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Modal Analysis on the Basis of the Finite Element Method in the Problem of Studying the Vibration Stability of a Steam Miniturbine

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Abstract:
This article deals with the rotor of a three-stage axial steam miniturbine with a power of 250 kW to ensure the functional performance of the turbine at the speed of 12,000 rpm. High speeds of rotation of the turbine rotor make increased demands on the mechanical strength of this element, both static and dynamic. The forced oscillations of the turbine rotor were studied. The search and analysis of the natural frequencies of torsional and flexural oscillations of rotor elements with the use of modal analysis on the basis of the finite element method using the ANSYS mathematical package were carried out. The Campbell diagram was constructed. The frequencies of the first thirty harmonics of the bending and torsional oscillations of the rotor design elements were determined. The dependence of the natural frequencies of oscillations on the turbine rotation frequency was revealed. The main critical rotation frequencies were determined from the point of view of the danger of occurrence of resonance.

Keywords: steam turbine, rotor, natural oscillation frequencies, resonance, finite element method, modal analysis, Campbell diagram.

1 Introduction

The main and most critical element of the turbine unit is its rotor, which ensures the functional performance of the turbine. At the same time, turbine rotors rotate with high speeds, which introduces increased demands on the mechanical strength of this element, both static and dynamic.

Regarding the dynamics of the turbounit, it should be taken into account that the majority of the processes occurring in it under transitional operating modes are of an oscillatory nature. So, most of the details are designed in such a way that the equivalent mechanical stresses in them do not exceed the yield strength of the material; one can assume that the internal elastic forces in the body of the part are proportional to its deformations and opposite to them in the direction. Such forces are called restoring forces. A mechanical system moving under the action of restoring forces is somehow inclined to make trasition into oscillatory modes of motion. The dynamics of the transitional modes of operation of the machine, including the turb characteristics of the oscillation processes occurring in it [1]-[5].

The rotating turbine rotor is subject to various periodic perturbations associated with the imperfection of the bearing supports, technological manufacturing tolerances resulting in a non-uniform distribution of the rotor mass and displacement of its center of mass from the rotation axis, as well as with the inhomogeneity of loading of the rotor working surfaces interacting with the moving gas flow. These disturbances lead to the development of forced oscillations of the rotor. When designing a turboset, it is important to take into account the characteristics of the forced oscillations and to provide for the possibility of appearance of resonant phenomena leading to the occurrence of extreme loads and destruction of the rotor parts [6]-[9].

In the light of the above factors, the problem of studying the characteristics of free and forced oscillations of the rotors of a turbomachine at the stage of its design is a topical one. In solving this problem, the turbine rotor should be considered as a continuum mechanical system, since the intrinsic elastic properties of the rotor of the turbine exert a significant influence on the vibrations arising during its operation.

A modern approach to solving such problems is modal analysis based on the study of the equations of oscillations of machine parts using the finite element method (FEM) [10]-[14].

In this paper, we consider the application of the universal ANSYS package to the modal analysis of the oscillations of the rotor of a steam miniturbine PMT-250 with a power of 250 kW, developed by LLC NPP Donskie Tekhnologii.

The work is carried out within the framework of the agreement with the Ministry of Education and Science of the Russian Federation No. 14.579.21.0123 on granting a subsidy of October 27, 2015. The topic is: "Creation of a highly efficient steam turbine for liquid and solid organic waste processing technologies for energy production for small distributed power engineering", Federal target program "Research and development in the priority areas of development of the scientific and technological complex of Russia for 2014-2020". The unique identifier of applied scientific research (project) is RFMEFI57915X0123.

2 Method of Modal Analysis of Turbine Rotor Oscillations Based on FEM

Consider a mechanical system having a finite number of degrees of freedom [15], equal to n. The equations of its motion can be represented in the vector-matrix form

$$\mathbf{M}(\mathbf{q},t)\mathbf{q}+\mathbf{k}(\mathbf{q},\mathbf{q},t)=\mathbf{Q}(t)$$

