Circulant Fuzzy Matrix In Human Body Disease

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Abstract

In this paper, we present some operations on circulant fuzzy matrices (CFMs). The first row of the circulant fuzzy matrices play important role in this Study. We apply circulant fuzzy technology through the well known Sanchez’s approach for HUMAN BODY DISEASE diagnosis. Finally, Sanchez’s formulated the diagnostic models involving circulant fuzzy matrices representing the medical knowledge between the symptoms and diseases.

Key Words and Phrases: Fuzzy matrices, Circulant fuzzy matrices, medical knowledge.

1 Introduction

The field of medicine is one of the most fruitful and absorbing areas of applications for fuzzy set theory. In most of cases of our life, the data obtained for decision making are only approximate. In 1965, Zadeh introduce the concept of fuzzy set theory to meet those
problems. The fuzzyness can be represented by different ways. One of the most useful representation is membership function. In the discrimination analysis, the symptoms are ranked according to the grade of discrimination of each disease by a particular symptom and is represented in the form of a matrix called a frequency distribution matrix \( F = (f_{ij}) \) where \( f_{ij} \) is the ratio of the patients with disease \( d_i \). This matrix model may not yield more accurate diagnosis in such cases where several diseases affect a single patient or when a single disease manifests quite differently in different patients and at different disease stages. Moreover, with the increased volume of information available to doctors from new medical technologies, the process of classifying different sets of symptoms under a single name of disease and determining the expropriate healing actions becomes increasingly difficult. Recently, there are varieties of models of medical diagnosis under the general structure of fuzzy sets theory involving fuzzy matrices to deal with different complicating aspects of medical diagnosis.

In this article, I extend Sanchez’s method for human disease diagnosis using the notion of circulant matrix theory. The notion of circulant and their applications are described comprehensively in Cao, Kim and Rough. The method of circulant medical diagnosis involves circulant order relations.

2 Preliminaries

2.1 Circulant Matrix

An \( n \times n \) circulant matrix has the form

\[
\begin{pmatrix} 
    a_1 & a_2 & \cdots & a_{n-1} & a_n \\
    a_n & a_1 & \cdots & a_{n-2} & a_{n-1} \\
    \vdots & \vdots & \ddots & \vdots & \vdots \\
    a_3 & a_4 & \cdots & a_1 & a_2 \\
    a_2 & a_3 & \cdots & a_n & a_1
\end{pmatrix}
\]

Thus a circulant matrix is determined by its first row.
2.2 Fuzzy matrices

Let \( F_{mn} \) denote the set of all \( m \times n \) Matrices over \( F \), if \( m = n \), in short, we write \( F_n \). Elements of \( F_{mn} \) are called as membership value matrices, binary fuzzy relation matrices (or) in short, fuzzy matrices.

2.3 Circulant fuzzy matrix

A Fuzzy matrix \( \tilde{A} = \left[ a_{ij} \right] \) is said to be circulant fuzzy matrix if all the elements of \( \tilde{A} \) can be determined completely by its first row. Suppose the first row of \( \tilde{A} \) is \( [a_1, a_2, \ldots, a_n] \) Then any element \( a_{ij} \) of \( \tilde{A} \) can be determined throughout the element of the first row as \( a_{ij} = a_{1(n-i+j+1)} \) with \( a_{1(n+k)} = a_{1k} \).

A Circulant fuzzy matrix is the form of

\[
\begin{pmatrix}
a_1 & a_2 & \cdots & a_{n-1} & a_n \\
a_n & a_1 & \cdots & a_{n-2} & a_{n-1} \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
a_3 & a_4 & \cdots & a_1 & a_2 \\
a_2 & a_3 & \cdots & a_n & a_1
\end{pmatrix}
\]

With entries in \([0, 1]\).

2.4 Complement of circulant fuzzy matrix

Let \( \tilde{A} = [a_{ij}] \in \text{circulant } FM_{m \times n} \), according to the definition in the representation of the complement of the fuzzy matrix \( \tilde{A} \) which is denoted by \( \tilde{A}^c \) and then \( \tilde{A}^c \) is called circulant fuzzy complement matrix if \( \tilde{A}^c = [(1 - a_{ij} \tilde{A})] m \times n \) for all \( a_{ij} \tilde{A}[0, 1] \). Then the matrix obtained from so called membership value would be the following \( \tilde{A}^c = a_{ij} \tilde{A} = [(1 - a_{ij} \tilde{A})] \) for all \( i \) and \( j \).

