International Journal of Pure and Applied Mathematics
Volume 117 No. 22 2017, 137-141
ISSN: 1311-8080 (printed version); ISSN: 1314-3395 (on-line version)
url: http://www.ijpam.eu
Special Issue
ijpam.eu

Engineering and Computer Aided Calculations upon Determination of Quality Characteristics of Dynamic Systems

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Abstract - This work analyzes application of a technique of calculations and computations in ideal models of dynamic systems considering for possible alterations of properties of their solutions in the case of equivalent transformations. Ideal sign models of actual dynamic systems are presented in the form of various mathematical expressions (sets of equations), interrelating physical variables, which describe quantitatively the state of these systems. The investigation is based on consideration of models of actual dynamic systems in various notations of their equations and determination of parameters, which upon minor variations can lead to variation of behavior quality of dynamic system. The main research results include detection of parameters, which upon minor variations lead to stability loss or overcontrol of dynamic systems upon their operation. The obtained conclusions of this study include the necessity to verify all types of models of dynamic system already at the stage of its mathematical simulation. Global increase in technogenic emergencies and catastrophes with sophisticated technologies, quite often assigned to human factor, resulted in necessity to perform this study.

Keywords: Model, engineering computations, stability, control, stabilization, ill-posed systems.

1 Introduction

Consideration of ideal sign models of dynamic systems without their comprehensive analysis and idealization of mathematics as universal means of obtaining knowledge of the World can lead in real life to negative consequences in the form of emergencies and technogenic catastrophes [1-2]. Human factor, which is usually associated with such cases, can be nearly completely excluded in most future cases, if mathematical models will be considered in combination with their modifications obtained by equivalent transformations [3]. As it happens, in certain cases these transformations (without variation of solutions of original dynamic systems as such) can change important properties of these solutions (such as continuous dependence of solutions on parameters, retention of solution stability upon minor variations of parameters, etc.). Such models are commonly known, the solutions of which vary to final and even higher values at arbitrary small (and inevitable in practice) variations of parameters.

Definition: Ill-posed systems are such systems, the s

small variations of coefficients and parameters of mathematical mode [2].

2 Methods

Designing and computations of engineering systems up to the late 1990-s were considered as well developed and reliable technique. In recent decade, probably as a consequence of operation of engineering systems under marginal conditions when their stability factor has been exhausted, technogenic emergencies and catastrophes started to occur. Investigation into these emergencies and catastrophes requires for thorough analysis of all stages of creation and operation of engineering system including its designing stage.

2.1 Parallel analysis of the system upon equivalent transformations

It has been demonstrated [1-3] how dangerous is unidentified upon computations contact not with common but with "ill-posed system ", or technical object, or with "ill-posed equivalent transformation " ("ill-posed" objects and transformations occur more rarely than conventional ones, thus, they were not taken into account, nevertheless, they occur) [4-6]. Each system, unidentified at its designing stage, nearly inevitably will lead to emergency or even to catastrophe. In order to prevent such emergencies and catastrophes it is required to apply improved computations which consider for possible variations of solution properties upon equivalent transformations.

2.2 Additional analysis of the system in Cauchy normal form

Another aspect of additional analysis of ill-posed system, related with necessity to account for possible errors in computations, is referred to the theory of ordinary differential equations. Sets of equations are well-known, the solutions of which continuously depend on the parameters, therefore, small (inevitable in real life) variations of parameters correspond to small variations of solutions. In addition, other systems exist without such continuous dependence. The use of such other systems in computations leads to errors and, as a consequence, to emergencies. These systems can be determined on the basis of well-known theorem of continuous dependence of solution of differential equation parameters, this theorem is the foundation of practical applications. According to this

s necessary

and sufficient that the right-hand parts are restricted and satisfy the Lipschitz conditions. However, in general case this theorem is erroneous. We will reveal later, how to avoid the error upon designing of specific engineering system.

3 Results

As an example of violation of the theorem of continuous dependence of solutions of differential equation on parameters let us consider the following system:

$$\begin{cases} [mD^3+(2+2m)D^2+(4+m)D+2]x_1=(D+1)^2x_2 \\ (D^2+4D+5)x_1=(D+1)x_2 \end{cases} \qquad (1)$$

where $D=\dfrac{d}{dt}$, m is the parameter.

This system has the characteristic polynomial:

$$\Delta_1=\begin{vmatrix} m\lambda^3+\lambda^2(2+2m)+\lambda(4+m)+2 & -\lambda^2-2\lambda-1 \\ \lambda^2+4\lambda+5 & -\lambda-1 \end{vmatrix}=$$

$$=-m\lambda^4+\lambda^4-3m\lambda^3+4\lambda^3-3m\lambda^2+8\lambda^2-m\lambda+8\lambda+3=$$

$$=-(\lambda+1)^2((m-1)\lambda^2+(m-2)\lambda-3)$$

at m = 1 the polynomial roots are $\lambda_1=\lambda_2=-1$, $\lambda_3=-3$.