where $\mathbf{q}$ is the vector-column of generalized coordinates of the system; $\mathbf{M}(\mathbf{q},t)$ is the matrix of inertia of the system; $\mathbf{k}(\mathbf{q},\mathbf{q},t)$ is a nonlinear vector function, determined by centrifugal and Coriolis forces of inertia, applied to the system; $\mathbf{Q}(t)$ is the vector-column of generalized external forces. The solution of this equation determines the trajectory $\mathbf{q}=\mathbf{q}(t)$ of the motion of the mechanical system, which can be represented as the sum of two motions

$$\mathbf{q}=\mathbf{q}_0(t)+\Delta\mathbf{q}(t), \quad (1)$$

where $\mathbf{q}_0(t)$ is the steady mode of motion under the action of forces $\mathbf{Q}(t)$, in our case, the rotation of the turbine rotor around the longitudinal axis; $\Delta\mathbf{q}(t)$ is the perturbed motion, expressed in terms of deviations of generalized coordinates from the trajectory $\mathbf{q}_0(t)$. Assuming the system parameters not to be explicitly dependent on time and considering the equations of motion in the form

$$\mathbf{M}(\mathbf{q})\mathbf{q}+\mathbf{k}(\mathbf{q},\mathbf{q})=\mathbf{Q}(t), \quad (2)$$

we expand (2) in a Taylor series in a neighborhood of the steady-state trajectory

$$\left[\mathbf{M}(\mathbf{q}_0)+\frac{\partial\mathbf{M}}{\partial\mathbf{q}}\Delta\mathbf{q}+\mathrm{K}\right](\mathbf{q}_0+\Delta\mathbf{q})+\mathbf{k}(\mathbf{q}_0,\mathbf{q}_0)+\frac{\partial\mathbf{k}}{\partial\mathbf{q}}\Delta\mathbf{q}+\frac{\partial\mathbf{k}}{\partial\mathbf{q}}\Delta\mathbf{q}+\mathrm{K}=\mathbf{Q}(t) \quad (3)$$

Assuming the deviations from the steady-state regime to be small, we open the brackets in (3) and, neglecting small quantities of the second order and above, in view of (2), we obtain

$$\mathbf{M}(\mathbf{q}_0)\Delta\mathbf{q}+\frac{\partial\mathbf{k}}{\partial\mathbf{q}}\Delta\mathbf{q}+\left[\frac{\partial\mathbf{M}}{\partial\mathbf{q}}\mathbf{q}_0+\frac{\partial\mathbf{k}}{\partial\mathbf{q}}\right]\Delta\mathbf{q}=0 \quad (4)$$

Equation (4) is reduced to the form

$$\mathbf{M}\Delta\mathbf{q}+\mathbf{B}\Delta\mathbf{q}+\mathbf{C}\Delta\mathbf{q}=0$$

where $\mathbf{B}=\dfrac{\partial\mathbf{k}}{\partial\mathbf{q}}$ is the damping matrix;

$\mathbf{C}=\left[\dfrac{\partial\mathbf{M}}{\partial\mathbf{q}}\mathbf{q}_0\,\dfrac{\partial\mathbf{k}}{\partial\mathbf{q}}\right]$ is the rigidity matrix of the system under consideration; this dependence is called the differential equation of small oscillations of the system. Assuming that the damping is insignificant, we obtain the equation of free small oscillations of the system, the solution of which has the form

$$\Delta\mathbf{q}(t)=\sum_{i=1}^{n}\mathbf{s}_i\cos\omega_i t \quad (5)$$

where ω_i are the natural frequencies of the system's oscillations, defined as the imaginary part of the roots of the characteristic equation [7] $\det\left(\mathbf{C}-\omega^2\mathbf{M}\right)=0,$ while $\mathbf{s}_i$ are the eigenvectors of the system, which are nontrivial solutions of the system of linear equations

$$\left(\mathbf{C}-\omega^2\mathbf{M}\right)\mathbf{s}_i=0.$$

The component $\Delta\mathbf{q}_i(t)=\mathbf{s}_i\cos\omega_i t$ of the general solution (5) is called the harmonic or mode of oscillation of the system. The number of modes equals the number of degrees of freedom of the system under cons

If we take into account the oscillation equations obtained using the apparatus of the theory of elasticity [16], then the rotor of a steam turbine is a continuum system with an infinite number of degrees of freedom. In the numerical solution of the equations of the theory of elasticity by the FEM, one applies the approximation of the general solution of the system by piecewise linear functions, which leads to the appearance of equations of the form

$$\mathbf{M}\mathbf{u}+\mathbf{B}\mathbf{u}+\mathbf{C}\mathbf{u}=0,$$

where $\mathbf{u}$ is the displacement of the nodes of the finite element mesh. Thus, the continual system is reduced to solving the problem of small oscillations of a discrete system with a finite number of degrees of freedom and the application of the above mathematical apparatus.

