

Fuzzy e^* -continuity and e^* -open Mappings in $\hat{S}$ ostak's Fuzzy Topological Spaces

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Abstract

We introduce e^* -continuous mapping and e^* -open (resp. e^* -closed) mapping in fuzzy topological spaces in the function of $\hat{S}$ ostak's.

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1 Introduction and preliminaries

$\hat{S}$ ostak [?] introduced the fuzzy topology as an extension of Chang's fuzzy topology [?]. It has been developed in many directions [?, ?, ?]. Weaker forms of fuzzy continuity between fuzzy topological spaces have been considered by many authors [?, ?, ?, ?, ?, ?, ?] using the concepts of fuzzy semi-open sets [?], fuzzy regular open sets [?], fuzzy preopen sets, fuzzy strongly semiopen sets [?], fuzzy γ -open sets [?], fuzzy δ -semiopen sets [?], fuzzy δ -preopen sets [?], fuzzy semi δ -preopen sets [?] and fuzzy e -open sets [?]. Recently, Bin Shahna [?] introduced and investigated fuzzy strong semi-continuity and fuzzy precontinuity between fuzzy topological spaces, one of which was independent and the other strictly stronger than fuzzy semi-continuity [?]. Ganguly and Saha [?] introduced the notions of fuzzy δ -cluster points in fuzzy topological spaces in the sense of Chang [?]. Kim and Park [?] introduced r - δ -cluster points and δ -closure operators in fuzzy topological spaces in view of the definition of $\hat{S}$ ostak. It is a good extension of the notions of Ganguly

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and Saha [?]. Park et al. [?] introduced the concept of fuzzy semi-preopen sets which is weaker than any of the concepts of fuzzy semi-open or fuzzy preopen sets. Using these concepts he defined and studied fuzzy semi-precontinuous mappings between fuzzy topological spaces in Chang's sense. Sobana et al. [?], defined r -fuzzy e -open and r -fuzzy e -closed sets in a fuzzy topological space in the sense of Šostak. In 2008, the initiations of e^* -open sets in topological spaces was introduced by Erdal Ekici [?]. Recently, Vadivel et. al. [?], defined the concept of r -fuzzy e^* -open and r -fuzzy e^* -closed sets in Šostak's fuzzy topological spaces. In this paper, we define fuzzy e^* -continuous and fuzzy e^* -open mappings in a fuzzy topological space in the sense of Šostak [?].

Throughout this paper, nonempty sets will be denoted by X, Y etc., $I = [0, 1]$ and $I_0 = (0, 1]$. For $\alpha \in I$, $\bar{\alpha}(x) = \alpha$ for all $x \in X$. A fuzzy point x_t for $t \in I_0$ is an element of I^X such that $x_t(y) = \begin{cases} t & \text{if } y = x \\ 0 & \text{if } y \neq x. \end{cases}$ The set of all fuzzy points in X is denoted by $Pt(X)$. A fuzzy point $x_t \in \lambda$ iff $t < \lambda(x)$. A fuzzy set λ is quasi-coincident with μ , denoted by $\lambda q \mu$, if there exists $x \in X$ such that $\lambda(x) + \mu(x) > 1$. If λ is not quasi-coincident with μ , we denoted $\lambda \bar{q} \mu$. If $A \subset X$, we define the characteristic function χ_A on X by $\chi_A(x) = \begin{cases} 1 & \text{if } x \in A, \\ 0 & \text{if } x \notin A. \end{cases}$

All other notations and definitions are standard, for all in the fuzzy set theory.

Lemma 1.1. [?] Let X be a nonempty set and $\gamma, \delta \in I^X$. Then

- (i) $\gamma q \delta$ iff there exists $x_t \in \gamma$ such that $x_t q \delta$.
- (ii) $\gamma q \delta$, then $\gamma \wedge \delta \neq \underline{0}$.
- (iii) $\gamma \bar{q} \delta$ iff $\gamma \leq \underline{1} - \delta$.
- (iv) $\gamma \leq \delta$ iff $x_t \in \gamma$ implies $x_t \in \delta$ iff $x_t q \gamma$ implies $x_t q \delta$ implies $x_t \bar{q} \gamma$.
- (v) $x_t \bar{q} \bigvee_{i \in \Gamma} \mu_i$ iff there exists $i_0 \in \Gamma$ such that $x_t \bar{q} \mu_{i_0}$.

Definition 1.1. [?] A function $\tau : I^X \rightarrow I$ is called a fuzzy topology on X if it satisfies the following conditions:

- (1) $\tau(\underline{0}) = \tau(\underline{1}) = 1$,
- (2) $\tau(\bigvee_{i \in \Gamma} \mu_i) \geq \bigwedge_{i \in \Gamma} \tau(\mu_i)$, for any $\{\mu_i\}_{i \in \Gamma} \subset I^X$,
- (3) $\tau(\mu_1 \wedge \mu_2) \geq \tau(\mu_1) \wedge \tau(\mu_2)$, for any $\mu_1, \mu_2 \in I^X$.

The pair (X, τ) is called a fuzzy topological space or smooth topological space (for short, fts or sts).

Remark 1.1. [?] Let (X, τ) be a sts. Then, every $r \in I_0$, $\tau_r = \{\delta \in I^X : \tau(\delta) \geq r\}$ is a Change's fuzzy topology on X .

Theorem 1.1. [?] Let (X, τ) be a sts. Then for each $\gamma \in I^X$, $r \in I_0$ we define an operator $C_\tau : I^X \times I_0 \rightarrow I^X$ as follows: $C_\tau(\gamma, r) = \bigwedge \{\delta \in I^X : \gamma \leq \delta, \tau(\underline{1} - \delta) \geq r\}$. For $\gamma, \delta \in I^X$ and $r, s \in I_0$, the operator C_τ satisfies the following conditions: (1) $C_\tau(\underline{0}, r) = \underline{0}$, (2) $\gamma \leq C_\tau(\gamma, r)$, (3) $C_\tau(\gamma, r) \vee C_\tau(\delta, r) = C_\tau(\gamma \vee \delta, r)$, (4) $C_\tau(\gamma, r) \leq C_\tau(\gamma, s)$ if $r \leq s$, (5) $C_\tau(C_\tau(\gamma, r), r) = C_\tau(\gamma, r)$.

