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FUZZY RELIABILITY ANALYSIS FOR THE EFFECT OF OXYTOCIN USING GENERALIZED RAYLEIGH DISTRIBUTION

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Abstract

This paper presents the reliability analysis using fuzzy generalized Rayleigh distribution for the effect of oxytocin. We deliver substantial representation of fuzzy properties of the generalized Rayleigh distribution along with the consequence of 10u doses of Oxytocin by measuring the reliability and failure rate value of cardiac output for lower and upper alpha cuts. The results show that for the lower alpha cut the reliability values are decreases and failure rate values are increases, and for the upper alpha cut the reliability values are increases and the failure rate values are decreases.

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Key Words: Oxytocin, Generalized Rayleigh distribution, Reliability and Failure rate value

1 Introduction

The problem of approximating the reliability and failure rate values in statistical distributions used to learning a certain occurrence is one of the significant problems facing constantly those who are interested in life time data analysis. The exponential distribution is often used in reliability theory and applications. The cause for that it has a constant failure rate. The exponential distribution is not guarantees to fit well a given set of real data. Other distributions have been used in reliability theory. Some were taken from the twelve different forms of distributions introduced by Burr [?] to model data. Among those different distributions, Burr-Type X and Burr-Type XII received the most attention. There is an exhaustive analysis of Burr-Type XII distribution in Rodriguez [?], see also Wingo [?] for a sufficient description of it.
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Surles and Padgett [?] introduced two-parameter Burr Type X distribution, which can also be described as generalized Rayleigh distribution (GRD). It is observed that the two-parameter generalized Rayleigh distribution can be used quite effectively in modelling strength and general lifetime data. Burr Type X distribution was also discussed by Raqab, M Z and Kundu D [?]. The parameter estimation for GRD by different method was discussed by Kundu D and Raqab M Z [?].

The peptide hormone oxytocin plays important roles in mammalian social behavior, including maternal care,pair bonding, and social memory. Oxytocin is given normally at cesarean segment to decrease the occurrence and harshness of post-partum haemorrhage, but it produce adverse haemodynamic effects. The potential risks of oxytocin boluses in women with significant cardiovascular diseases were studied by Camann W R [?], Mayer D

[?].

In this paper we estimate the reliability and failure rate values of the fuzzy generalized Rayleigh distribution to compute the effects of oxytocin in cesarean segment underneath spinal anesthesia.

2 Notation

φ –Shape parameter
γ –Scale parameter
$\beta(\alpha)$ –Alpha cut of shape parameter
$\gamma(\alpha)$ –Alpha cut of scale parameter
$R(t)$ –Reliability rate
$H(t)$ –Failure rate rate
$\overline{R}(t)(\alpha)$ –Fuzzy Reliability rate
$\overline{H}(t)(\alpha)$ –Fuzzy Failure rate rate

3 Fuzzy Generalized Rayleigh Distribution

A random variable T follows the GRD has probability density function (p.d.f.) $f(t:\ \varphi,\gamma) = 2\varphi\gamma^2 t e^{-(\gamma t)^2}\left(1-e^{-(\gamma t)^2}\right)^{\varphi-1},\ \ t>0,$ where $\varphi \geq 0$is the shape parameter and $\gamma > 0$ is the scale parameter. When $\varphi = 0$and set $\gamma = 2\phi^2$the GRD reduce to Rayleigh distribution with scale parameterϕ. The cumulative distribution function (c.d.f.) of T is given by

$$F(t;\ \varphi,\ \gamma) = \left(1-e^{-(\gamma t)^2}\right)^{\varphi}.$$

The GRD has the rth moment,

$$E(T^r) = \int_{-\infty}^{\infty} t^r f(t)\,dt$$
$$= \int_{-\infty}^{\infty} t^r \frac{2}{\Gamma(\varphi+1)\gamma^{\varphi+1}} t^{2\varphi+1} e^{\frac{t^2}{\varphi}}\,dx \ = \ \frac{\Gamma(\varphi+r/2+1)}{\Gamma(\varphi+1)}\gamma^{r/2} \quad r = 1, 2, ...$$