2.5 Product of circulant fuzzy matrices

Let \( \tilde{A} = [a_{ij}] \in \text{circulant } FM_{m \times n} \), represent the fuzzy membership function of \( u_i \). Also let \( \tilde{B} = [b_{jk}]_{n \times p} \) represent the fuzzy membership function \( u_j \). We now define \( \tilde{A} \circ \tilde{B} \) the product of \( \tilde{A} \) and \( \tilde{B} \) as \( \tilde{A} \circ \tilde{B} = [d_{ik} \tilde{A} \tilde{B}]_{m \times p} = [\max \min(\mu_{ij} \tilde{A}, \mu_{jk} \tilde{B})]_{m \times p} \), \( 1 \leq i \leq m \), \( 1 \leq k \leq p \) for \( j = 1, 2, 3, \ldots, n \).
3 Application Of Circulant Fuzzy Matrix In Human Body Disease

In a given pathology, suppose $S$ is a set of symptoms, $D$ a set of diagnosis and $P$ a set of patients. Analogous to Sanchez’s notion of medical Knowledge as an circulant order relation $R$ from the set of symptoms $S$ to the set of diagnosis $D$.

The methodology involves mainly the following 3 jobs.

1. Determination of symptoms

2. Formulation of medical knowledge based in ciculant order relations.

3. Determination of diagnosis on the basis of composition of circulant order relations.

Suppose $S$ is a set of symptoms of certain diseases, $D$ is a set of diseases and $P$ is a set of patients, construct an circulant matrix $S$ over $D$. A relation matrix say, $\tilde{R}_1$ is constructed from the circulant matrix $(F,D)$ and called symptom-disease matrix. Similarly, write an another relation matrix $S$ over $D$, say $\tilde{R}_2$, called non symptom diseases matrix. Analogous to Sanchez’s notion of medical knowledge, I refer to each of the matrices $\tilde{R}_1$ and $\tilde{R}_2$ as medical knowledge of an circulant matrix. Again I construct another circulant matrix $(F,P)$ over $S$, where $F$ is a mapping given by $F : P \rightarrow F(S)$. This circulant matrix gives another relation matrix $Q$ called patient-symptom matrix.

3.1 Algorithm

Step 1: Input the circulant matrix value over the set of Symptom $S$ over diseases $D$ and write the input value over the set of symptoms $S$ over $D$ denoted by the knowledge matrix $\tilde{R}_1$ and $\tilde{R}_2$ respectively.

Step 2: Input the circulant matrix over the set $P$ of patients over $S$ and write its relation $Q$.

Step 3: Compute the relation matrices under the composition $(\circ)$.
i. $\tilde{T}_1 = Q \circ \tilde{R}_1$

ii. $\tilde{T}_2 = Q \circ \tilde{R}_2$

iii. $\tilde{T}_3 = Q \circ (J - \tilde{R}_1)$

Where $J$ is the matrix with all its entries 1, which is the greatest element of $F$.

iv. $\tilde{T}_4 = Q \circ (J - \tilde{R}_2)$

v. Compute the diagnosis score $S_{\tilde{R}_1}$ and $S_{\tilde{R}_2}$.

\[
ST_1 = \max \{T_1(p_i, d_j), T_3(p_i, d_j)\} \text{ for } i = 1, 2, 3; j = 1, 2, 3
\]

\[
ST_2 = \max \{T_2(p_i, d_j), T_4(p_i, d_j)\} \text{ for } i = 1, 2, 3; j = 1, 2, 3
\]

**Step 4:** Find $S_k = \max[ST_1(p_i, d_j) - ST_2(p_i, d_j)]$ then we conclude the patient $p_i$ is suffering from the disease $d_k$.

**Step 5:** If $S_k$ has more than one value then go to Step 1 and repeat the process by reassessing the symptoms for the patient.

