

Location-2-Domination for Special kinds of Simple Graphs

G.Rajasekar¹ and A.Venkatesan²

¹Associate Professor, Department of Mathematics,
Jawahar Science College, Neyveli
grsmaths@gmail.com

²Assistant Professor, Department of Mathematics,
St. Josephs College of Arts and Science College (Autonomous), Cuddalore
suresh11venkat@gmail.com

Abstract

In this paper to Study the Location-2-Dominating set, Location-2-Dominating set is denoted by $R_D^2(G)$, and a theorem to find Location-2-dominating set for complete Graph, Wheel Graph, Star Graph, Complete Bipartite Graph, Helm Graph, Gear Graph, Book graph, Friendship Graph.

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1 Introduction

Throughout this Paper let us follow the terminology and notation of Harary [5]. Thus graph G is undirected and without loops with vertex set V and edge set E , an acyclic graph is called forest and tree is a connected acyclic graph, for safeguard analysis of facility, such as a fire protection study of nuclear power plants [6], the facility can be modeled by a graph or network. For such application a vertex can represent a room, hallway, etc., each edge can

connect two areas that are physically adjacent or perhaps two that are within sight or sound of each other. One primary function of safeguard system is detection of some object-perhaps detection of a fire, or perhaps detection of an intruder. Suppose detection device to be located at a vertex and the device can supply three outputs: There is a object at that vertex, there is an object on one the vertex adjacent to that vertex or no objects occupies that vertex or any adjacent one. What is necessary is to determine a collection of location at which to place the detection device so that if an object is placed at any vertex in the graph, then the object can be detected and its position uniquely identified. In order to detect an object which be placed at any vertex of V , its necessary to have a dominating set [7, 8, 9, 10, 11]. There exists the further problem of precisely determining the location of such an object. The concepts of locating sets were introduced in [10], Motivated by the safeguard analysis problem the idea of dominating and locating were merged in [11] as follows.

E. J. Cockayne and S. T. Hedetniemi [8] introduce the concept dominating set A subset S of vertices from V is called a dominating set for G if every vertex of G is either a member of S or adjacent to a member of S . A dominating set of G is called a minimum dominating set if G has no dominating set of smaller cardinality. The cardinality of minimum dominating set of G is called the dominating number for G and it is denoted by $\gamma(G)$ [2].

F.Harary and T.W.Haynes [3] introduce the concepts of double domination in graphs. A dominating set S of G is called double dominating set if every vertex in $V - S$ is adjacent to at least two vertices in S . Given a dominating set S for graph G , for each u in $V - S$ let $S(u)$ denote the set of vertices in S which are adjacent to u . The set S is called locating dominating set, if for any two vertices u and w in $V - S$ one has $S(u)$ not equal to $S(w)$ and the minimum cardinality of Location Domination number is denoted by $RD(G)$ [4].

2 Location-2-Domination

Definition 1. [1] A subset $S \subseteq V$ is Location2-Dominating set of G if S is 2-Dominating set of G and if for any two vertices

$u, v \in V - S$ such that $N(u) \cap S \neq N(v) \cap S$. The minimum cardinality of Location2-Dominating set is denoted by $R_D^2(G) = |S|$

3 Location-2-Domination for Simple Graphs

Theorem 2. [1] Location-2-Domination number of a Path P_n is $R_D^2(P_n) = \begin{cases} \frac{n-1}{2} + 1, & n \text{ is odd} \\ \frac{n}{2} + 1, & n \text{ is even} \end{cases}$

Theorem 3. [1] For any cycle C_n , $R_D^2(C_n) = \begin{cases} \frac{n}{2}, & n \text{ is even} \\ \frac{n-1}{2} + 1, & n \text{ is odd} \end{cases}$ for $n \neq 4$

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Theorem 4. In Location-2-Domination For any graph the vertex $\{v\}$ is an pendent vertex then $\{v\} \in R_D^2(G)$ only.