It will often be suitable to work with the complement of the c.d.f, the reliability function

$$R(t) = P(T > t)$$
$$R(t) = 1 - F(t)$$
$$R(t) = 1 - \left(1-e^{-(\gamma t)^2}\right)^{\varphi}, \quad t \geq 0.$$

which gives the reliability function of GRD. The failure rate function of GRD is given by

$$H(t) = \frac{f(t)}{R(t)}$$

$$H(t) = \frac{2\varphi\gamma^2 t e^{-(\gamma t)^2}\left(1-e^{-(\gamma t)^2}\right)^{\varphi-1}}{1-\left(1-e^{-(\gamma t)^2}\right)^{\varphi}}, \quad t \geq 0.$$

There are numerous approaches and examples in classical reliability theory, which assume that all parameters of lifetime density functions are accurate. Though, in the reality randomness and fuzziness are often mixed up in the lifetimes of systems. But, the parameters sometimes cannot record precisely due to machine faults, trial, individual judgment, approximation or certain unexpected situations. When parameter in the lifetime distribution is fuzzy, the conventional reliability system may have trouble for handling reliability and failure rate functions. The theory of fuzzy reliability was proposed and development by several authors Cai et al. [4], [5], Cai [3], Karpisek Z[9], Hammer [8], Garg H[7], Baloui Jamkhaneh. E. [1]. Now we are consider the GRD with fuzzy parameters $\overline{\varphi}$, $\overline{\gamma}$ that is swapped with φ, γ. A random variable T follows the fuzzy generalized distribution is denoted by$T \sim FGRD\,(\overline{\varphi}, \overline{\gamma})$. The probability density function of a random variable $T \sim FGRD\,(\overline{\varphi}, \overline{\gamma})$ is given by

$$f(t:\, \overline{\varphi}, \overline{\gamma}) = 2\overline{\varphi}\,\overline{\gamma}^2 t e^{-(\overline{\gamma}t)^2}\left(1-e^{-(\overline{\gamma}t)^2}\right)^{\overline{\varphi}-1}, \;\; t > 0,\; \overline{\varphi} \in \overline{\varphi}\,(\alpha),\, \overline{\gamma} \in \overline{\gamma}\,(\alpha).$$

The cumulative distribution function (c.d.f.) of $T \sim FGRD\,(\overline{\varphi}, \overline{\gamma})$ is given by

$$F(t;\, \overline{\varphi}, \overline{\gamma}) = \left(1-e^{-(\overline{\gamma}t)^2}\right)^{\overline{\varphi}}, \overline{\varphi} \in \overline{\varphi}\,(\alpha), \overline{\gamma} \in \overline{\gamma}\,(\alpha).$$

The fuzzy reliability function of the FGRD distribution is defined as

$$\overline{R}(t)(\alpha) = \overline{P}(T > t)$$

$$\overline{R}(t)(\alpha) = [R_L(t)(\alpha),\; R_U(t)(\alpha)]$$

$$R_L(t)(\alpha) = \min\left\{1-\left(1-e^{-(\overline{\gamma}t)^2}\right)^{\overline{\varphi}}, \quad t \geq 0,\, \overline{\varphi} \in \overline{\varphi}\,(\alpha),\, \overline{\gamma} \in \overline{\gamma}\,(\alpha)\right\}$$

$$R_U(t)(\alpha) = \max\left\{1-\left(1-e^{-(\overline{\gamma}t)^2}\right)^{\overline{\varphi}}, \quad t \geq 0,\, \overline{\varphi} \in \overline{\varphi}\,(\alpha),\, \overline{\gamma} \in \overline{\gamma}\,(\alpha)\right\}$$

$$\overline{R}(t)(\alpha) = 1-\left(1-e^{-(\overline{\gamma}t)^2}\right)^{\overline{\varphi}}, \quad t \geq 0,\, \overline{\varphi} \in \overline{\varphi}\,(\alpha),\, \overline{\gamma} \in \overline{\gamma}\,(\alpha).$$

