Analysis of Cost on an Univariate Policy of Recruitment in Manpower Planning

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Abstract

This research paper analyses the cost on an univariate policy of recruitment in manpower planning. A mathematical model is built and suitable univariate CUM recruitment policy, based on shock model approach involving thresholds for the loss of manpower in the organization is suggested. The explicit expression for the mean time for recruitment and the long-run average cost are obtained. A different probabilistic analysis is used to derive the analytical result. Numerical illustrations are briefed by assuming specific distributions.

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1 Introduction

Manpower planning supports to make the links between strategy, structure and people more efficient. The purpose of manpower planning is to get a better match between manpower requirement and manpower availability. Manpower planning is particularly suitable for the application of statistical techniques. To prevent business failure, co-ordination of demand and supply is required, together with the monitoring and assessment of productivity and technological changes. Different types of manpower model plays an important role in efficient design and control of system. These models incorporate several factors such as recruitment, training, promotion, demotion and wastage.

Attrition is a common phenomenon in many organizations which leads to the depletion of manpower. Recruitment on every occasion of depletion of manpower is not advisable since every recruitment involves cost. Hence, the cumulative depletion of manpower is permitted till it reaches a level called the threshold. If the total loss of manpower exceeds this threshold, the activities in the organization will be affected and hence recruitment becomes necessary. An univariate recruitment policy, usually known as CUM policy of recruitment in the literature, is based on the replacement policy associated with the shock model approach in reliability theory and is stated as follows: “Recruitment is made whenever the cumulative loss of man hours exceeds threshold level $T$”. As the loss of manpower is unpredictable, a suitable recruitment policy has to be designed to overcome this loss.

A full-fledged discussion about the statistical approach to manpower planning is done by Bartholomew [3] in which a description of manpower system along with historical development of the subject is discussed. A descriptive survey of the manpower planning models and technique can be seen in Bryant [5]. The basis for the present study of this paper is from Sathiymoorthi and Elangovan [10], [11]. Expected time to recruitment and its variance using shock model approach have been considered by Srinivasan...
and Mariappan [13]. Arokkia Saibe and Esther Clara [2] and Srinivasan and Arokkia Saibe [12] have discussed estimation of mean time to recruitment for a manpower system when threshold has two components. Venkatesh and Srinivasan [15] and Amudha and Srinivasan [1] have discussed mean and variance of the time to recruitment in a single graded manpower system using an univariate policy of recruitment.

2 Model Description

- The organization takes decisions at k random epochs, and at every decision making epoch, a random number of persons quit the organization.
- There is an associated loss of man hour to the organization if a person quits.
- If the total number of persons who leave the organization exceeds a threshold level, the organization faces a breakdown, and so recruitment is necessary.
- The loss of a man hours at any decision form a sequence of identically and independently distributed random variables.
- Survival time process and loss of man hour process are independent.
- Recruitment time is negligible.
- The cost associated with the loss of man hour is linear.
- Recruitments take place only at decision points.

3 Notations

4 Main Results

4.1 Long-Run Average Cost for CUMT Policy

In this section, an analytical expression for the long-run average cost for the CUMT policy is obtained.
\( Y_n \): a continuous random variable denoting the loss of man hours at the \( n^{th} \) decision.

\( T \): a non-negative constant denoting the threshold level.

\( W \): time to recruitment under the given recruitment policy.

\( r \): the fixed cost for each recruitment.

\( C(y) \): cost incurred whenever there is a loss of man hour of magnitude \( y \).

\( T_n \): loss of man hours in the first \( n \) decision with \( T_0 = 0, T_n = \sum_{i=1}^{n} Y_i \)

\( G(.) \): common distribution of \( Y_n, n = 1, 2, 3, ... \)

\( G_n(.) \): distribution function of \( T_n \)

\( C(CUMT) \): the long-run average cost per unit time for the CUMT policy.

\( L(t) \): cumulative distribution function of \( W \).

\( V_k(T) \): probability that there are exactly \( k \) decision making epochs in \((0, T]\)

\( f(.) \): the probability density function of the inter arrival times between successive decisions making epochs.

\( f_k(.) \): \( k \) convolution of \( f(.) \).

\( F(.) \): cumulative distribution function corresponding to \( f(.) \).

\( F_k(.) \): \( k \) Stieltjes convolution of \( F(.) \)

Under this policy, “the recruitment is made whenever the cumulative loss of man hours exceeds the threshold \( T \)”.

### 4.2 Expected Time to Recruitment

The distribution of the time to recruitment is

\[
P(W > t) = 1 - L(t)
\]
Now the probability that the threshold level is not reached till \( t \) is given by
\[
1 - L(t) = P(W > t) = \sum_{k=0}^{\infty} \Pr\{V_k(t) \text{ and the threshold level is not crossed}\}
\]
\[
= \sum_{k=0}^{\infty} V_k(t)Pr\left\{ \sum_{i=1}^{k} Y_i < T \right\}
\]
\[
1 - L(t) = P(W > t) = \sum_{k=0}^{\infty} \{F_k(t) - F_{k+1}(t)\} \Pr\left\{ \sum_{i=1}^{k} Y_i < T \right\}
\]

The mean of \( W \) is given by
\[
E(W) = -\left[ \frac{dL^*(s)}{ds} \right]_{s=0}
\]
(2)

Where \( L^*(s) = \text{Laplace-Stieltjes transform of } L(t) \).