Theorem 1.2. [?] Let (X, τ) be a sts. Then for each $r \in I_0$, $\gamma \in I^X$ we define an operator $I_\tau : I^X \times I_0 \rightarrow I^X$ as follows: $I_\tau(\gamma, r) = \bigvee \{\delta \in I^X : \gamma \geq \delta, \tau(\delta) \geq r\}$. For $\gamma, \delta \in I^X$ and $r, s \in I_0$, the operator I_τ satisfies the following conditions: (1) $I_\tau(\underline{1}, r) = \underline{1}$, (2) $\gamma \geq I_\tau(\gamma, r)$, (3) $I_\tau(\gamma, r) \wedge I_\tau(\delta, r) = I_\tau(\gamma \wedge \delta, r)$, (4) $I_\tau(\gamma, r) \leq I_\tau(\gamma, s)$ if $s \leq r$, (5) $I_\tau(I_\tau(\gamma, r), r) = I_\tau(\gamma, r)$, (6) $I_\tau(\underline{1} - \gamma, r) = \underline{1} - C_\tau(\gamma, r)$ and $C_\tau(\underline{1} - \gamma, r) = \underline{1} - I_\tau(\gamma, r)$.

Definition 1.2. [?] Let (X, τ) be a sts, every $\delta \in I^X$, $x_t \in P_t(X)$ and $r \in I_0$,

- (i) δ is called r -open Q_τ -neighbourhood of x_t if $x_t q \delta$ with $\tau(\delta) \geq r$.
- (ii) δ is called r -open R_τ -neighbourhood of x_t if $x_t q \delta$ with $\delta = I_\tau(C_\tau(\gamma, r), r)$.
We denote $Q_\tau(x_t, r) = \{\delta \in I^X : x_t q \delta, \tau(\delta) \geq r\}$, $R_\tau(x_t, r) = \{\delta \in I^X : x_t q \delta = I_\tau(C_\tau(\gamma, r), r)\}$.

Definition 1.3. [?] Let (X, τ) be a sts, every $\gamma \in I^X$, $x_t \in P_t(X)$ and $r \in I_0$, x_t is called

- (i) r - τ cluster point of γ if for every $\delta \in Q_\tau(x_t, r)$, we have $\delta q \gamma$.
- (ii) r - δ cluster point of γ if for every $\delta \in R_\tau(x_t, r)$, we have $\delta q \gamma$.
- (iii) An δ -closure operator is a mapping $D_\tau : I^X \times I \rightarrow I^X$ defined as follows:
 $\delta - C_\tau(\gamma, r)$ or $D_\tau(\gamma, r) = \bigvee \{x_t \in P_t(X) : x_t \text{ is } r\text{-}\delta\text{-cluster point of } \gamma\}$

Definition 1.4. Let (X, τ) be a sts. For $\gamma \in I^X$ and $r \in I_0$, γ is called an

- (i) r -fuzzy semiopen (resp. r -fuzzy semi-closed)(in short r -fso)(resp. r -fsc) [?]
set if $\gamma \leq C_\tau(I_\tau(\gamma, r), r)$ (resp. $I_\tau(C_\tau(\gamma, r), r) \leq \gamma$).

- (ii) r -fuzzy preopen (resp. r -fuzzy preclosed)(in short r -fpo)(resp. r -fpc) [?] set if $\gamma \leq I_\tau(C_\tau(\gamma, r), r)$ (resp. $C_\tau(I_\tau(\gamma, r), r) \leq \gamma$).
- (iii) r -fuzzy δ -closed [?] iff $\gamma = D_\tau(\gamma, r)$.

The complement of r -fso (resp. r -fpo, r -fspc and r -f δ c) is r -fsc (resp. r -fpc, r -fspc and r -f δ o).

Definition 1.5. Let (X, τ) be a sts. $\gamma, \delta \in I^X$ and $r \in I_0$, γ is called an

- (i) r -f δ so (resp. r -f δ sc) [?] set if $\gamma \leq C_\tau(\delta - I_\tau(\gamma, r), r)$ (resp. $I_\tau(\delta - C_\tau(\gamma, r), r) \leq \gamma$).
- (ii) r -f δ po (resp. r -f δ pc) [?] set if $\gamma \leq I_\tau(\delta - C_\tau(\gamma, r), r)$ (resp. $C_\tau(\delta - I_\tau(\gamma, r), r) \leq \gamma$).
- (iii) r -f e o (resp. r -f e c) [?] set if $\gamma \leq C_\tau(\delta - I_\tau(\gamma, r), r) \vee I_\tau(\delta - C_\tau(\gamma, r), r)$ (resp. $C_\tau(\delta I_\tau(\gamma, r), r) \wedge I_\tau(\delta C_\tau(\gamma, r), r) \leq \gamma$).
- (iv) r -f β o (resp. r -f β c) [?] set if $\gamma \leq C_\tau(I_\tau(C_\tau(\gamma, r), r), r)$ (resp. $I_\tau(C_\tau(I_\tau(\gamma, r), r), r) \leq \gamma$).

Definition 1.6. [?] Let (X, τ) be a sts. For $\gamma, \delta \in I^X$ and $r \in I_0$, γ is called an r -fuzzy e^* -open (resp. r -fuzzy e^* -closed) set if $\gamma \leq C_\tau(I_\tau(\delta - C_\tau(\gamma, r), r), r)$ (resp. $I_\tau(C_\tau(\delta - I_\tau(\gamma, r), r), r) \leq \gamma$).

Definition 1.7. [?] Let (X, τ) be a sts. $\gamma, \delta \in I^X$ and $r \in I_0$,

- (i) $e^*I_\tau(\gamma, r) = \bigvee \{ \delta \in I^X : \delta \leq \gamma, \delta \text{ is a } r\text{-fe}^*\text{o set} \}$ is called the r -fuzzy e^* -interior of γ .
- (ii) $e^*C_\tau(\gamma, r) = \bigwedge \{ \delta \in I^X : \delta \geq \gamma, \delta \text{ is a } r\text{-fe}^*\text{c set} \}$ is called the r -fuzzy e^* -closure of γ .

