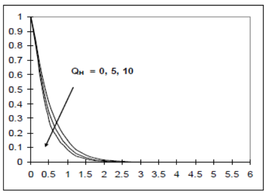

Figure 16: Profiles of θ with QH

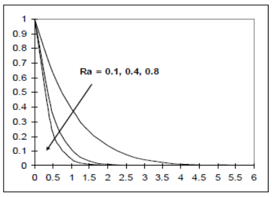

Figure 17: Profiles of θ with Ra

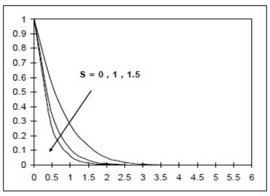

Figure 18: Profiles of θ with S

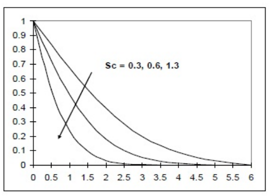

Figure 19: Profiles of C with Sc

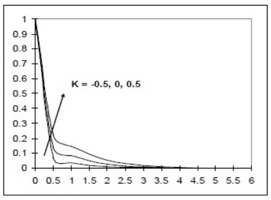

Figure 20: Profiles of C with K

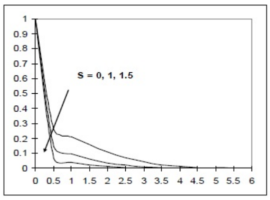

Figure 21: Profiles of C with S

Table:1 depicts the rate of heat transfer for various parameters , dp, QH and S. Each column of the table signifies the importance of metal particles for heat transfer. Altogether 4-5% of heat transfer enhancement is noticed for every 1% enhancement of the cu particles in water. The heat transfer rate is more significant about average size of the particle (60 nm), for large size $K > 60nm$ the heat transfer rate slightly increases. The natural increase in heat transfer is observed for enhancement of heat source. The suction enhances the heat transfer rate due to the quick movement of the cu particles.

	d_p	20	60	100	60	60	60	60
	Q_H	5	5	5	10	15	5	5
ϕ	S	0.5	0.5	0.5	0.5	0.5	0.5	0.5
0		1.7426	1.7426	1.7426	2.1265	2.4546	1.4212	2.1079
0.05		1.7362	1.7359	1.7362	2.1207	2.4493	1.4176	2.0978
0.1		1.7298	1.7298	1.7295	2.1155	2.4445	1.4144	2.0873

Table 1: Nusselt Number Values

Table-2 depicts the Sherwood number (Mass Transfer coefficient) for variation of K, Sc and S. The rate of mass transfer is significantly high for generative $K < 0$ chemical reaction on the other hand the rate of mass transfer is significantly low for destructive $K > 0$ chemical reaction when both the cases are compared with no chemical reaction. Naturally the rate of mass transfer is more for more diffusivity (Sc) for all kinds of chemical reactions. The increase in suction (S) shows a gradual enhancement of the diffusivity during all kinds of chemical reaction.

K	Sc=0.3,S=0.5	Sc=0.6,S=0.5	Sc=0.3,S=0.2	Sc=0.3,S=0.8
-0.5	0.53187	0.80307	0.48272	0.58453
0	0.38990	0.60345	0.34005	0.44384
0.5	0.22224	0.36791	0.17144	0.2779

Table 2: Sherwood Number Values

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