Proof. Let S denote the Location-2-Domination set. Suppose $\{v\} \notin S$, therefore $\{v\} \in V - S$ only now $N(v)$ is possibly one vertex only because $d_G(v) = 1$. Which is contradicts the hypothesis of 2-Domination. Therefore there is no Question for Location-2-Domination. ie; $\{v\} \in S$ only. $\square$

Theorem 5. For any complete graph with ' n ' vertices then $R_D^2(G) = n - 1$. for $n \geq 2$

Proof. Let G be complete graph with n vertices are $\{v_1, v_2, \dots, v_n\}$ each vertex degree $n - 1$. So the Location-2-Dominating set contains $n - 1$ vertices Therefore $R_D^2(G) = n - 1$. for $n \geq 2$ Suppose $|S| < n - 1$. Let $S = \{v_3, v_4, \dots, v_n\}$ and Therefore $V - S = \{v_1, v_2\}$, $N(v_1) = \{v_2, v_3, v_4, \dots, v_n\}$, $N(v_1) \cap S = \{v_3, v_4, \dots, v_n\}$ and $N(v_2) = \{v_1, v_3, v_4, \dots, v_n\}$, $N(v_2) \cap S = \{v_3, v_4, \dots, v_n\}$ Therefore $N(v_1) \cap S = \{v_3, v_4, \dots, v_n\} = N(v_2) \cap S$. Which is contradiction to Location -2-Domination. Therefore only one vertex in V only belongs to $V - S$ ie; $|S| = n - 1$. Therefore $R_D^2(G) = n - 1$, for $n \geq 2$ $\square$

Theorem 6. For any Wheel graph W_n , $n \geq 6$ then

$$R_2^D(G) = \begin{cases} \frac{n}{2} & \text{for } n \text{ is even} \\ \frac{(n-1)}{2} & \text{for } n \text{ is odd} \end{cases}$$

Proof. Let us consider the Wheel graph W_n , $n \geq 6$ with n vertices are $\{v_1, v_2, \dots, v_n\}$, $d_G(v_1) = n - 1, d_G(v_2) = d_G(v_3) = \dots d_G(v_n) = 3$

Case(i): For n is even, the vertices of W_n are $\{v_1, v_2, \dots, v_n\}$, now the resultant graph eliminate the vertex $\{v_1\}$, then $\{v_2, \dots, v_n\}$ forms a cycle of odd length $n - 1$, so by the Theorem 3 $R_2^D(G) = R_2^D(C_{n-1}) = \frac{(n-1)-1}{2} + 1 = \frac{n}{2}$

Case(ii): For n is odd, the vertices of W_n are $\{v_1, v_2, \dots, v_n\}$, now the resultant graph eliminate the vertex $\{v_1\}$, then $\{v_2, \dots, v_n\}$ forms a cycle of even length $n - 1$, so by the Theorem 3 $R_2^D(G) = R_2^D(C_{n-1}) = \frac{n-1}{2}$ $\square$

Theorem 7. For any Star graph Therefore $K_{1,n}(G)$ then Therefore $R_2^D(G) = n$

Proof. In any Star graph $K_{1,n}(G)$ has $n+1$ vertices are $\{v, v_1, v_2, \dots, v_n\}$. Clearly $d_G(v) = n$ and $d_G(v_1) = d_G(v_2) = \dots = d_G(v_n) = 1$. Let S be the Collection of Location-2-Dominating set, by the Theorem 4 the S -set Containing all the pendent vertices.

Therefore $|S| = n$; ie; $R_2^D(G) = n$

Suppose any one $\{v_1, v_2, \dots, v_n\} \in V - S$. If Possibly Assume that $v_1 \in V - S$. Then $N(v_1) = \{v\}$ only which contradiction to Theorem 4, so there is no question about Location-2-Domination. The only Possibility all the pendent vertices $\{v_1, v_2, \dots, v_n\} \in V - S$. Therefore $|S| = n$. ie; $R_2^D(G) = n$. $\square$

Theorem 8. For any Helm graph H_n , then $R_2^D(H_n) = n + 1$.