Lemma 1.2. [?] The following hold for a subset γ of a sts X .

- (i) $e^*C_\tau(\gamma, r)$ is r -fe * c.
- (ii) $1 - e^*C_\tau(\gamma, r) = e^*I_\tau(1 - (\gamma, r))$.

Theorem 1.3. [?] Let (X, τ) be a sts and $r \in I_0$.

- (i) Any union of r -fe * o sets is an r -fe * o set.

(ii) Any intersection of r -fe*c sets is an r -fe*c set.

Theorem 1.4. [?] Let (X, τ) be a sts, For $\gamma, \delta \in I^X$ and $r \in I_0$.

- (i) γ is r -fe*o iff $\underline{1} - \gamma$ is r -fe*c.
- (ii) If $\tau(\gamma) \geq r$ then γ is r -fe*o set.
- (iii) $I_\tau(\gamma, r)$ is an r -fe*o set.
- (iv) $C_\tau(\gamma, r)$ is an r -fe*c set.

Theorem 1.5. [?] Let (X, τ) be a sts. Let $\gamma \in I^X$ and $r \in I_o$.

- (i) γ is r -fe*o iff $\gamma = e^*I_\tau(\gamma, r)$.
- (ii) γ is r -fe*c iff $\gamma = e^*C_\tau(\gamma, r)$.

Theorem 1.6. [?] Let (X, τ) be a sts. Let $\gamma \in I^X$ and $r \in I_o$, then

- (i) $e^*C_\tau(0, r) = 0, e^*I_\tau(1, r) = 1$.
- (ii) $I_\tau(\gamma, r) \leq e^*I_\tau(\gamma, r) \leq \gamma \leq e^*C_\tau(\gamma, r) \leq C_\tau(\gamma, r)$.
- (iii) $\gamma \leq \delta \Rightarrow e^*I_\tau(\gamma, r) \leq e^*I_\tau(\delta, r)$ and $e^*C_\tau(\gamma, r) \leq e^*C_\tau(\delta, r)$.
- (iv) $e^*C_\tau(\gamma, r) \vee e^*C_\tau(\delta, r) \leq e^*C_\tau(\gamma \vee \delta, r)$.
- (v) $e^*C_\tau(e^*C_\tau(\gamma, r), r) = e^*C_\tau(\gamma, r)$ and $e^*I_\tau(e^*I_\tau(\gamma, r), r) = e^*I_\tau(\gamma, r)$.
- (vi) $C_\tau(e^*C_\tau(\gamma, r), r) = e^*C_\tau(C_\tau(\gamma, r), r) = C_\tau(\gamma, r)$.
are holds.

Theorem 1.7. [?] Let (X, τ) be a sts, having $\gamma \in I^X$ and $r \in I_0$ we have

- (i) $e^*I_\tau(1 - \gamma, r) = 1 - (e^*C_\tau(\gamma, r))$.
- (ii) $e^*C_\tau(1 - \gamma, r) = 1 - (e^*I_\tau(\gamma, r))$.

Definition 1.8. Let (X, τ_1) and (X, τ_2) be sts's and $f : (X, \tau_1) \rightarrow (Y, \tau_2)$ a mapping. Then f is called

- (i) fuzzy continuous(in short f-cts) [?] iff $\tau_2(\delta) \leq \tau_1(f^{-1}(\delta))$ for each $\delta \in I^Y$.

- (ii) δ -semi (resp. δ -pre, e and β)-continuous (in short $f\delta s$ -cts, $f\delta p$ -cts, fe -cts and $f\beta$ -cts [?]) iff $f^{-1}(\delta)$ is r - $f\delta so$ (resp. r - $f\delta po$, r - feo and r - $f\beta o$) of X for each $\delta \in I^X$, $r \in I_0$ with $\tau_2(\delta) \geq r$.

Definition 1.9. Let (X, τ_1) and (Y, τ_2) be sts's and $f : (X, \tau_1) \rightarrow (Y, \tau_2)$ be a mapping. Then f is called fuzzy

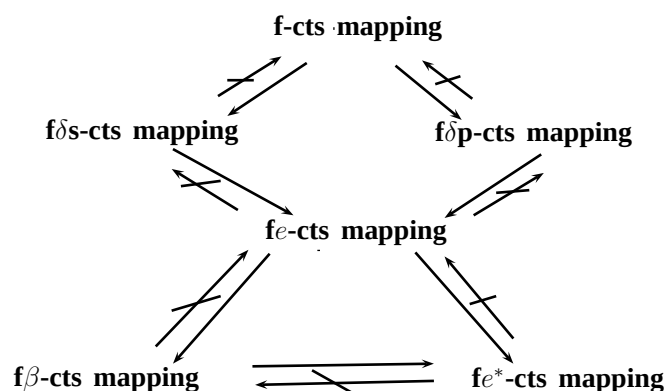
- (i) δ -semiopen (resp. fuzzy δ -preopen, fuzzy e -open and β -open)(in short, $f\delta so$, $f\delta po$, feo and $f\beta o$) [?] map iff $f(\gamma)$ is r - $f\delta so$ (resp. r - $f\delta po$, r - feo and r - $f\beta o$) set of Y for every $\gamma \in I^X$, $r \in I_0$ with $\tau_1(\gamma) \geq r$
- (ii) δ -semiclosed (resp. fuzzy δ -preclosed, fuzzy e -closed and β -closed)(in short, $f\delta sc$, $f\delta pc$, fec and $f\beta c$) [?] map iff $f(\gamma)$ is r - $f\delta sc$ (resp. r - $f\delta pc$, r - fec and r - $f\beta c$) set of Y for every $\gamma \in I^X$, $r \in I_0$ with $\tau_1(1 - \gamma) \geq r$.

2 Fuzzy e^* -continuous mappings

We introduce the following definition namely fuzzy e^* -continuous.

