

MEAN AND VARIANCE OF TIME TO RECRUITMENT IN A TWO GRADED MANPOWER SYSTEM WITH INTER-DECISION TIMES HAVING GEOMETRIC PROCESS AND DEPLETION HAVING INDEPENDENT AND NON-IDENTICALLY DISTRIBUTED RANDOM VARIABLES

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Abstract

In this paper, an organization subjected to a random exit of personnel due to policy decisions taken by the organization is considered; there is an associated loss of manpower if a person quits the organization. As the exit of personnel is unpredictable, a recruitment policy involving two thresholds, optional and mandatory is suggested to enable the organization to plan its decision on appropriate univariate policy of recruitment. Based on shock model approach, a mathematical model is constructed using an appropriate univariate policy of recruitment. The analytical expressions for mean and variance of time to recruitment is obtained when i) the loss of manpower forms a sequence of independent and non-identically distributed exponential random variables ii) inter-decision times are geometric process iii) the optional and mandatory thresholds having exponential distribution.

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1 Introduction

The problem of time to recruitment is studied by several authors both for a single and multigraded system for different types of thresholds, according as the inter-decision times are independent and identically distributed random variables (or)

correlated random variables. For a multi-graded system, in [5] the author has obtained the performance measures namely mean and variance of the time to recruitment for a two graded system, when the loss of manpower, the threshold and the inter-decision times are independent and identically distributed exponential random variables forming the same renewal process for both grades.

In [9] the authors have obtained the performance measures when the loss of manpower follows poisson distribution and thresholds for the loss of manpower in the two grades are geometric random variables. In [7] the author has extended the results in [3] for geometric thresholds when the inter-decision times for the two grades form two different renewal processes. In [8] is studied when the loss of manpower and thresholds are geometric random variables and the inter-decision times are correlated random variables or forming two different renewal processes. In [2] the author has considered a new univariate recruitment policy involving two thresholds in which one is optional and the other a mandatory and obtained the mean time to recruitment under different conditions on the thresholds according as the inter-decision times are independent and identically distributed random variables or the inter-decision times are exchangeable and constantly correlated exponential random variables and also obtained the mean time to recruitment when the optional and mandatory thresholds are geometric random variables. For a single graded manpower systems, in [6] the authors have obtained the mean and variance of time to recruitment when the loss of manpower form a sequence of independent and identically distribution poisson random variables and the thresholds follows geometric distribution. In [4] the authors have obtained the mean time to recruitment for a single grade manpower system by assuming that the inter-decision times form a geometric process, the loss of manpower is a sequence of independent and identically distributed exponential random variables and the thresholds are exponential. In [1] the authors have studied the results of [4] for a two graded systems when the thresholds are exponential thresholds or SCBZ property possessing thresholds or extended exponential thresholds or, geometric thresholds. In [10] the authors have obtained the mean time to recruitment for a two graded manpower system with a univariate policy of recruitment involving combined thresholds using geometric process for inter-decision times. Recently in [11] the authors have obtained mean and variance of time to recruitment for a two graded manpower system with a univariate policy of recruitment involving (i) the loss of manpower and inter-decisions times are independent and non-identically distributed exponential random variables (ii) thresholds optional and mandatory follows exponential random variables. The objectives of the present paper is to study the problem of time to recruitment for a two graded manpower systems

and to obtain the mean and variance of time to recruitment using CUM univariate recruitment policy for exponential thresholds with loss of manpower having independent and non-identically distributed exponential random variables, inter-decision times having geometric process. The analytical results are numerically illustrated and the influence of nodal parameters on the mean and variable of time to recruitment is studied.