Proof. Let us Consider the Helm graph H_n , has $2n + 1$ vertices are $\{v, v_1, v_2, \dots, v_n, u_1, \dots, u_n\}$. Then $d_G(v) = n, d_G(v_1) = d_G(v_2) = \dots = d_G(v_n) = 4$ and $d_G(u_1) = d_G(u_2) = \dots = d_G(u_n) = 1$. So the S -set contains the vertices are $\{v, u_1, \dots, u_n\}$ only ie; $|S| = n + 1$. Therefore $R_2^D(H_n) = n + 1$. Suppose any one of the Pendent vertices $\{u_1, \dots, u_n\} \notin S$ therefore the only Possibility they should present in $V - S$. In this case could not find 2-domination for the vertices $\{u_1, \dots, u_n\}$ therefore there is no question for Location-2-

Domination. So only Possibility $\{u_1, \dots, u_n\} \in S$. ie; $|S| = n + 1$.
Therefore $R_2^D(H_n) = n + 1$. $\square$

Theorem 9. For any Gear graph G_n , then $R_2^D(G_n) = n$.

Proof. Let us consider the Gear graph G_n has $2n + 1$ vertices are $\{v, v_1, v_2, \dots, v_n, u_1, \dots, u_n\}$ with $3n$ edges, Then $d_G(v) = n$, $d_G(v_1) = d_G(v_2) = \dots = d_G(v_n) = 3$ and $d_G(u_1) = d_G(u_2) = \dots = d_G(u_n) = 2$. Clearly the S -set containing the vertices are $\{v_1, v_2, \dots, v_n\}$ ie; $|S| = n$ Therefore $R_2^D(G_n) = n$
Suppose S -set containing the vertices are $\{u_1, u_2, \dots, u_n\}$ it must also need to contain the vertex $\{v\}$. Because there is no adjacency from $\{u_1, u_2, \dots, u_n\}$ to $\{v\}$ therefore $|S| = n + 1$ which is not good? Therefore $R_2^D(G_n) = n$. $\square$

Theorem 10. For any Book graph $B_n, (S_{m+1}P_2)$ then $R_2^D(B_n) = n + 1$.

Proof. Let us consider the B_n graph with $2n + 2$ vertices are $\{u, u_1, \dots, u_n, v, v_1, v_2, \dots, v_n\}$ clearly $d_G(u) = d_G(v) = n$ and $d_G(u_1) = \dots = d_G(u_n) = d_G(v_1) = \dots = d_G(v_n) = 2$. The collection of Location-2-Dominating set is followed by either $\{v, u_1, u_2, \dots, u_n\}$ or $\{u, v_1, v_2, \dots, v_n\}$ only.
ie; $|S| = n + 1$. Therefore $R_2^D(B_n) = n + 1$
Suppose the collection of S - set is either $\{u, u_1, u_2, \dots, u_n\}$ or $\{v, v_1, v_2, \dots, v_n\}$ then $\{v, v_1, v_2, \dots, v_n\} \in V - S$ or $\{u, u_1, u_2, \dots, u_n\} \in V - S$, in this case $N(v) = \{u\}$, $N(v_1) = \{u_1\}$, $\dots$, $N(v_n) = \{u_n\}$ and $N(u) = \{v\}$, $N(u_1) = \{v_1\}$, $\dots$, $N(u_n) = \{v_n\}$ are only exists which is contradiction to 2-domination , so there is no question about Location-2-Domination. Therefore the S - set contains either $\{v, u_1, u_2, \dots, u_n\}$ or $\{u, v_1, v_2, \dots, v_n\}$ set only.
ie; $|S| = n + 1$. Therefore $R_2^D(B_n) = n + 1$. $\square$

Theorem 11. For any Friendship graph F_n then $R_2^D(F_n) = n + 1$.