The fuzzy hazard rate function of the FGRD distribution is defined as

$$\overline{H}(t)(\alpha) = \frac{f(t;\overline{\varphi},\overline{\gamma})}{R(t;\overline{\varphi},\overline{\gamma})},\ \overline{\varphi} \in \overline{\varphi}(\alpha),\ \overline{\gamma} \in \overline{\gamma}(\alpha)$$

$$\overline{H}(t)(\alpha) = [H_L(t)(\alpha),\ H_U(t)(\alpha)]$$

$$H_L(t)(\alpha) = \min\left\{ \frac{2\overline{\varphi}\,\overline{\gamma}^2 t e^{-(\overline{\gamma}t)^2}\left(1-e^{-(\overline{\gamma}t)^2}\right)^{\overline{\varphi}-1}}{1-\left(1-e^{-(\overline{\gamma}t)^2}\right)^{\overline{\varphi}}},\quad t \geq 0,\ \overline{\varphi} \in \overline{\varphi}(\alpha),\ \overline{\gamma} \in \overline{\gamma}(\alpha) \right\}$$

$$H_U(t)(\alpha) = \max\left\{ \frac{2\overline{\varphi}\,\overline{\gamma}^2 t e^{-(\overline{\gamma}t)^2}\left(1-e^{-(\overline{\gamma}t)^2}\right)^{\overline{\varphi}-1}}{1-\left(1-e^{-(\overline{\gamma}t)^2}\right)^{\overline{\varphi}}},\quad t \geq 0,\ \overline{\varphi} \in \overline{\varphi}(\alpha),\ \overline{\gamma} \in \overline{\gamma}(\alpha) \right\}$$

$$\overline{H}(t)(\alpha) = \frac{2\overline{\varphi}\,\overline{\gamma}^2 t e^{-(\overline{\gamma}t)^2}\left(1-e^{-(\overline{\gamma}t)^2}\right)^{\overline{\varphi}-1}}{1-\left(1-e^{-(\overline{\gamma}t)^2}\right)^{\overline{\varphi}}},\quad t \geq 0,\ \overline{\varphi} \in \overline{\varphi}(\alpha),\ \overline{\gamma} \in \overline{\gamma}(\alpha)$$

4 Results and Discussion

Consider the trail conducted by Pinder. A.J. [**?**] parturient were undergo the elective cesarean section under spinal anesthesia. Patients were received the 10 u of oxytocin by rapid i.v. in a double blinded fashion. The study was taken for thirty four patients. The cardiac output were shown in the following table 1.

Table 1: Cardiac output after the administration of 10 u oxytocin

Time (sec.)	0	15	30	45	60	75	90	105	120	135	150	175	180
10u	10	16	17	17	18	16	15	14	13	13	12	11	11

The parameters for FGRD for the above data are $\varphi = 3.6876$, $\gamma = 8.06$. The equivalent fuzzy triangular numbers are [2.9246, 3.6876, and 4.4826] and [7.2050, 8.0600, 8.8440]. The corresponding $\alpha - cuts$ are $[3.6876 - 0.7630\alpha\ 3.6876, 3.6876 + 0.7950\alpha]$ and $[8.0600 - 0.8550\alpha,\ 8.0600,\ 8.0600 + 0.7840\alpha]$. The fuzzy reliability values are given in the table 2, table 3 and failure rate values are given in the table 4 and 5 respectively.