2 Notations

X_i	The loss of manpower due to the i^{th} decision epoch $i = 1, 2, 3 \dots$ forming a sequence of independent and non-identically distributed exponential random variables with parameters $\alpha_i, (\alpha_i > 0)$.
S_k	Cumulative loss of manpower in the first k -decisions ($k = 1, 2 \dots$)
$G_k(\cdot)$	The distribution function of sum of k independent and non-identically distributed exponential random variables.
$g_k(\cdot)$	The probability density function of S_k
	$G_k(t) = \sum_{i=1}^k c_i (1 - e^{-\alpha_i t}), g_k(t) = \sum_{i=1}^k c_i \alpha_i e^{-\alpha_i t}$ $g_k^*(s) = \sum_{i=1}^k c_i \frac{\alpha_i}{\alpha_i + s} \text{ where } c_i = \prod_{j=1, j \neq i}^k \frac{\alpha_j}{\alpha_j + \alpha_i}$
U_1	An exponential random variables.
$\sqrt{}$	Exponential parameters of U_1
$F(\cdot)$	Probability distribution function of U_1
$f(\cdot)$	Probability distribution function of U_1
U_k	A continuous random variables denoting the inter-decision times between $(k-1)^{th}$ & the k^{th} decision epochs, $k = 1, 2, 3 \dots$
a	Parameter for geometric process $U_k, (a > 0)$
$T_k(\cdot)$	Probability distribution function of U_k , the k^{th} term of geometric process
$t_k(\cdot)$	Probability density function of U_k
$F_k(\cdot)$	Probability distribution function of U_k
$F_k^*(\cdot)$	Laplace stieltjes transform f convolution of distribution function of U_k
$f_k^*(\cdot)$	Laplace transform of convolution of density function of U_k
Y_1, Y_2	The continuous random variables denoting the optional thresholds levels for the grade1 and grade 2 follows exponential distribution with parameters λ_1, λ_2 respectively.

Z_1, Z_2	The continuous random variables denoting the mandatory thresholds levels for the grade1 and grade 2 follows exponential distribution with parameters μ_1, μ_2 respectively.
W	The continuous random variable denoting the time to recruitment in the organization.
p	The probability that the organization is not going for recruitment whenever the total loss of manpower crosses the optional threshold Y
$V_k(t)$	The probability that exactly k -decisions are taken in $[0, t)$
$L(\cdot)$	Distribution function of W
$l(\cdot)$	The probability density function of W
$l^*(\cdot)$	The Laplace transform of $l(\cdot)$
$E(W)$	The expected time to recruitment
$V(W)$	The variance of the time to recruitment

CUM policy : Recruitment is done whenever the cumulative loss of manpower crosses the mandatory threshold. The organization may or may not go for recruitment if the cumulative loss of manpower crosses the optional threshold.

Main Results

The survival function of W is given by

$$P(W > t) = \sum_{k=0}^{\infty} V_k(t) P(S_k < Y) + \sum_{k=0}^{\infty} V_k(t) P(S_k \geq Y) P(S_k < Z) p \quad (1)$$

For maximum case

$$P(S_k < Y) = \int_0^{\infty} P(S_k < y | S_k = x) g_k(x) dx = g_k^*(\lambda_1) + g_k^*(\lambda_2) - g_k^*(\lambda_1 + \lambda_2) \quad (2)$$

$$P(S_k < Y) = D_1 + D_2 - D_3 \quad (3)$$

Similarly,

$$P(S_k < Z) = g_k^*(\mu_1) + g_k^*(\mu_2) - g_k^*(\mu_1 + \mu_2) \quad (4)$$

$$P(S_k < Y) = D_4 + D_5 - D_6 \quad (5)$$

where $D_1 = g_k^*(\lambda_1)$ $D_2 = g_k^*(\lambda_2)$ $D_3 = g_k^*(\lambda_1 + \lambda_2)$
 $D_4 = g_k^*(\mu_1)$ $D_5 = g_k^*(\mu_2)$ $D_6 = g_k^*(\mu_1 + \mu_2)$

Sub (3) & (5) in (1), we get,

$$P(W > t) = \sum_{k=0}^{\infty} V_k(t) A_k \quad \text{where} \quad A_k = B_k(1 - pC_k) + pC_k \quad (6)$$

where $B_k = D_1 + D_2 - D_3$ and $C_k = D_4 + D_5 - D_6$

$$\begin{aligned}
 P(W > t) &= \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)] A_k \\
 L(t) = 1 - P(W > t) &= \sum_{k=0}^{\infty} F_{k+1}(t) A_k - \sum_{k=0}^{\infty} F_k(t) A_k \\
 l^*(s) &= \sum_{k=0}^{\infty} f_{k+1}^*(s) A_k - \sum_{k=0}^{\infty} f_k^*(s) A_k
 \end{aligned} \tag{7}$$

where

$$\begin{aligned}
 A_k &= [g_k^*(\lambda_1) + g_k^*(\lambda_2) - g_k^*(\lambda_1 + \lambda_2)] [1 - p(g_k^*(\mu_1) + g_k^*(\mu_2) - g_k^*(\mu_1 + \mu_2))] \\
 &\quad + p[(g_k^*(\mu_1) + g_k^*(\mu_2) - g_k^*(\mu_1 + \mu_2))]
 \end{aligned}$$

