

Vertex Polynomial of Cycle related graphs

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Abstract

The vertex polynomial of the graph $G = (V, E)$ is defined as $V(G, x) = \sum_{k=0}^{\Delta(G)} v_k x^k$, where $\Delta(G) = \max\{d(v)/v \in V\}$ and v_k is the number of vertices of degree k . In this paper I seek to find the Vertex Polynomial of Cycle related graphs.

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1 Introduction

In this paper I consider only simple undirected graphs. The terms not defined here we can refer Frank Harary [2]. The vertex set is denoted by V and the edge set by E . For $v \in V$, $d(v)$ is the number of edges incident with v , the maximum degree of G is defined as $\Delta(G) = \max\{d(v)/v \in V\}$. Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two graphs, the union $G_1 \cup G_2$ is defined to be $G = (V, E)$ where $V = V_1 \cup V_2$ and $E = E_1 \cup E_2$, the sum $G_1 + G_2$ is defined as

$G_1 \cup G_2$ together with all the lines joining points of V_1 to V_2 . If G is of order n , the corona of G with H , $G \odot H$ is the graph obtained by taking one copy of G and n copies of H and joining the i^{th} vertex of G with an every vertex in the i^{th} copy of H . The graph G with $V = S_1 \cup S_2 \cup \dots \cup S_i \cup T$, where each S_i is a set of vertices having at least two vertices and having the same degree and $T = V \setminus \cup S_i$. The degree splitting graph of G denoted by $DS(G)$ and is obtained from G by adding the vertices $w_1, w_2, \dots, w_t$ and joining w_i to each vertex of S_i , $1 \leq i \leq t$ [5]. For each vertex v of a graph G , take a new vertex v , join v to all the vertices of G which are adjacent to v . The graph $S(G)$ thus obtained is called splitting graph of G [1]. The Cycle consisting of order n is denoted by C_n . The graph $G = (V, E)$ is simply denoted by G .

2 Main Results

Theorem 1. *The Vertex Polynomial of $C_m \cup C_n$ is given by $V(C_m \cup C_n, x) = (m + n)x^2$.*

Proof. Let C_m and C_n be two cycles with order m and n respectively. Each vertex of the graphs C_m and C_n has degree 2. Therefore, $C_m \cup C_n$ has order $m + n$ and each of its vertex has degree 2. This gives $V(C_m \cup C_n, x) = (m + n)x^2$. $\square$

Theorem 2. *The Vertex Polynomial of the graph $S(C_m \cup C_n)$ is given by $V(S(C_m \cup C_n), x) = (m + n)x^4 + (m + n)x^2$.*

Proof. Let C_m and C_n be two cycles with order m and n respectively. Each vertex of the graphs C_m and C_n has degree 2. Therefore, $C_m \cup C_n$ has order $m + n$ and each of its vertex has degree 2. From the definition of Splitting graph, the graph $S(C_m \cup C_n)$ has order $2(m + n)$. Among this, $m + n$ vertices have degree 2 and $m + n$ have degree 4. Hence, $V(S(C_m \cup C_n), x) = (m + n)x^4 + (m + n)x^2$. $\square$

Theorem 3. *The Vertex Polynomial of the graph $DS(C_m \cup C_n)$ is $V(DS(C_m \cup C_n), x) = x^{m+n} + (m + n)x^3$.*

Proof. Let C_m and C_n be two cycles with order m and n respectively. Each vertex of the graphs C_m and C_n has degree 2. Therefore, $C_m \cup C_n$ has order $m + n$ and each of its vertex has de-

gree 2. From the definition of Degree Splitting graph, the graph $DS(C_m \cup C_n)$ has order $m + n + 1$. Among this, $m + n$ vertices have degree 3 and one vertex has degree $m + n$. This gives, $V(DS(C_m \cup C_n), x) = x^{m+n} + (m + n)x^3$. $\square$