Table 2: Fuzzy reliability rate values for Lower and Upper alpha cuts at t=0.5 and t=0.75

α	**t=0.5**		**t=0.75**	
	$R_L(t)(\alpha)$	$R_U(t)(\alpha)$	$R_L(t)(\alpha)$	$R_U(t)(\alpha)$
0	0.99118	0.96950	0.78668	0.52323
0.1	0.99053	0.97112	0.77472	0.53664
0.2	0.98985	0.97269	0.76258	0.55017
0.3	0.98915	0.97419	0.75028	0.56381
0.4	0.98840	0.97564	0.73784	0.57755
0.5	0.98763	0.97704	0.72527	0.59137
0.6	0.98682	0.97838	0.71260	0.60526
0.7	0.98597	0.97966	0.69984	0.61920
0.8	0.98509	0.98090	0.68700	0.63318
0.9	0.98417	0.98208	0.67410	0.64717
1	0.98321	0.98321	0.66116	0.66116

Table 3: Fuzzy reliability rate values for Lower and Upper alpha cuts at t=1.0 and t=1.25

α	**t=1**		**t=1.25**	
	$R_L(t)(\alpha)$	$R_U(t)(\alpha)$	$R_L(t)(\alpha)$	$R_U(t)(\alpha)$
0	0.32806	0.09565	0.07230	0.00800
0.1	0.31063	0.10232	0.06514	0.00898
0.2	0.29392	0.10943	0.05867	0.01007
0.3	0.27793	0.11699	0.05281	0.01129
0.4	0.26263	0.12503	0.04753	0.01266
0.5	0.24803	0.13358	0.04275	0.01420
0.6	0.23410	0.14266	0.03845	0.01591
0.7	0.22083	0.15229	0.03456	0.01783
0.8	0.20821	0.16251	0.03106	0.01998
0.9	0.19620	0.17334	0.02791	0.02238
1	0.18480	0.18480	0.02507	0.02507

Table 4: Fuzzy failure rate values for Lower and Upper alpha cuts at t=0.5 and t=0.75

α	**t=0.5**		**t=0.75**	
	$H_L(t)(\alpha)$	$H_U(t)(\alpha)$	$H_L(t)(\alpha)$	$H_U(t)(\alpha)$
0	0.17409	0.89076	2.04979	11.37135
0.1	0.18716	0.81215	2.16461	10.09011
0.2	0.20085	0.73967	2.28158	8.94500
0.3	0.21520	0.67291	2.40061	7.92225
0.4	0.23022	0.61146	2.52160	7.00941
0.5	0.24591	0.55496	2.64446	6.19525
0.6	0.26230	0.50306	2.76908	5.46965
0.7	0.27941	0.45542	2.89537	4.82349
0.8	0.29724	0.41174	3.02325	4.24854
0.9	0.31581	0.37174	3.15262	3.73740
1	0.33513	0.33513	3.28340	3.28340

Table 5: Fuzzy failure rate values for Lower and Upper alpha cuts at t=1.0 and t=1.25

α	**t=1**		**t=1.25**	
	$H_L(t)(\alpha)$	$H_U(t)(\alpha)$	$H_L(t)(\alpha)$	$H_U(t)(\alpha)$
0	4.89716	40.70488	7.07739	127.39631
0.1	5.08316	34.10216	7.28602	98.05602
0.2	5.26876	28.55429	7.49346	75.44449
0.3	5.45391	23.89472	7.69976	58.02413
0.4	5.63852	19.98295	7.90498	44.60763
0.5	5.82254	16.70051	8.10918	34.27833
0.6	6.00593	13.94749	8.31241	26.32875
0.7	6.18863	11.63971	8.51474	20.21297
0.8	6.37062	9.70619	8.71621	15.50985
0.9	6.55186	8.08719	8.91689	11.89460
1	6.73234	6.73234	9.11681	9.11681

5 Conclusion

The GRD and its reliability and failure rate function was successfully established in the fuzzy state. The reliability values and failure rate values were calculate for the doses of 10 u oxytocin. The results show that the reliability values are decreases for lower alpha cuts and increases for upper alpha cuts. In the meantime the failure rate values are increases for the lower alpha cuts and decreases for upper alpha cut.

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