**Case Study**

Let us consider 3 patients Harish, Kumar and Ram are denoted by the set $P = \{\text{Harish, Kumar, Ram}\}$ and the set of symptoms $S = \{\text{Chest pain, Vomiting and Nausea, Shortness of breath}\}$. Let the set of diseases be $D = \{\text{Heart disease, Brain disease, lungs disease}\}$.

**Step 1**

$\tilde{R}_1 = \begin{bmatrix}
S_1 & S_2 & S_3 \\
1 & 0.5 & 0.8 \\
0.8 & 1 & 0.5 \\
0.5 & 0.8 & 1 \\
\end{bmatrix}$

$\tilde{R}_2 = \begin{bmatrix}
S_1 & S_2 & S_3 \\
0.5 & 1 & 0.4 \\
0.4 & 0.5 & 1 \\
1 & 0.4 & 0.5 \\
\end{bmatrix}$

**Step 2**

$Q = \begin{bmatrix}
P_1 & P_2 & P_3 \\
1 & 0.5 & 0.6 \\
0.6 & 1 & 0.5 \\
0.5 & 0.6 & 1 \\
\end{bmatrix}$

**Step 3**

$\tilde{T}_1 = \begin{bmatrix}
P_1 & P_2 & P_3 \\
1 & 0.6 & 0.8 \\
0.8 & 1 & 0.6 \\
0.6 & 0.8 & 1 \\
\end{bmatrix}$

$\tilde{T}_2 = \begin{bmatrix}
P_1 & P_2 & P_3 \\
0.6 & 1 & 0.5 \\
0.5 & 0.6 & 1 \\
1 & 0.5 & 0.6 \\
\end{bmatrix}$
\[ \tilde{T}_3 = \begin{array} {ccc} P_1 & 0.5 & 0.5 \\ P_2 & 0.5 & 0.5 \\ P_3 & 0.5 & 0.5 \end{array} \]

\[ \tilde{T}_3 = \begin{array} {ccc} P_1 & 0.5 & 0.6 \\ P_2 & 0.6 & 0.5 \\ P_3 & 0.6 & 0.6 \end{array} \]

**Step 4** \( S\tilde{T}_1 = \max(\tilde{T}_1, \tilde{T}_3) \)

\[ \tilde{T}_2 = \begin{array} {ccc} P_1 & 1 & 0.6 \\ P_2 & 0.8 & 1 \\ P_3 & 0.6 & 0.8 \end{array} \]

\[ \tilde{T}_4 = \begin{array} {ccc} P_1 & 0.6 & 1 \\ P_2 & 0.6 & 0.6 \\ P_3 & 1 & 0.6 \end{array} \]

\[ S\tilde{T}_2 = \max(\tilde{T}_2, \tilde{T}_4) \]

**Step 5** Now we have the difference for and against the diseases:

\[
\begin{array}{ccc|ccc}
S\tilde{T}_1 - S\tilde{T}_2 & d_1 & d_2 & d_3 \\
\hline
P_1 & 0.4 & -0.4 & 0.2 \\
P_2 & 0.2 & 0.4 & -0.4 \\
P_3 & -0.4 & 0.2 & 0.4 \\
\end{array}
\]

From the above Table, it is obvious that, if the doctor agrees, then Harish (\( P_1 \)) suffer from Heart disease and Kumar (\( P_2 \)) suffer from Brain disease and Ram (\( P_3 \)) suffer from Lungs disease.

### 4 Conclusion

Medicine is one of the field in which the workable of fuzzy set theory was recognized quite early. In this paper, we clarify the theory circulant fuzzy matrices in the field of human diseases diagnosis. We improve some new notions such as complement of max of circulant fuzzy matrix. The Doctor generally gathers knowledge about the patient from the past history, laboratory test result and other investigative procedures such as x-rays and ultra sonic rays etc. The knowledge provided by each of these sources carries with it varying degrees of uncertainty. Thus the best and most useful descriptions
of disease entities often use linguistic terms that are vague. Hence in this paper, Fuzzy set structure has been utilized in several different approaches to model the medical diagnostic process and decision making process. At the conclusion, From the above analysis it is obvious that, Harish ($P_1$) suffer from Heart disease and Kumar ($P_2$) suffer from Brain disease and Ram ($P_3$) suffer from Lungs disease. Future work in this regard would be required to study whether the concepts put forward in this paper yield a fruitful result.

References


