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Computation of System Availability Using Fifth Order Runge−Kutta Method for Furnace Draft Air Cycle in a Thermal Power Plant

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Abstract

In any industry, the reliability of hardware components plays an important role which has more contribution in terms of higher productivity along with lower maintenance cost. Availability is also another primary performance measure which depends on both failure and repair rates of the equipments. Availability integrates the parameters of both reliability and maintainability which depends on the number of failure that occurs and how immediately these faults are rectified. In this paper, the system availability is obtained using fifth order Runge – Kutta (R-K) method based on Markov model. The operating behaviour of any system can be better captured through Markov model. Based on the Markov model a system of first order ordinary differential equations is derived. Laplace Transform technique is used in general to solve system of differential equations. Laplace Transform technique cannot be applied to large systems with more components as it is time consuming and not always practical. Instead of conventional Laplace transform technique, Fifth order R-K method is adopted to solve the system of differential equations through which system availability is quantified. The results are compared with fourth order R-K method. Fifth order R-K method has high accuracy and convergence when compared to fourth order R-K method. High accuracy and precision is required in safety critical systems. In this work, the availability of furnace draft air cycle in a thermal power plant (TPP) is discussed as a case study. The system availability is calculated based on the completely active states. This proposed method gives component reliability of active states which yields to the quantification of more accurate system availability.

Key Words: Markov chain, availability, thermal power plant, fourth and fifth order runge-kutta methods.

1. Introduction

Thermal power plants (TPP) are one of the main sources of electricity in both industrialized and developing countries. The function of the coal fired TPP is to convert the energy available in the coal to electricity. Coal power plants work by using several steps to convert stored energy in coal to usable electricity that we find in our home that powers our lights, computers, and sometimes, back into heat for our homes. Since the availability problems may even cause the productivity losses in TPP, it becomes inevitable to avoid the computation of system availability. At current rates of usage of fuel, it is also expected that coal will run out by 2088 [1]. So, the optimum utilization of resources is very much important for meeting the demands of the future generation. System availability has become one of the major parameters for the performance of TPP.

The present investigation of this paper embodies the evaluation of system availability through the vector version of 4^{th} and 5^{th} order R-K method. This proposed R−K method yields the component reliability, system reliability and availability, since it obeys the Markov Chain property. The method is used to solve the system of first order ordinary differential equation that yields an accurate solution, good rate of convergence and less time-consuming. In 4^{th} and 5^{th} order R−K method, the selection of an initial probability vector depends on the operating performance of the highly reliable Markov states.

In this present study, the furnace draft air cycle in TPP is taken as case study for evaluating the system availability using 4^{th} and 5^{th} order R-K method. Moreover, this method yields an accurate reliability, availability and component reliability values are more useful for reliability and TPP engineers. Matrix form of the vector version of the fourth order R−K method is used to solve the system of first order ordinary differential equations that depends on the initial probability vector. The analytical solution obtained using Laplace Transform technique cannot be applied to large systems with more components [2]. Also, solution by conventional method using Laplace transform technique is time consuming and hence not practically feasible in industries. To avoid this situation, R−K method is preferably used for solving huge set of system of differential equations. The rest of the paper is structured as follows: In section 2, the work related to various standby systems is discussed. In section 3, the basic concepts of reliability, availability, Markov Chain and fourth order R-K method are discussed. The proposed approach and a schematic representation for detailed workflow for evaluating system availability using 4^{th} and 5^{th} order R-K method is discussed in section 4. In section 5, general R-K method, vector version along with their pseudo-code representation is presented. In section 6, the proposed approach is discussed through a case study of furnace draft air cycle in TPP by fifth order R-K method and compared with fourth order R-K method. The results and discussions are dealt in section 7 and finally, conclusion is presented in section 8.

2. Related Work

Computation of availability of cattle feed plant system [3], and a methodology to study the transient and steady state behavior of the feeding system in the sugar industry is discussed in [4]. In both these applications, system availability was estimated using matrix method and their calculations were implemented by C program. System availability evaluation for the process of a paper production industry consisting of four subsystems based on the probability considerations and the governing differential equations are solved using R-K method of order four presented in [5].

A simulation model on the availability optimization of CO shift conversion system of a fertilizer plant [6], analysis of urea synthesis system of a fertilizer plant [7], performance analysis of sole lasting system availability of a shoe industry [8]; are based on mathematical representation of the problem by the probabilistic approach. In all these applications, the system of differential equations is derived based on Markov birth-death process and the system availability is evaluated by steady state behaviour.