Definition 2.1. Let (X, τ_1) and (Y, τ_2) be sts's and the mapping $g : (X, \tau_1) \rightarrow (Y, \tau_2)$. Then g is called fuzzy e^* -continuous (in short fe^* -cts) iff $g^{-1}(\delta)$ is r - fe^*o of X for each $\delta \in I^X$, $r \in I_0$ with $\tau_2(\delta) \geq r$.

Remark 2.1. The below implications are true for $r \in I_0$.



where f -cts, $f\delta s$ -cts, $f\delta p$ -cts, fe -cts, $f\beta$ -cts and fe^* -cts are abbreviated by fuzzy-cts, $f\delta$ -semi-cts, $f\delta$ -pre-cts, fe -cts, $f\beta$ -cts and fe^* -cts mappings respectively.

Remark 2.2. From the above definitions, it is evident that every $f\delta p$ -cts and $f\delta s$ -cts is fe -cts. Also, it is clear that each fe -cts is $f\beta$ -cts and fe^* -cts. Also, each $f\beta$ -cts is fe^* -cts. In general it may not be true for converse.

Below examples represents that the inverse is not true.

Example 2.1 Having μ_1, μ_2, μ_3 and μ_4 be fuzzy subsets of $X = \{p, q\}$ represented as

$$\begin{aligned}\mu_1(p) &= 0.2, \mu_1(q) = 0.1; \\ \mu_2(p) &= 0.3, \mu_2(q) = 0.5; \\ \mu_3(p) &= 0.7, \mu_3(q) = 0.7; \\ \mu_4(p) &= 0.2, \mu_4(q) = 0.8.\end{aligned}$$

Then $\tau, \eta : I^X \rightarrow I$ defined as

$$\tau(\delta) = \begin{cases} 1, & \text{if } \gamma = \underline{0} \text{ or } \underline{1}, \\ \frac{1}{2}, & \text{if } \delta = \mu_1, \mu_2, \mu_3, \\ 0, & \text{or else,} \end{cases} \quad \eta(\gamma) = \begin{cases} 1, & \text{if } \gamma = \underline{0} \text{ or } \underline{1}, \\ \frac{1}{2}, & \text{if } \gamma = \mu_4, \\ 0, & \text{or else.} \end{cases}$$

to be sts on X . $g : (X, \tau) \rightarrow (Y, \eta)$ is an identity mapping. Take $r = \frac{1}{2}$. For any $\frac{1}{2}$ -fuzzy open set (in short, fo set) μ_4 in (Y, η) , $g^{-1}(\mu_4) = \mu_4$ is fuzzy $\frac{1}{2}$ - β -open set in (X, τ) . Then g is $f\beta$ -cts, however g is not fe -cts, considering $g^{-1}(\mu_4)$ is not $\frac{1}{2}$ -feo in (X, τ) .

Example 2.2 Having γ and δ be fuzzy subsets of $X = \{p, q, r\}$ represented as

$$\begin{aligned}\gamma(p) &= 0.4, \gamma(q) = 0.5, \gamma(r) = 0.5; \\ \delta(p) &= 0.4, \delta(q) = 0.5, \delta(r) = 0.4.\end{aligned}$$

Then $\tau, \eta : I^X \rightarrow I$ defined as

$$\tau(\gamma) = \begin{cases} 1, & \text{if } \gamma = \underline{0} \text{ or } \underline{1}, \\ \frac{1}{2}, & \text{if } \gamma = \gamma, \\ 0, & \text{or else,} \end{cases} \quad \eta(\gamma) = \begin{cases} 1, & \text{if } \gamma = \underline{0} \text{ or } \underline{1}, \\ \frac{1}{2}, & \text{if } \gamma = \delta, \\ 0, & \text{or else.} \end{cases}$$

to be sts on X . $g : (X, \tau) \rightarrow (Y, \eta)$ is an identity mapping. Take $r = \frac{1}{2}$. For any $\frac{1}{2}$ -fo set δ in (Y, η) , $g^{-1}(\delta) = \delta$ is fuzzy $\frac{1}{2}$ - e -open set in (X, τ) . Then g is fe -cts, however g is not $f\delta s$ -cts, considering $g^{-1}(\delta)$ is not fuzzy $\frac{1}{2}$ - $f\delta so$ set in (X, τ) .

Example 2.3 Having γ and δ be fuzzy subsets of $X = \{p, q, r\}$ represented as

$$\begin{aligned}\gamma(p) &= 0.5, \gamma(q) = 0.3, \gamma(r) = 0.2; \\ \delta(p) &= 0.5, \delta(q) = 0.4, \delta(r) = 0.4.\end{aligned}$$

Then $\tau, \eta : I^X \rightarrow I$ defined as

$$\tau(\gamma) = \begin{cases} 1, & \text{if } \gamma = \underline{0} \text{ or } \underline{1}, \\ \frac{1}{2}, & \text{if } \gamma = \gamma, \\ 0, & \text{or else,} \end{cases} \quad \eta(\gamma) = \begin{cases} 1, & \text{if } \gamma = \underline{0} \text{ or } \underline{1}, \\ \frac{1}{2}, & \text{if } \gamma = \delta, \\ 0, & \text{or else.} \end{cases}$$

to be sts on X . $g : (X, \tau) \rightarrow (Y, \eta)$ is an identity mapping. Take $r = \frac{1}{2}$. For any $\frac{1}{2}$ -fo set δ in (Y, η) , $g^{-1}(\delta) = \delta$ is $\frac{1}{2}$ -feo in (X, τ) . Then g is fe-cts, however g is not fdp-cts, having $g^{-1}(\delta)$ is not $\frac{1}{2}$ -fdpo set in (X, τ) .