This problem is solved using ANSYS mathematical package, which contains a specialized block of modal analysis of structures.

3 The Results of the Study on Determining the Frequency and Shape of Free Oscillations of the Parts of the Turbine Rotor

We present a solution to the problem of modal analysis for determining the natural frequencies and oscillation modes of the rotor parts of a steam miniturbine obtained in ANSYS. The torsional and flexural oscillations of the rotor were studied (Figure 1).

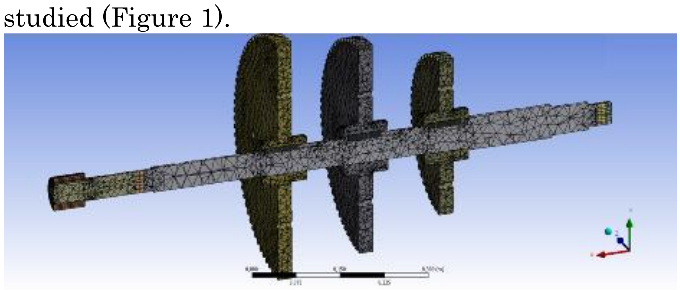

Figure 1. Finite element mesh for calculating stresses in a model of the moving part of the turbine

Being continual, this system has an infinite spectrum of oscillations. The analysis of the resonance frequencies of oscillations of the rotor parts is limited to the determination of thirty basic harmonics. The minimum number of harmonics is determined by the parameter of the effective equivalent mass of the structure, which, to ensure the required accuracy, should be at least 90% of the real mass of the rotor [17],[18].

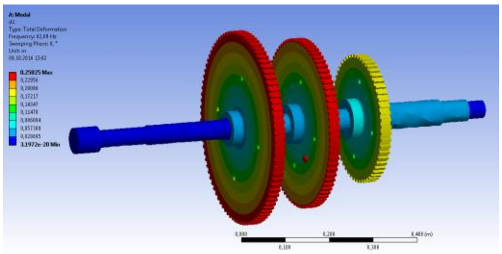

Figure 2. The first form of the torsional oscillations of the shaft

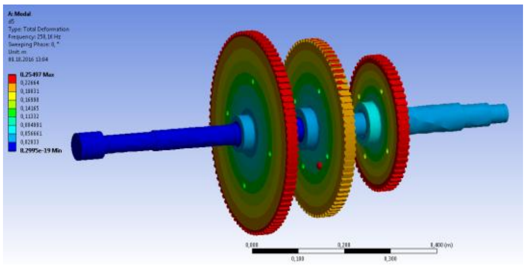

Figure 3. The second form of the torsional oscillations of the shaft

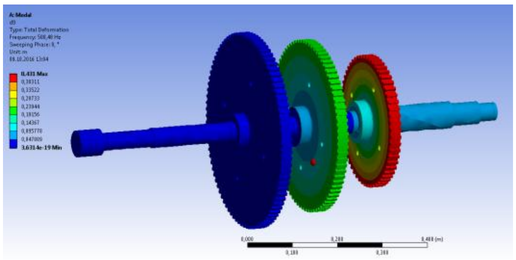

Figure 4. The third form of the torsional oscillations of the shaft

The torsional oscilations of the rotor are represented by three main harmonics (Figures 2-4, Table 1)

Form of oscillations	f_i, Hz	A_m, mm
1	62.69	0.25
2	258.16	0.25
3	508.48	0.43

Table 1: Frequencies and amplitudes of torsional oscillations of the turbine shaft

As will be shown below, the torsional oscillations of the rotor shaft lying in the low frequency region are most dangerous from the point of view of the occurrence of resonance in the range of operating frequencies of the turbine rotor rotation. The

are

shaft at the maximum moment of inertia of the structure relative to the longitudinal axis of the shaft.