Since $\{U_k\}$ is a geometric process, $T_k(u) = P(T_k \leq u) = F(a^{(k-1)}U)$, $k = 1, 2, 3, \dots$

$$t_k(u) = \frac{d}{du} [T_k(u)] = a^{k-1} f(a^{k-1}u), t_k^*(s) = a^{k-1} f^*(a^{k-1}s) = f^*\left(\frac{s}{a^{k-1}}\right)$$

$$\text{Hence } f_k^*(s) = \prod_{r=1}^k f^*\left(\frac{s}{a^{r-1}}\right), f_k^*(s) = \prod_{r=1}^{k+1} f^*\left(\frac{s}{a^{r-1}}\right), k = 1, 2, 3, \dots \tag{8}$$

It is known that

$$E(W) = - \left[\frac{d}{ds} l^*(s) \right]_{s=0}, E(W^2) = \left[\frac{d^2}{ds^2} l^*(s) \right]_{s=0}, Var(W) = E(W^2) - (E(W))^2 \tag{9}$$

$$\text{Consider } \prod_{r=1}^k f^*\left(\frac{s}{a^{r-1}}\right) = f^*\left(\frac{s}{a^0}\right) f^*\left(\frac{s}{a^1}\right) f^*\left(\frac{s}{a^2}\right) \dots f^*\left(\frac{s}{a^{k-1}}\right)$$

$$f^*(0) = 1, f^{*1}(0) = \frac{d}{ds} f^*(0) = 1$$

$$\frac{d}{ds} \left[\prod_{r=1}^k f^*\left(\frac{s}{a^{r-1}}\right) \right]_{s=0} = f^{*1}(0) \left[\frac{a^k - 1}{a^{k-1}(a - 1)} \right] \tag{10}$$

$$\text{Consider } \prod_{r=1}^{k+1} f^*\left(\frac{s}{a^{r-1}}\right) = f^*\left(\frac{s}{a^0}\right) f^*\left(\frac{s}{a^1}\right) f^*\left(\frac{s}{a^2}\right) \dots f^*\left(\frac{s}{a^k}\right)$$

$$\frac{d}{ds} \left[\prod_{r=1}^{k+1} f^* \left(\frac{s}{a^{r-1}} \right) \right]_{s=0} = f^{*1}(0) \left[\frac{a^{k+1} - 1}{a^k(a-1)} \right] \quad (11)$$

From (7)

$$\left[\frac{d}{ds} l^*(s) \right]_{s=0} = \sum_{k=0}^{\infty} \frac{d}{ds} \left(\prod_{r=1}^{k+1} f^* \left(\frac{s}{a^{r-1}} \right) \right)_{s=0} A_k - \sum_{k=0}^{\infty} \frac{d}{ds} \left(\prod_{r=1}^k f^* \left(\frac{s}{a^{r-1}} \right) \right)_{s=0} A_k \quad (12)$$

Substitute equations (10) , (11) and (12) in $E(W)$, we get

$$E(W) = E(U_1) \sum_{k=0}^{\infty} \frac{1}{a^k} A_k \quad (13)$$

$$\begin{aligned} \text{Consider } \frac{d^2}{ds^2} \left[\prod_{r=1}^k f^* \left(\frac{s}{a^{r-1}} \right) \right] &= \frac{d}{ds} \left[\frac{d}{ds} \left(\prod_{r=1}^k f^* \left(\frac{s}{a^{r-1}} \right) \right) \right] \\ \frac{d^2}{ds^2} \left[\prod_{r=1}^k f^* \left(\frac{s}{a^{r-1}} \right) \right]_{s=0} &= \sigma_{u_1}^2 \left[\frac{(a^{2k} - 1)}{(a^2 - 1)a^{2(k-1)}} \right] + E(u_1) \left(\frac{(a^k - 1)^2}{(a - 1)^2 a^{2(k-1)}} \right) \end{aligned} \quad (14)$$

where $f^{*11}(0) - (f^{*1}(0))^2 = \text{var}(f^{*1}(0)) = \sigma_{u_1}^2$ and $-(f^{*1}(0)) = E(u_1)$