Example 2.4 Having γ, δ and ω be fuzzy subsets of $X = \{p, q, r\}$ represented as

$$\begin{aligned} \gamma(p) &= 0.3, \quad \gamma(q) = 0.5, \quad \gamma(r) = 0.2; \\ \delta(p) &= 0.4, \quad \delta(q) = 0.5, \quad \delta(r) = 0.5; \\ \omega(p) &= 0.7, \quad \omega(q) = 0.4, \quad \omega(r) = 0.8. \end{aligned}$$

Then $\tau, \eta : I^X \rightarrow I$ defined as

$$\tau(\gamma) = \begin{cases} 1, & \text{if } \gamma = \underline{0} \text{ or } \underline{1}, \\ \frac{1}{2}, & \text{if } \gamma = \gamma, \delta, \\ 0, & \text{or else,} \end{cases} \quad \eta(\gamma) = \begin{cases} 1, & \text{if } \gamma = \underline{0} \text{ or } \underline{1}, \\ \frac{1}{2}, & \text{if } \gamma = \omega, \\ 0, & \text{or else.} \end{cases}$$

be a sts on X . $g : (X, \tau) \rightarrow (Y, \eta)$ is an identity mapping. Take $r = \frac{1}{2}$. For any $\frac{1}{2}$ -fo set ω in (Y, η) , $g^{-1}(\omega) = \omega$ is $\frac{1}{2}$ -fe*o set in (X, τ) . Then g is fe*-cts, however g is not f β -cts, having $g^{-1}(\omega)$ is not $\frac{1}{2}$ -f β o set in (X, τ) .

Example 2.5 Having γ and δ be fuzzy subsets of $X = \{p, q, r\}$ represented as

$$\begin{aligned} \gamma(p) &= 0.4, \quad \gamma(q) = 0.5, \quad \gamma(r) = 0.2; \\ \delta(p) &= 0.5, \quad \delta(q) = 0.4, \quad \delta(r) = 0.7. \end{aligned}$$

Then $\tau, \eta : I^X \rightarrow I$ defined as

$$\tau(\gamma) = \begin{cases} 1, & \text{if } \gamma = \underline{0} \text{ or } \underline{1}, \\ \frac{1}{2}, & \text{if } \gamma = \gamma, \\ 0, & \text{or else,} \end{cases} \quad \eta(\gamma) = \begin{cases} 1, & \text{if } \gamma = \underline{0} \text{ or } \underline{1}, \\ \frac{1}{2}, & \text{if } \gamma = \delta, \\ 0, & \text{or else.} \end{cases}$$

be a sts on X . $g : (X, \tau) \rightarrow (Y, \eta)$ identity mapping. Take $r = \frac{1}{2}$. For any $\frac{1}{2}$ -fos δ in (Y, η) , $g^{-1}(\delta) = \delta$ is $\frac{1}{2}$ -fe*o set in (X, τ) . Then g is fe*-cts

(i) however g is not fe-cts, since $g^{-1}(\delta)$ is not $\frac{1}{2}$ -feo set in (X, τ) .

(ii) however g is not f-cts, since $\delta \in I^Y$, $\eta(\delta) \not\leq \tau(g^{-1}(\delta))$.

Theorem 2.1. Let (X, τ_1) and (Y, τ_2) be sts's and the mapping $g : X \rightarrow Y$. Then

- (i) g is fe*-cts mapping.

- (ii) $g^{-1}(\delta)$ is r -fe*c in X for every $\gamma \in I^Y$, $r \in I_0$ with $\tau_2(1 - \delta) \geq r$.
- (iii) $g(e^*C_{\tau_1}(\gamma, r)) \leq C_{\tau_2}(g(\gamma), r) \forall \gamma \in I^X$ and $r \in I_0$.
- (iv) $e^*C_{\tau_1}(g^{-1}(\delta), r) \leq g^{-1}(C_{\tau_2}(\delta, r))$, $\forall \delta \in I^Y$ and $r \in I_0$.
- (v) $I_{\tau_1}(C_{\tau_1}(\delta - I_{\tau_1}(g^{-1}(\delta), r), r), r) \leq g^{-1}(C_{\tau_2}(\delta, r))$, $\forall \delta \in I^Y$ and $r \in I_0$. are equivalent.

Proof. (i) $\Rightarrow$ (ii): Let $\delta \in I^Y$, $r \in I_0$ with $\tau_2(1 - \delta) \geq r$. Then by (i), $g^{-1}(1 - \delta)$ is an r -fe*o set of X . But $g^{-1}(1 - \delta) = 1 - g^{-1}(\delta)$. Therefore $g^{-1}(\delta)$ is an r -fe*c set of X .

(ii) $\Rightarrow$ (iii): Let $\gamma \in I^X$, $r \in I_0$, since $\tau_2(1 - C_{\tau_2}(g(\gamma), r)) \geq r$. From the definition of C_{τ_2} in Theorem ?? and from Definition ??(3),

$$g^{-1}(1 - C_{\tau_2}(g(\gamma), r)) = 1 - g^{-1}(C_{\tau_2}(g(\gamma), r))$$

is an r -fe*o set of X . By Theorem ??(i) $g^{-1}(C_{\tau_2}(g(\gamma), r))$ is an r -fe*c set of X . Since $\gamma \leq g^{-1}(g(\gamma)) \leq g^{-1}(C_{\tau_2}(g(\gamma), r))$, we have $eC_{\tau_1}(\gamma, r) \leq g^{-1}(C_{\tau_2}(g(\gamma), r))$. Hence $g(e^*C_{\tau_1}(\gamma, r)) \leq C_{\tau_2}(g(\gamma), r)$.

(iii) $\Rightarrow$ (iv): each $\delta \in I^Y$, $r \in I_0$, let $\gamma = g^{-1}(\delta)$. By (iii)

$$g(e^*C_{\tau_1}(g^{-1}(\delta), r)) \leq C_{\tau_2}(g(g^{-1}(\delta)), r) \leq C_{\tau_2}(\delta, r).$$

$$\Rightarrow e^*C_{\tau_1}(g^{-1}(\delta), r) \leq g^{-1}(C_{\tau_2}(\delta, r)).$$

(iv) $\Rightarrow$ (i): Let $\delta \in I^Y$, $r \in I_0$ with $\tau_2(\delta) \geq r$. By (iv)

$$e^*C_{\tau_1}(g^{-1}(1 - \delta), r) \leq g^{-1}(C_{\tau_2}(1 - \delta, r)) = g^{-1}(1 - \delta).$$

By Theorem ??, we have

$$g^{-1}(1 - \delta) \geq e^*C_{\tau_1}(g^{-1}(1 - \delta), r) = 1 - (e^*I_{\tau_1}(g^{-1}(\delta), r))$$

. Therefore $g^{-1}(\delta)$ is r -fe*o set in X .