The flexural oscillations of the rotor shaft are represented by a wider spectrum (Figure 5, Table 2).

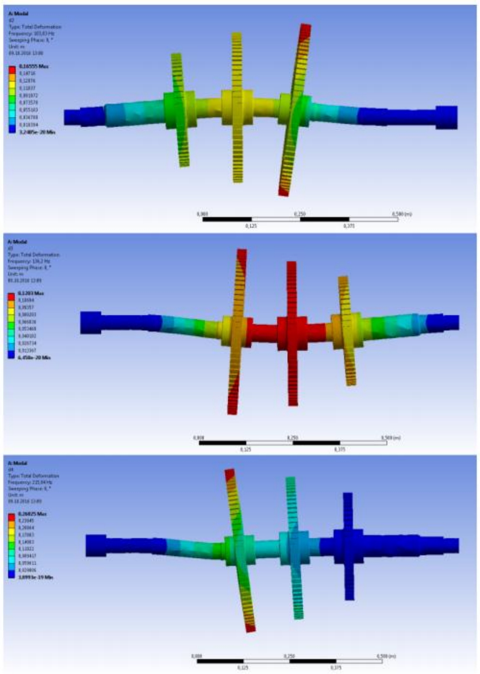

Figure 5. The first three forms of bending oscillations of the rotor shaft

Forms of oscillations	f_i, Hz	A_m, mm
1	136.2	0.12
2	215.94	0.27
3	268.42	0.14
4	450.67	0.12
5	688.12	0.12
6	698.1	0.17
7	867.3	0.14
8	995.68	0.09
9	1,082.6	0.17
10	1,448.6	0.13
11	1,713.9	0.12

Table 2. Frequencies and amplitudes of flexural oscillations of the turbine shaft

Harmonics from the first to the ninth lie in the working frequency range of the turbine rotor rotation, and also can be dangerous from the point of view of resonance occurrence.

4 Determination of the Critical Rotation Frequencies of the Turbine Rotor Based on the Campbell Diagram

The modal analysis solution block of ANSYS allows taking into account the gyroscopic effects associated with the rotation of the rotor around the longitudinal axis and allows determining the dependence of the frequencies of the fundamental

ne rotation

frequency. The results of this analysis are conveniently presented in the form of the so-called Campbell diagram (Figure 6).

The diagram presents the dependences of the frequencies of the first fifteen harmonics on the rotation frequency of the turbine rotor. The range of rotation frequencies from 5,000 to 15,000 rpm was considered. Three critical rotation frequencies were revealed: 6,961.7 rpm, 7,909.7 rpm and 13,464 rpm. At these frequencies, resonance oscillations of the rotor are observed, mainly manifested by the torsional oscillations of the shaft. The flexural oscillations of the shaft in the range of operating shaft rotation frequencies do not manifest themselves.

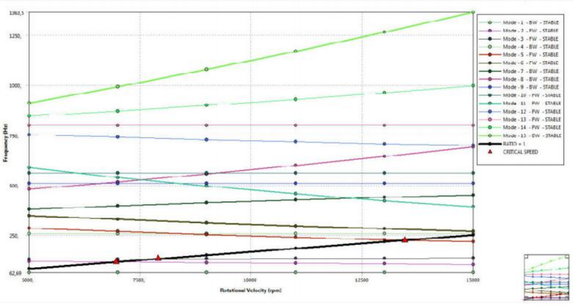

Figure 6. Campbell diagram for the rotor of the steam turbine

5 Conclusion

The modal analysis using the FEM and the apparatus of the theory of oscillations applied to the rotor of the steam miniturbine with a power of 250 kW demonstrated a complex form of torsional and bending oscillations of the shaft. The frequencies of the first thirty harmonics of the natural bending and the torsional oscillations of the rotor design elements are determined. The dependence of the natural frequencies of oscillations on the turbine rotation frequency is revealed.

The shaft of the miniturbine turned out to be flexible, as evidenced by the critical rotation frequencies of the rotor of 6,961.7 rpm, 7,909.7 rpm and 13,464 rpm, which are dangerous from the point of view of appearance of resonance. Taking into account that the first two critical frequencies of the turbogenerator are located significantly below the design speed of 12,000 rpm, it is recommended to pass them in the double-quick acceleration mode.

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