Similarly,

$$\begin{aligned} \text{Consider } \frac{d^2}{ds^2} \left[\prod_{r=1}^{k+1} f^* \left(\frac{s}{a^{r-1}} \right) \right] &= \frac{d}{ds} \left[\frac{d}{ds} \left(\prod_{r=1}^{k+1} f^* \left(\frac{s}{a^{r-1}} \right) \right) \right] \\ \frac{d^2}{ds^2} \left[\prod_{r=1}^{k+1} f^* \left(\frac{s}{a^{r-1}} \right) \right]_{s=0} &= \sigma_{u_1^2}^2 \left(\frac{a^{2(k+1)} - 1}{(a^2 - 1)a^2} \right) + E(u_1) \left(\frac{(a^{k+1} - 1)^2}{(a - 1)^2 a_2 k} \right) \end{aligned} \quad (15)$$

where $\sigma_{u_1^2} = f^{*11}(0) - (f^{*1}(0))^2$, $E(u_1) = -(f^{*1}(0))$

From equation(7),we get

$$\frac{d^2}{ds^2} [l^*(s)]_{s=0} = \sum_{k=0}^{\infty} \frac{d^2}{ds^2} \left(\prod_{r=1}^{k+1} f^* \left(\frac{s}{a^{r-1}} \right) \right)_{s=0} A_k - \sum_{k=0}^{\infty} \frac{d^2}{ds^2} \left(\prod_{r=1}^k f^* \left(\frac{s}{a^{r-1}} \right) \right)_{s=0} A_k \quad (16)$$

Sub (14),(15) and (16) in $E(W^2)$,we get

$$E(W^2) = \sum_{k=0}^{\infty} \frac{\sigma U_1^2 A_k}{a^{2k}} + \sum_{k=0}^{\infty} \frac{E(U_1)}{((a-1))a^{2k}} (a+1-2a^{k+1}) A_k \quad (17)$$

Using (13) and (17) with $\text{var}(W)$, we get the variance of maximum model.

3 Conclusion

The analytical expressions for the performance measures namely mean and variance of the time to recruitment are analyzed numerically by varying one parameter and keeping other parameters fixed. The effect of nodal parameter a on the performance measures are shown in the following tables.

Effect of a on performance measures

$\alpha_1 = 0.001$, $\alpha_2 = 0.00$, $\alpha_3 = 0.003$, $\alpha_4 = 0.004$, $\alpha_5 = 0.005$, $E(U_1) = 0.9$, $\sigma_{U_1}^2 = 4$, $\lambda_1 = 0.01$, $\lambda_2 = 0.02$, $\mu_1 = 0.03$, $\mu_2 = 0.04$, $p = 0.06$.

Table:1

a	2	3	4	5	6
$E(W)$	0.0533	0.0345	0.0255	0.0202	0.0167
$E(W^2)$	0.0676	0.0288	0.0159	0.0100	0.0069
$V(W)$	0.0648	0.0276	0.0152	0.0096	0.0066

Table:2

a	0.97	0.825	0.625	0.451	0.25
$E(W)$	0.1229	0.1509	0.2209	0.3730	1.4377
$E(W^2)$	0.3491	0.5366	1.3295	6.9829	929.5116
$V(W)$	0.3340	0.5138	1.2807	6.8437	927.4445

4 Findings

From the Table : 1 we observe that

1. As $a > 1$, the U'_k 's, $k = 1, 2, 3 \dots n$ form a decreasing sequence and hence the inter-arrival times between decision epochs will decrease. Consequently, the expected time to recruitment and the variance of the time to recruitment decrease.

From the Table : 2 we observe that

2. As $a < 1$, the U'_k 's, $k = 1, 2, 3 \dots n$ form a increasing sequence and hence the inter-arrival times between decision epochs will increase. Consequently, the expected time to recruitment and the variance of the time to recruitment increase.

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