(ii) $\Rightarrow$ (v): For all $\delta \in I^Y$, $r \in I_0$, since $\tau_2(1 - C_{\tau_2}(\delta, r)) \geq r$. Then by (ii) we see that $g^{-1}(C_{\tau_2}(\delta, r))$ is r -fe*c in X . Therefore

$$g^{-1}(C_{\tau_2}(\delta, r)) \geq I_{\tau_1}(C_{\tau_1}(g^{-1}(\delta - I_{\tau_2}(\delta, r)), r), r) \geq I_{\tau_1}(C_{\tau_1}(\delta - I_{\tau_2}(g^{-1}(\delta), r), r), r).$$

(v) $\Rightarrow$ (ii): Since all $\delta \in I^Y$, $r \in I_0$, with $\tau_2(1 - \delta) \geq r$. Then by (v)

$$I_{\tau_1}(C_{\tau_1}(\delta - I_{\tau_2}(g^{-1}(\delta), r), r), r) \leq g^{-1}(C_{\tau_2}(\delta, r)) = g^{-1}(\delta).$$

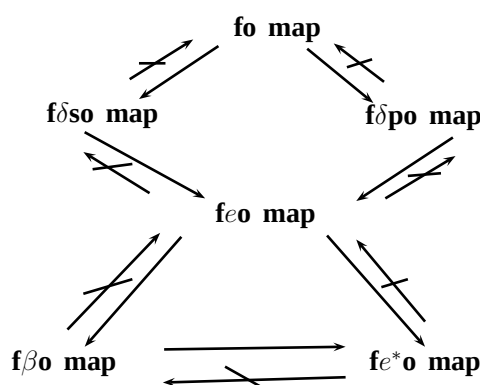
Hence $g^{-1}(\delta)$ is r -fe*c in X .

3 Fuzzy e^* -open and fuzzy e^* -closed mappings

Definition 3.1. Let (X, τ_1) and (Y, τ_2) be sts's and $g : (X, \tau_1) \rightarrow (Y, \tau_2)$ be a mapping. Then g is

- (i) fe^*o map iff $g(\gamma)$ is r - fe^*o set of Y for each $\gamma \in I^X$, $r \in I_0$ with $\tau_1(\gamma) \geq r$.
- (ii) fe^*c map iff $g(\gamma)$ is r - fe^*c set of Y for each $\gamma \in I^X$, $r \in I_0$ with $\tau_1(1 - \gamma) \geq r$.

Remark 3.1. The below implications is true for $r \in I_0$. by Definition ??



where fo , $f\delta so$, $f\delta sc$, $f\delta po$, $f\delta pc$, feo , fec , $f\beta o$, $f\beta c$, fe^*o , fe^*c are abbreviated by fuzzy open, fuzzy δ -semiopen, fuzzy δ -semiclosed, fuzzy δ -preopen, fuzzy δ -preclosed, fuzzy e -open, fuzzy e -closed, fuzzy β -open, fuzzy β -closed, fuzzy e^* -open, fuzzy e^* -closed mappings respectively.

Remark 3.2. Bt Definition ?? it is evident that every $f\delta po$ (resp. $f\delta pc$) map is feo (resp. fec) map and every $f\delta so$ (resp. $f\delta sc$) map is feo (resp. fec) map. Also, it is clear that every feo (resp. fec) map is $f\beta o$ (resp. $f\beta c$) map and fe^*o (resp. fe^*c) map. Also, every $f\beta o$ (resp. $f\beta c$) map is fe^*o (resp. fe^*c) map. The converses need not be true in general.

The below example represents that the inverse is not true.

Example 3.1 Having λ_1 , λ_2 , λ_3 and λ_4 be fuzzy subsets of $X = \{p, q\}$ represented as

$$\lambda_1(p) = 0.2, \lambda_1(q) = 0.1;$$

$$\begin{aligned}\lambda_2(p) &= 0.3, \quad \lambda_2(q) = 0.5; \\ \lambda_3(p) &= 0.7, \quad \lambda_3(q) = 0.7; \\ \lambda_4(p) &= 0.2, \quad \lambda_4(q) = 0.8.\end{aligned}$$

Then $\tau, \eta : I^X \rightarrow I$ defined as

$$\tau(\gamma) = \begin{cases} 1, & \text{if } \gamma = \underline{0} \text{ or } \underline{1}, \\ \frac{1}{2}, & \text{if } \gamma = \lambda_4, \\ 0, & \text{or else,} \end{cases} \quad \eta(\gamma) = \begin{cases} 1, & \text{if } \gamma = \underline{0} \text{ or } \underline{1}, \\ \frac{1}{2}, & \text{if } \gamma = \lambda_1, \lambda_2, \lambda_3, \\ 0, & \text{or else,} \end{cases}$$

be a sts on X . $g : (X, \tau) \rightarrow (Y, \eta)$ is an identity mapping. Take $r = \frac{1}{2}$. For any $\frac{1}{2}$ -fo set λ_4 in (X, η) , $g(\lambda_4) = \lambda_4$ is $\frac{1}{2}$ -f β o set in (Y, η) . Hence g is f β o mapping. However g is not feo mapping, as $f(\lambda_4) = \lambda_4$ is not $\frac{1}{2}$ -feo set in (Y, η) .