### 4.3 Expected Total Cost

The total cost \( TC \) is given by
\[
TC = r + \sum_{i=0}^{\infty} \sum_{j=0}^{i} C(Y_{j+1}) \chi(T_i \leq T < T_{i+1})
\]
(3)

The expected total cost is given by
\[
E(TC) = E\left[ r + \sum_{i=0}^{\infty} \sum_{j=0}^{i} C(Y_{j+1}) \chi(T_i \leq T < T_{i+1}) \right]
\]
\[
= r + E\left[ \sum_{i=0}^{\infty} C(Y_1 + Y_2 + \ldots + Y_{j+1}) \chi(T_i \leq T < T_{i+1}) \right]
\]
\[
= r + E\left[ \sum_{i=0}^{\infty} C(T_{i+1}) \chi(T_i \leq T < T_{i+1}) \right]
\]
\[
= r + E\left[ \frac{E\{C(T_{i+1})\}}{T_i} \chi(T_i \leq T < T_{i+1}) \right] / T_i
\]
\[
E(TC) = r + \sum_{i=0}^{\infty} \int_{0}^{T} \left[ C(t) + \int_{T-t}^{\infty} C(y) dG(y) \right] dG_i(t) \quad (4)
\]
\[
\therefore C(CUMT) = \frac{E(TC)}{E(W)}
\]

Where \(E(W)\) and \(E(TC)\) are given by (2) and (4).

5 Special Case

In this section an explicit expression for the long-run average cost is obtained assuming specific distributions and linear cost.

Let \(Y_i, i = 1, 2, 3, \ldots\) follow exponential distribution with parameter \(\theta\).

Assume that \(G(y) = P(Y_i \leq y) = 1 - e^{-\theta y}, \theta > 0\).

\[\therefore G_k(y) = P[\sum_{i=1}^{k} Y_i \leq y]\]
is a Gamma distribution with parameter \(\theta \geq 0, k > 0\), for a fixed \(k\).

Its density function is \(\frac{\theta^k k! x^{k-1} e^{-\theta x}}{(k-1)!}\).

The mean time to recruitment for the CUMT policy is

\[E(W) = \sum_{k=1}^{\infty} \frac{k}{\delta_1} \left[ \int_{0}^{T} \frac{\theta^k k! x^{k-2} e^{-\theta x}}{(k-2)!} dx - \int_{0}^{T} \frac{\theta^k k! x^{k-1} e^{-\theta x}}{(k-1)!} dx \right] \]

Let \(C(y) = A + By\), where \(A\) and \(B\) are real constants.

\[E(TC) = r + \sum_{i=0}^{\infty} \int_{0}^{T} \left\{ (A + Bt) \frac{\theta^i t^{i-1} e^{-\theta t}}{(i-1)!} dt \right\} + A + \left( \frac{B}{\theta} \right)(2 - e^{-\theta T})\]

The explicit expression for the long-run average cost per unit time for the CUMT policy for the special case is

\[C(CUMT) = \frac{r + \sum_{i=0}^{\infty} \int_{0}^{T} \left( A + Bt \right) \frac{\theta^i t^{i-1} e^{-\theta t}}{(i-1)!} dt + A + \left( \frac{B}{\theta} \right)(2 - e^{-\theta T})}{\sum_{k=1}^{\infty} \frac{k}{\delta_1} \left[ \int_{0}^{T} \frac{\theta^k k! x^{k-2} e^{-\theta x}}{(k-2)!} dx - \int_{0}^{T} \frac{\theta^k k! x^{k-1} e^{-\theta x}}{(k-1)!} dx \right]}\]

6 Numerical Illustrations and Conclusion

Case (i): The values of \(r, A, B, T\) and \(\delta_1\) are fixed and variations are with respect to \(\theta\).

For fixed \(r = Rs.5,000, A = 5, B = 20, T = 80, \delta_1 = 5\)
<table>
<thead>
<tr>
<th>$\theta$</th>
<th>Long-run average cost (in Rs.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>5144</td>
</tr>
<tr>
<td>0.1</td>
<td>3424</td>
</tr>
<tr>
<td>0.15</td>
<td>2905</td>
</tr>
<tr>
<td>0.2</td>
<td>2669</td>
</tr>
<tr>
<td>0.25</td>
<td>2534</td>
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<tr>
<td>0.3</td>
<td>2447</td>
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<td>0.35</td>
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<tr>
<td>0.4</td>
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</tr>
<tr>
<td>0.45</td>
<td>2303</td>
</tr>
<tr>
<td>0.5</td>
<td>2274</td>
</tr>
</tbody>
</table>

The Table 1 gives the long-run average cost for CUMT policy, when variations are allowed in $\theta$, keeping $r, A, B, T$ and $\delta_1$ fixed. As $\theta$ increases, $\frac{1}{\theta}$, the mean loss of man hours decreases and hence the long-run average cost decreases.

**Case (ii):** The values of $\theta, r, A, B$ and $T$ are fixed and variations are with respect to $\delta_1$.
For fixed $\theta = 0.5, r = Rs.5,000, A = 5, B = 20, T=80$

<table>
<thead>
<tr>
<th>$\delta_1$</th>
<th>Long-run average cost (in Rs.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>455</td>
</tr>
<tr>
<td>2</td>
<td>910</td>
</tr>
<tr>
<td>3</td>
<td>1364</td>
</tr>
<tr>
<td>4</td>
<td>1819</td>
</tr>
<tr>
<td>5</td>
<td>2274</td>
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<tr>
<td>6</td>
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<td>7</td>
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<tr>
<td>9</td>
<td>4093</td>
</tr>
<tr>
<td>10</td>
<td>4548</td>
</tr>
</tbody>
</table>

From the Table 2 it is found that as $\delta_1$ increases, the mean time between consecutive decisions decreases. This, in turn decreases the
average time to recruitment and consequently the long-run average cost increases.

References