The performance optimization for the paper making system in a paper plant [9], performance analysis of the screening unit in a paper plant [10], performance optimization of CO_2 cooling system of a fertilizer plant, [11], performance optimization for the digesting system of a paper plant [12] and the performance enhancement for crystallization unit of a sugar plant [13] are dealt with genetic algorithm and Markov approach. For these applications, the first order ordinary differential equations have been developed using the probabilistic approach through the failure and repair rates of the Markov transition diagram. Furthermore, genetic algorithm has been used to select the various feasible values of the system failure and repair parameters. This method is very useful to the plant management for the timely execution of proper maintenance decisions and hence to enhance the performance of all applications. Stochastic analysis and performance evaluation of a complex TPP using probabilistic approach which shows the relation between availability and failure rate is discussed in [14]. The availability of the pulping system in the paper industry in transient state and the performance of the availability were estimated using correlation and regression [15].

For solving the system of first order ordinary differential equations in any dimension, the implementation and error bound using the explicit MATLAB pseudo code for 4^{th} and 5^{th} order R- K method are discussed. Furthermore, these two methods are compared through running time and maximum errors are discussed through the Rossler nonlinear system which arises in chemical kinetics in [16]. Analysis and continuous time Markov chain are combined to estimate the reliability of space tracking, Fault tree telemetry and command (TT&C) system (a small TT&C system with two mission phases). The reliability model of phased-mission system to calculate the probability of performing the entire mission at a desired time was discussed through 4th order R-K method in [17]. The reliability estimation of the state dependent system using 4th order R-K method for standby system, standby system with repair and availability of standby system with repair were discussed through a suitable case study. Furthermore, general Markov structure for standby systems [2], $3-out-of-4{:}G$ warm standby system [18] and the reliability of emergency diesel generators $1-\mathrm{out}-\mathrm{of}-2{:}\,\mathrm{DG}, 1-\mathrm{out}-\mathrm{of}\,3{:}\,\mathrm{DG}$ and $2-\mathrm{out}-\mathrm{of}-3{:}\,\mathrm{DG},\ 2-\mathrm{out}-\mathrm{of}-4{:}\,\mathrm{DG}$ configurations using 4th order R-K method were discussed. Moreover, these reliability and availability values are compared to the analytic solution obtained using Laplace transform technique in [19-21].

The current study is an attempt to address the issue of the evaluation of the performance modeling of the analysis of furnace draft air cycle in TPP. This system consists of three main subsystems such as primary air fan, furnace and induces draft fan. In furnace draft air cycle, the hot air from primary air fan enters into primary air distribution headers and coal is transported from primary air feeders to coal bunkers with the help of air medium in the furnace. Flue gases from furnace are passed to chimney through induced draft fans. In order to find the availability of the system, a mathematical formation of the problem is developed using Markov birth-death process based upon probabilistic approach. The failure and repair rates for all these subsystems have been taken from maintenance history sheets of TPP. The interrelationship among the completely working, reduced capacity and failed states of these subsystems are modeled into Markov transition diagram [22].

3. Concepts of Reliability and Availability

In this section, the fundamentals of reliability, availability, concepts of Markov Chain and Markov transition probability matrix (TPM) are discussed.

Fundamentals of Reliability, Mean Time to Failure

In general, the reliability of any component / system is defined as the probability that an item will perform a required function without failure under stated conditions for a specified period of time [2, 21]. Indeed, the system may be required to perform various functions, and each of them may have a different reliability values.

In different times, the system may have a different probability without fail to perform the required function under the specified condition. Moreover, term failure indicates that, the system is not capable of performing a function when required. Mathematically, the reliability of a system/component is expressed as

$$R(t) = P(T > t), t \geq 0 \qquad (1)$$

Here, the continuous random variable T represents the time-to-failure/failure time has the probability density function

$$R(t) = \int_t^\infty f(t)dt \qquad (2)$$

Similarly, the unreliability of the component is defined as

$$F(t) = P(T < t) = 1 - R(t),$$
$$\exists \ 0 \leq R(t), F(t) \leq 1 \quad (3)$$

Therefore, the unreliability is given by

$$F(t) = \int_0^t f(t)dt \quad (4)$$

The probability density function is defined as

$$f(t) = \frac{d}{dt}[F(t)] = -\frac{d}{dt}[R(t)] \quad (5)$$

The expected failure time/average failure is described for non-repairable systems during which a component is expected to perform without fail. Thus, mean time to failure *(MTTF)* of a component, is expressed by

$$MTTF = E(T) = \int_0^\infty R(t)dt \quad (6)$$

As we define in Eq. (6), it is a statistical value that deals with how long a product can reasonably be expected to perform without fail over a long period of time for a large number of components.