Example 3.2 Having γ and δ be fuzzy subsets of $X = \{p, q, r\}$ represented as

$$\begin{aligned}\gamma(p) &= 0.4, \quad \gamma(q) = 0.5, \quad \gamma(r) = 0.5; \\ \delta(p) &= 0.4, \quad \delta(q) = 0.5, \quad \delta(r) = 0.4.\end{aligned}$$

Then $\tau, \eta : I^X \rightarrow I$ defined as

$$\tau(\gamma) = \begin{cases} 1, & \text{if } \gamma = \underline{0} \text{ or } \underline{1}, \\ \frac{1}{2}, & \text{if } \gamma = \delta, \\ 0, & \text{or else.} \end{cases} \quad \eta(\gamma) = \begin{cases} 1, & \text{if } \gamma = \underline{0} \text{ or } \underline{1}, \\ \frac{1}{2}, & \text{if } \gamma = \gamma, \\ 0, & \text{or else,} \end{cases}$$

be a sts on X . $g : (X, \tau) \rightarrow (Y, \eta)$ is an identity mapping. Take $r = \frac{1}{2}$. For any $\frac{1}{2}$ -fo set δ in (X, η) , $g(\delta) = \delta$ is $\frac{1}{2}$ -feo set in (Y, η) . Hence g is feo mapping. However g is not f δ so mapping, as $g(\delta) = \delta$ is not $\frac{1}{2}$ -f δ so set in (Y, η) .

Example 3.3 Having γ and δ be fuzzy subsets of $X = \{p, q, r\}$ defined as follows

$$\begin{aligned}\gamma(p) &= 0.5, \quad \gamma(q) = 0.3, \quad \gamma(r) = 0.2; \\ \delta(p) &= 0.5, \quad \delta(q) = 0.4, \quad \delta(r) = 0.4.\end{aligned}$$

Then $\tau, \eta : I^X \rightarrow I$ defined as

$$\tau(\gamma) = \begin{cases} 1, & \text{if } \gamma = \underline{0} \text{ or } \underline{1}, \\ \frac{1}{2}, & \text{if } \gamma = \delta, \\ 0, & \text{or else,} \end{cases} \quad \eta(\gamma) = \begin{cases} 1, & \text{if } \gamma = \underline{0} \text{ or } \underline{1}, \\ \frac{1}{2}, & \text{if } \gamma = \gamma, \\ 0, & \text{or else,} \end{cases}$$

be a sts on X . $g : (X, \tau) \rightarrow (Y, \eta)$ is an identity mapping. Take $r = \frac{1}{2}$. For any $\frac{1}{2}$ -fo set δ in (X, η) , $g(\delta) = \delta$ is $\frac{1}{2}$ -feo set in (Y, η) . Hence g is feo mapping. However g is not f δ po mapping, as $g(\delta) = \delta$ is not $\frac{1}{2}$ -f δ po set in (Y, η) .

Example 3.4 Having γ, δ and ω be fuzzy subsets of $X = \{p, q, r\}$ represented as

$$\begin{aligned}\gamma(p) &= 0.3, \quad \gamma(q) = 0.5, \quad \gamma(r) = 0.2; \\ \delta(p) &= 0.4, \quad \delta(q) = 0.5, \quad \delta(r) = 0.5; \\ \omega(p) &= 0.7, \quad \omega(q) = 0.4, \quad \omega(r) = 0.8.\end{aligned}$$

Then $\tau, \eta : I^X \rightarrow I$ defined as

$$\tau(\gamma) = \begin{cases} 1, & \text{if } \gamma = \underline{0} \text{ or } \underline{1}, \\ \frac{1}{2}, & \text{if } \gamma = \omega, \\ 0, & \text{or else,} \end{cases} \quad \eta(\gamma) = \begin{cases} 1, & \text{if } \gamma = \underline{0} \text{ or } \underline{1}, \\ \frac{1}{2}, & \text{if } \gamma = \gamma, \delta, \\ 0, & \text{or else,} \end{cases}$$

be a sts on X . $g : (X, \tau) \rightarrow (Y, \eta)$ is an identity mapping. Take $r = \frac{1}{2}$. For any $\frac{1}{2}$ -fo set ω in (X, τ) , $g(\omega) = \omega$ is $\frac{1}{2}$ -fe*o set in (Y, η) . Hence g is fe*o mapping. However g is not f β o mapping, as $g(\omega) = \omega$ is not $\frac{1}{2}$ -f β o set in (Y, η) .

Example 3.5 Having γ and δ be fuzzy subsets of $X = \{p, q, r\}$ defined as follows

$$\begin{aligned}\gamma(p) &= 0.4, \quad \gamma(q) = 0.5, \quad \gamma(r) = 0.2; \\ \delta(p) &= 0.5, \quad \delta(q) = 0.4, \quad \delta(r) = 0.7.\end{aligned}$$

Then $\tau, \eta : I^X \rightarrow I$ defined as

$$\tau(\gamma) = \begin{cases} 1, & \text{if } \gamma = \underline{0} \text{ or } \underline{1}, \\ \frac{1}{2}, & \text{if } \gamma = \delta, \\ 0, & \text{or else,} \end{cases} \quad \eta(\gamma) = \begin{cases} 1, & \text{if } \gamma = \underline{0} \text{ or } \underline{1}, \\ \frac{1}{2}, & \text{if } \gamma = \gamma, \\ 0, & \text{or else,} \end{cases}$$

be an sts on X . $g : (X, \tau) \rightarrow (Y, \eta)$ is an identity mapping. Take $r = \frac{1}{2}$. For any $\frac{1}{2}$ -fo set δ in (X, τ) , $g(\delta) = \delta$ is $\frac{1}{2}$ -fe*o set in (Y, η) . Hence g is fe*o mapping. However g is not feo mapping, as $g(\delta) = \delta$ is not $\frac{1}{2}$ -feo set in (Y, η) .

Theorem 3.1. Let (X, τ_1) and (Y, τ_2) be sts's and $f : X \rightarrow Y$ be a mapping. Then

- (i) g is fe*o mapping.
- (ii) $g(I_{\tau_1}(\gamma, r)) \leq e^*I_{\tau_2}(g(\gamma), r)$ for each $\gamma \in I^X$ and $r \in I_0$.
- (iii) $I_{\tau_1}(g^{-1}(\delta), r) \leq g^{-1}(e^*I_{\tau_2}(\delta, r))$ for each $\delta \in I^Y$ and $r \in I_0$.