Theorem 4. *The Vertex Polynomial of the graph $C_m + C_n$ is given by $V(C_m + C_n, x) = mx^{n+2} + nx^{m+2}$.*

Proof. Let C_m and C_n be two cycles with order m and n respectively. Each vertex of the graphs C_m and C_n has degree 2. Then the graph $C_m + C_n$ has order $m + n$. Among this, m vertices have degree $n + 2$ and n vertices have degree $m + 2$. This gives, $V(C_m + C_n, x) = mx^{n+2} + nx^{m+2}$. $\square$

Theorem 5. *The Vertex Polynomial of $S(C_m + C_n)$ is given by $V(S(C_m + C_n), x) = mx^{2(n+2)} + nx^{2(m+2)} + mx^{n+2} + nx^{m+2}$.*

Proof. The graph $C_m + C_n$ has order $m + n$. From the Splitting graph definition, the graph $S(C_m + C_n)$ has order $2(m + n)$, among this, m vertices have degree $n + 2$, n vertices have degree $m + 2$, m vertices have degree $2(n + 2)$ and n vertices have degree $2(m + 2)$. Hence, $V(S(C_m + C_n), x) = mx^{2(n+2)} + nx^{2(m+2)} + mx^{n+2} + nx^{m+2}$. $\square$

Theorem 6. *The Vertex Polynomial of $DS(C_m + C_n)$ is given by $V(DS(C_m + C_n), x) = mx^{n+3} + nx^{m+3} + x^m + x^n$.*

Proof. The graph $C_m + C_n$ has order $m + n$. From the Degree Splitting graph definition, the graph $DS(C_m + C_n)$ has order $m + n + 2$, among this, m vertices have degree $n + 3$, n vertices have degree $m + 3$, one vertex has degree m and one vertex has degree n . This gives, $V(DS(C_m + C_n), x) = mx^{n+3} + nx^{m+3} + x^m + x^n$. $\square$

Theorem 7. *The Vertex Polynomial of the graph $C_m \odot C_n$ is given by $V(C_m \odot C_n, x) = mn x^3 + mx^{n+2}$.*

Proof. Let C_m and C_n be two cycles with order m and n respectively. Each vertex of the graphs C_m and C_n has degree 2. Using definition of corona, we have one copy of C_m and m copies C_n and joining the i^{th} vertex of C_m with an every vertex in the i^{th} copy of C_n . Therefore, the graph $C_m \odot C_n$ has order $m(n + 1)$, among this,

mn vertices have degree 3 and m vertices have degree $n+2$. Hence,
 $V(C_m \odot C_n, x) = mnx^3 + mx^{n+2}$. $\square$

Theorem 8. *The Vertex Polynomial of $S(C_m \odot C_n)$ is given by $V(S(C_m \odot C_n), x) = mnx^3 + mnx^6 + mx^{2(n+2)} + mx^{n+2}$.*

Proof. The graph $C_m \odot C_n$ has order $m(n+1)$, From the definition of Splitting graph, the graph $S(C_m \odot C_n)$ has order $2m(n+1)$. Among this, mn vertices have degree 3, m vertices have degree $n+2$, mn vertices have degree 6 and m vertices have degree $2(n+2)$. Therefore, $V(S(C_m \odot C_n), x) = mnx^3 + mnx^6 + mx^{2(n+2)} + mx^{n+2}$. $\square$

Theorem 9. *The Vertex Polynomial of the graph $DS(C_m \odot C_n)$ is given by $V(DS(C_m \odot C_n), x) = x^{mn} + x^m + mx^{n+3} + mnx^4$.*

Proof. The graph $C_m \odot C_n$ has order $m(n+1)$, From the definition of Degree Splitting graph, the graph $DS(C_m \odot C_n)$ has order $mn+m+2$. Among this, mn vertices have degree 4, m vertices have degree $n+3$, one vertex has degree mn and one vertex has degree m . Therefore, $V(DS(C_m \odot C_n), x) = x^{mn} + x^m + mx^{n+3} + mnx^4$. $\square$

References

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