Proof. For any F_n has $2n + 1$ vertices are $\{v, v_1, v_2, \dots, v_n, v_{n+1}, v_{n+2}, \dots, v_{2n}\}$, then $d_G(v) = 2n$, and $d_G(v_1) = d_G(v_2) = \dots = d_G(v_{2n}) = 2$. Clearly F_n is fusion of C_3 in n times with a single vertex $\{v\}$ By the Theorem 3 $R_2^D(C_3) = 2$, in C_3 the S -set are either $\{v, v_1\}$ or $\{v, v_2\}$ Now Define $R_2^D(F_m) = m$ times fussion of $R_2^D(C_3)$ with single vertex,

For $m = 1$, $R_2^D(F_1) = R_2^D(C_3) = 2$

For $m = 2$, $R_2^D(F_2) = R_2^D(C_3) + R_2^D(C_3) = 2 + 2 - 1 = 3$

Similarly $m = 3, \dots, n - 1$ For $m = n$, $R_2^D(F_n) = R_2^D(C_3) + \dots + R_2^D(C_3) = 2 + 2 + \dots + 2 - 1 - 1 - \dots - 1 = 2n - (n - 1) = n + 1$

Therefore $R_2^D(F_n) = n + 1$

Suppose, the Fusion vertex not a Location-2-Domination vertex in C_3 . Clearly the S -set Contains more than $n + 1$ vertices .which is not good, therefore Fusion vertex is a one the vertex in S -set .Therefore the S -set contains only $n + 1$ vertices. $\square$

Theorem 12. For any complete bipartite graph $K_{n,m}$ then

$$R_2^D(G) = \begin{cases} m + 1, & n = 2, m = 2, 3, \dots \\ n + m - 2, & n > 2, m = 2, 3, \dots \end{cases}$$

Proof. Let us consider the Complete Bipartite graph $K_{n,m}$, the vertex set are $\{v_1, v_2, v_3, \dots, v_n\}$ and $\{u_1, u_2, u_3, \dots, u_m\}$. Let S -set be the Location -2-Dominating Set. Case(i): For $n = 2, m = 2, 3, \dots$ the vertex set of $K_{n,m}$ are $\{v_1, v_2\}$ and $\{u_1, u_2, u_3, \dots, u_m\}$ then $d_G(v_1) = d_G(v_2) = m$ and $d_G(u_1) = d_G(u_2) = \dots d_G(u_m) = 2$ now collect S -set possibly by $\{v_1, v_2, u_1, u_3, \dots, u_m\}$ vertices are only. ie collect all the vertex in V - vertex set, because they have only two elements if omit any one vertex from V - vertex set we could not find 2-Domination for U - vertex set, and only one vertex is omit from U -vertex set possible, if omit more than its contradicts to 2-Domination . so there is no question for Location-2-Domination for these cases. Therefore S -set contains only $m + 1$ vertices. ie; $R_2^D(G) = m + 1$, $n = 2, m = 2, 3, \dots$ Case(ii): For $n > 2, m = 2, 3, \dots$ the vertex set of $K_{n,m}$ are $\{v_1, v_2, \dots, v_n\}$ and $\{u_1, u_2, u_3, \dots, u_m\}$ then $d_G(v_1) = \dots = d_G(v_n) = m$ and $d_G(u_1) = d_G(u_2) = \dots = d_G(u_m) = n$ now collect S -set possibly by $\{v_1, v_2, v_4, \dots, v_n, u_1, u_3, \dots, u_m\}$ only, ie collect the vertex in V -vertex set are $(n - 1)$ only. If omit more than one its contradicts the 2-Domination for U - vertex set, also only one vertex is omit from U -vertex set ,if omit more than its contradicts the 2-Domination for V - vertex set so there is no question for Location-2-Domination for these cases. Therefore S -set contains only $n - 1 + m - 2 = n + m - 2$ vertices .ie; $R_2^D(G) = n + m - 2$, $n > 2, m = 2, 3, \dots$ $\square$

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