Proof. (i) $\Rightarrow$ (ii) : For all $\gamma \in I^X$, $r \in I_0$ with $\tau_1(I_{\tau_1}(\gamma, r)) \geq r$. Since g is fe*o mapping, we see that $g(I_{\tau_1}(\gamma, r))$ is r -fe*o set in Y . From Theorem ??(i), $g(I_{\tau_1}(\gamma, r)) = e^*I_{\tau_2}(g(I_{\tau_1}(\gamma, r)), r) \leq e^*I_{\tau_2}(g(\gamma), r)$.

(ii) $\Rightarrow$ (iii) : Let $\delta \in I^Y$, $r \in I_0$. By (ii),

$$g(I_{\tau_1}(g^{-1}(\delta), r)) \leq e^*I_{\tau_2}(gg^{-1}(\delta), r) \leq e^*I_{\tau_2}(\delta, r).$$

$$\Rightarrow I_{\tau_1}(g^{-1}(\delta), r) \leq g^{-1}(e^*I_{\tau_2}(\delta, r)).$$

(iii) $\Rightarrow$ (i) : Every $\gamma \in I^X$, $r \in I_0$ with $\tau_1(\gamma) \geq r$. Let $\gamma = g^{-1}(\delta)$. By (iii), we have $I_{\tau_1}(\gamma, r) \leq g^{-1}(e^*I_{\tau_2}(g(\gamma), r))$. Since $I_{\tau_1}(\gamma, r) = \gamma$, we get $g(\gamma) \leq e^*I_{\tau_2}(g(\gamma), r) \leq g(\gamma)$. Thus $g(\gamma) = e^*I_{\tau_2}(g(\gamma), r)$. By Theorem ??(i), $g(\gamma)$ is r -fe*o set in Y . Thus g is fe*o mapping.

Theorem 3.2. Let $g : (X, \tau_1) \rightarrow (Y, \tau_2)$ be a fe*o (resp. fδso, fδpo) mapping. If $\delta \in I^Y$ and $\gamma \in I^X$, $\tau_1(1 - \gamma) \geq r$, $r \in I_0$ such that $g^{-1}(\delta) \leq \gamma$, then $\exists$ an r -fe*c (resp. r -fδsc, r -fδpc) set ν of Y such that $\delta \leq \nu, g^{-1}(\nu) \leq \gamma$.

Proof. Let $\nu = 1 - g(1 - \gamma)$. we know that $g^{-1}(\delta) \leq \gamma$ having $g(1 - \gamma) \leq 1 - \delta$. since g is fe*o mapping, then ν is r -fe*c set in Y such that $g^{-1}(\nu) = 1 - g^{-1}(g(1 - \gamma)) \leq 1 - (1 - \gamma) = \gamma$.

Theorem 3.3. If $g : (X, \tau_1) \rightarrow (Y, \tau_2)$ be a fe*o mapping. Then for each $\delta \in I^Y$, $r \in I_0$,

$$g^{-1}(I_{\tau_2}(C_{\tau_2}(\delta - I_{\tau_2}(\delta, r), r), r)) \leq C_{\tau_1}(g^{-1}(\delta), r).$$

Proof. Since $\tau_1(1 - C_{\tau_1}(g^{-1}(\delta), r)) \geq r$ and $g^{-1}(\delta) \leq C_{\tau_1}(g^{-1}(\delta), r)$ for each $\delta \in I^Y$, $r \in I_0$, it follows from Theorem ??, $\exists$ an r -fe*c set γ of Y , $\delta \leq \gamma$ such that $f^{-1}(\gamma) \leq C_{\tau_1}(g^{-1}(\delta), r)$. So

$$\begin{aligned} g^{-1}(\gamma) &\geq g^{-1}(I_{\tau_2}(C_{\tau_2}(\delta - I_{\tau_2}(\gamma, r), r), r)) \\ &\geq g^{-1}(I_{\tau_2}(C_{\tau_2}(\delta - I_{\tau_2}(\delta, r), r), r)) \\ \Rightarrow g^{-1}(I_{\tau_2}(C_{\tau_2}(\delta - I_{\tau_2}(\delta, r), r), r)) &\leq g^{-1}(\gamma) \leq C_{\tau_1}(g^{-1}(\delta), r). \end{aligned}$$

$$\text{Hence } g^{-1}(I_{\tau_2}(C_{\tau_2}(\delta - I_{\tau_2}(\delta, r), r), r)) \leq C_{\tau_1}(g^{-1}(\delta), r).$$

Theorem 3.4. If $g : (X, \tau_1) \rightarrow (Y, \tau_2)$ be a bijective mapping such that

$$g^{-1}(I_{\tau_2}(C_{\tau_2}(\delta - I_{\tau_2}(\delta, r), r), r)) \leq C_{\tau_1}(g^{-1}(\delta), r).$$

for every $\delta \in I^Y$, $r \in I_0$, then g is fe*o map.

Proof. Let $\gamma \in I^X$, $r \in I_0$ having $\tau_1(\gamma) \geq r$. Then, from the given condition,

$$\begin{aligned} g^{-1}(I_{\tau_2}(C_{\tau_2}(\delta - I_{\tau_2}(g(1 - \gamma), r), r), r)) &\leq C_{\tau_1}(g^{-1}(f(1 - \gamma)), r) \\ &= C_{\tau_1}(1 - \gamma, r) \\ &= 1 - \gamma \end{aligned}$$

and so $I_{\tau_2}(C_{\tau_2}(\delta - I_{\tau_2}(\delta, r), r), r) \leq g(1 - \gamma)$ which shows that $g(1 - \gamma)$ is an r -fe*c set of Y . Since g is bijective, $g(\gamma)$ is an r -fe*o set of Y . Therefore g is fe*o map.

Conclusion In this paper, fe^* -cts, fe^*o and fe^*c mappings are introduced in sts in the sense of $\hat{S}$ ostak's. We investigated some of their properties. Moreover, we have investigated the relationships between f-cts, $f\delta s$ -cts, $f\delta p$ -cts, fe -cts, $f\beta$ -cts and fe^* -cts. Also we have shown some theorems using fe^*o and fe^*c mappings and established the relationships between fo map, $f\delta so$ map, $f\delta po$ map, feo map, $f\beta o$ map and fe^*o map.

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