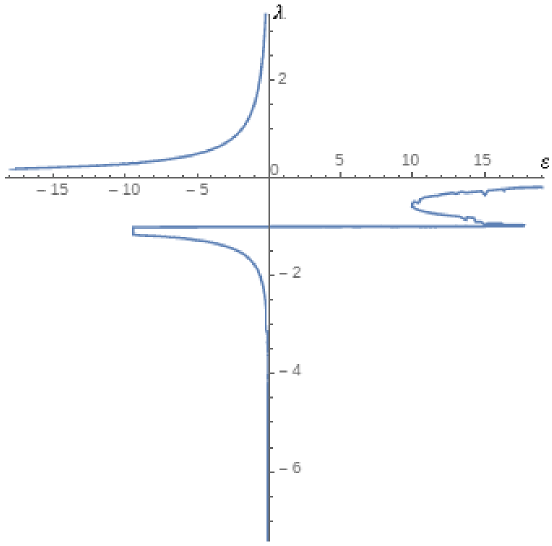

Figure 1. Characteristic polynomial graph

Figure 1 illustrates characteristic polynomial graph of Eq. (1) at $m=1-\varepsilon$.

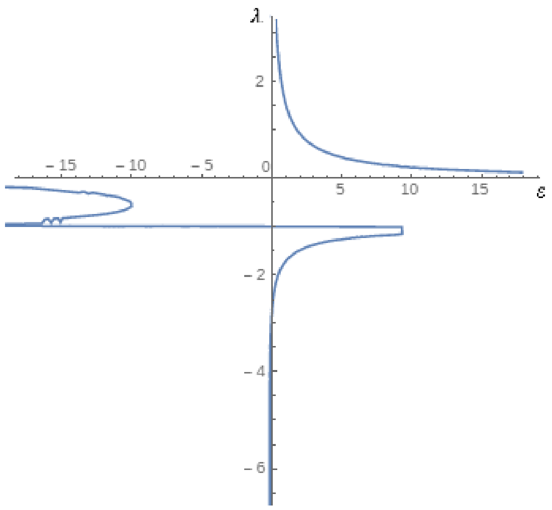

Figure 2. Characteristic polynomial graph

Figure 2 illustrates characteristic polynomial graph of Eq. (1) at $m=1+\varepsilon$.

The aforementioned Eq. (1) at m = 1 does not have continuous dependence of solutions on m. This example was discussed many times at various scientific conferences and was never argued [3].

As a second example let us consider a new system obtained from Eq. (1) by equivalent transformations (introducing new variables) which can be presented in normal form:

$$\begin{cases} m\dot{x}_1=-2x_1+x_2+x_3 \\ 0=x_1+x_2+2x_3+x_4 \\ \dot{x}_3=x_4 \\ \dot{x}_4=-x_3-2x_4 \end{cases} \qquad (2)$$

(the second equation was degenerated from differential to algebraic equation).

Equation (2) at m =1 has the same solutions as Eq. (1):

$$x_1=C_1e^{-3t}+(C_2\ t+C_3)e^{-t} \qquad (3)$$

However, these solutions (contrary to the solutions of Eq. (1)) depend on m continuously.

4 Discussion

The error upon proof of the theorem of continuous dependence of solutions of differential equation on parameters was caused by the fact that the proofs were given for sets of equations written in Cauchy normal form (and indeed, for such systems the theorem is valid). Since nearly any set of differential equations by means of equivalent transformations can be reduced to this or that Cauchy normal form, then an erroneous conclusion is made that the theorem is valid for all systems, including those in non-normal form. Herewith, it was not identified that exactly the equivalent transformation into normal form can vary such property of the system as continuous dependence of solutions on parameters and coefficients.

It is highly important, of course, to establish the consequences occurring with actual object described by mathematical model comprised of Eqs. (1) and (2) with the same solutions. This issue was discussed in [3]. Actually, all depends on the fact which system, Eq. (1) or Eq. (2), reflects better the features of specific object. If the object has three simple feedbacks, then its behavior is better described by Eq. (2). If it has one complex feedback (including derivatives of variables x_1 and x_2), then its behavior upon minor deviations from calculated parameters is described by Eq. (1) (if the parameters exactly equal to their calculated values, then both Eq. (1) and Eq. (2) describe the system equally well, since they are equivalent).

If, while designing specified engineering object, we at once transfer to analysis of mathematical model of the object in normal form, then an unreasonable ...le and will retain its stability upon inevitable minor deviations of the object parameters from calculations. But the behavior of the obtained object will be more complicated. If actual value of parameter m will be not exactly unity (m=1), but m= 1- ε, where ε is the small positive number (ε > 0), then the obtained engineering object will operate correctly and can be commissioned. However, since ε is the small number, minor variations of parameters upon operation are unavoidable. Thus, ε can vary from ε > 0 to ε < 0 and at this point the object loses stability, thus facilitating possible emergency or catastrophe.

Application of the aforementioned method to solution of the problem of stability of dynamic systems will indicate that upon development of actual engineering systems on the basis of Lyapunov functions for calculations of stability can lead errors, similar to the mentioned above. Existence of Lyapunov functions for the considered set of differential equations does not guarantee actual stability, it also requires additional verifications. The works devoted to solutions of the problem of stability of dynamic systems [7-14] present examples of actual engineering systems, they also require the mentioned additional verifications. Without such verifications it is possible to come to erroneous conclusion about retention of stability which later can be a hidden reason of emergencies and catastrophes.

Application of the above-mentioned method to solution of the problem of stabilization of dynamic systems will be characterized by the fact that solution of such problems and their practical implementation discussed in [7,15-16] should also be adjusted. Despite the fact that solution of the problem of stabilization of program motion or kinematic path of dynamic system has more general application than Lyapunov stability, nevertheless, all aforementioned factors will influence on the time of transitional processes and periods of occurring oscillations. Additional verifications according to the above procedure are also required, mainly for clarification of the scope of possible stabilization and its accuracy which later can prevent emergencies or catastrophes.

Application of the above method upon determination of oscillating and wave processes under dynamic conditions will be based on the fact that during operation of actual engineering systems oscillating processes can occur both in dynamic systems and in control systems. Oscillating processes, and in general case also wave processes, can occur also in systems describing medical and biological objects. In the latter case this oscillation around certain average is possibly unattainable point of the system equilibrium. In all these cases it is also possible to apply the Lyapunov stability and occurrence of the aforementioned procedural error. While considering dynamic systems according to the models described in [17-23], oscillating (wave) processes can exist physically (or analytically), and the Lyapunov stability in the systems will exist. Herewith, it is necessary to consider additional sets of differential equations according to the above procedure.

Application of the aforementioned method to solution of the problem of optimization of dynamic systems having description of their state at discrete times result in mathematical models in the form sets of linear algebraic equations. Similarly to determination of pseudo-inverse matrix. where one parameter varies, it is required to use algorithm which calculates the influence of simultaneous variations of many parameters of the considered object. Such recommendations should be made also for dynamic systems described according to the models in [24-26].

Application of the above method to solution of the problems of determination of various measure types of dynamic systems is characterized by the fact that upon operation of actual dynamic system it would be necessary to apply various measurements and various measure types, because certain parameters cannot be measured directly or we do not know yet, which measurements will be required further. In the limiting case, if variations of parameters of the considered object influence on the value of each coefficient, then the number of possible combinations of positive and negative variations is $W = 2^{(n^2)}$. For instance, at n =10 we will have $W = 2^{100}$, which is higher than 10^{30}, exceeding by far capabilities of both current and advanced fast computer facilities. It means that direct enumeration of parameter combinations of original system is impossible at present. Various measure types of dynamic systems and method of their determination are given in [27-34]. It is required to develop in details algorithms allowing to determine the most dangerous combinations of signs of variations on the basis of moderate computations (in real time), thus, to consider for maximum possible influence of simultaneous variations of various parameters on behavior of the considered object.

Application of the above method to solution of optimization problems of organizational systems is mainly determined by the fact that the organizational systems, contrary to mechanical ones, at the first glance have no concern with the equations of mechanics (interrelating such parameters of dynamic systems as applied force or restraining force, weight and position in space). However, upon their description we should nevertheless consider the points of phase space where they are located, characterizing their state. Conventional consideration of such organizational systems at discrete times and development of linear functionals characterizing their functioning leads to difference equations and, at decrease in observation period (time discrete), even to differential ones. Examples of organizational systems of various purposes are given elsewhere [35-44]. Upon such limiting process and application of a set of differential equations as a model, and upon consideration of difference equations as in the case of algebraic equations, the analytical procedure is similar to the aforementioned one.

5 Conclusions

Upon designing of actual engineering systems aiming at prevention of future possible emergencies

1. Upon designing of actual engineering systems not to apply only the theorem of continuous dependence of solutions on parameters.
2. To use original system upon equivalent transformations.
3. Among numerous equivalent forms of equations of the considered object, it is recommended to select such form, which considers for peculiarities of its structure more exactly.

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