System Availability

The availability of the component / system is described when both failure and repair distributions are known.

Precisely, the availability of components is interpreted as the probability that a system is operational at a given point in time / as the percentage of time specified over some interval; in which the component is in operation.

Mathematically, the availability is described as the probability that the system is working properly at time *t*. Then,

$$Availability = \frac{up\ time}{up\ time + down\ time} = \frac{MTTF}{MTTF+MTTR} \quad (7)$$

Let $A(T)$ be the availability at time t referred to as the point availability. Then the average availability over the time interval $[0, T]$ is given by

$$A(T) = \frac{1}{T}\int_0^T A(T)dt \quad (8)$$

The interval availability over the interval from t_1 to t_2 is given by

$$A_{t_2-t_1} = \frac{1}{t_2-t_1}\int_{t_1}^{t_2} A(t)dt \quad (9)$$

Finally, the long run equilibrium / steady state availability of the component is interpreted as

$$A = \lim_{T\to\infty} A \quad (10)$$

In general, the availability is same as the reliability for non-repairable systems and it is greater than or equal to the reliability for repairable components [20-21].

Markov Chain

Markov Chain is defined as a stochastic model with a sequence of possible events in which the probability of the future behaviour of the system depends only on the present state but not on the past states [20-21]. Now, mathematically we describe the Markov Chain as follows:

Let $\{X_n\}, n = 0,1,2, \ldots$ be a sequence of random variables where X_n denotes the state of a Markov system at the n^{th} finite step. If $X_n = j$ then the state of the system at time step n is j, X_0 is the initial state of the system. Markov property is given by

$$P(X_n = j / X_{n-1} = i_{n-1}, X_{n-2} = i_{n-2}, \ldots, X_0 = i_0) = P(X_n = j / X_{n-1} = i_{n-1}) \quad (11)$$

Let $P_j(n)$ denote the probability mass function of the random variable X_n, $P_j(n) = P(X_n = j)$, meaning the system is in state j at the n^{th} time step with probability $P_j(n)$ and the conditional probability is defined as $P_{ij}(m,n) = P(X_n = j / X_m = i), j = 1,\ 2, \ldots, n$. $P_{ij}(m,n)$ denotes the probability of being in state j at time step n given that the system is in state i at time step m. The one step transition probability is given by $P_{ij}(1) = P(X_n = j | X_{n-1} = i_{n-1}), n \geq 1$. If there are n states in the system then there are n^2 transition probabilities describing a Markov process which can most conveniently be given in the form of an $n \times n$ TPM denoted by $(P_{ij})_{n\times n}$.

Therefore,

$$p_{ij} = \begin{array}{c c} & \begin{array}{ccccc} 1 & 2 & 3 & \ldots & n \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ \vdots \\ n \end{array} & \begin{pmatrix} p_{11} & p_{12} & p_{13} & \cdots & p_{1n} \\ p_{21} & p_{22} & p_{23} & \cdots & p_{2n} \\ p_{31} & p_{32} & p_{33} & \cdots & p_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ p_{n1} & p_{n2} & p_{n3} & \cdots & p_{nn} \end{pmatrix} \end{array} \quad (12)$$

The 4th order R-K Method

The 4th order R-K method is one of the most important techniques for solving system of first order differential equations [2, 16, 21]. The first order ordinary differential equation of the form $\frac{dY}{dx} = f(x, Y)$ with initial condition $Y(X_0) = Y_0$ is considered. The fourth order R-K method for solving this system is given by

$$Y_{n+1} = Y_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4) \quad (13)$$

where

$$k_1 = f(x_n, Y_n) \quad (14)$$

$$k_2 = f(x_n + \frac{h}{2}, Y_n + \frac{k_1}{2}) \quad (15)$$

$$k_3 = f\left(x_n + \frac{h}{2}, Y_n + \frac{k_2}{2}\right) \quad (16)$$

$$k_4 = f(x_n + h, Y_n + k_3) \quad (17)$$

$O(h^5)$ is the order of truncation error. Here h denotes the step size.

4. Proposed Approach

This paper presents a study of Markov approach for determining the system availability of furnace draft air cycle in TPP using R-K method of fifth order. Initially, the real time application can be modelled into a schematic representation followed by the operating conditions thereby it can be easily modelled into Markov chain. Therefore, the state dependent systems can be converted into Markov Chain, and then the system of first order ordinary differential equation is obtained. These differential equations are solved by 5th order R-K method which provides accurate results compared with existing solutions. The primary advantage of the proposed method is that, the system having sufficiently large number components in a Markov Chain can be solved using a vector version of fifth order R-K method.

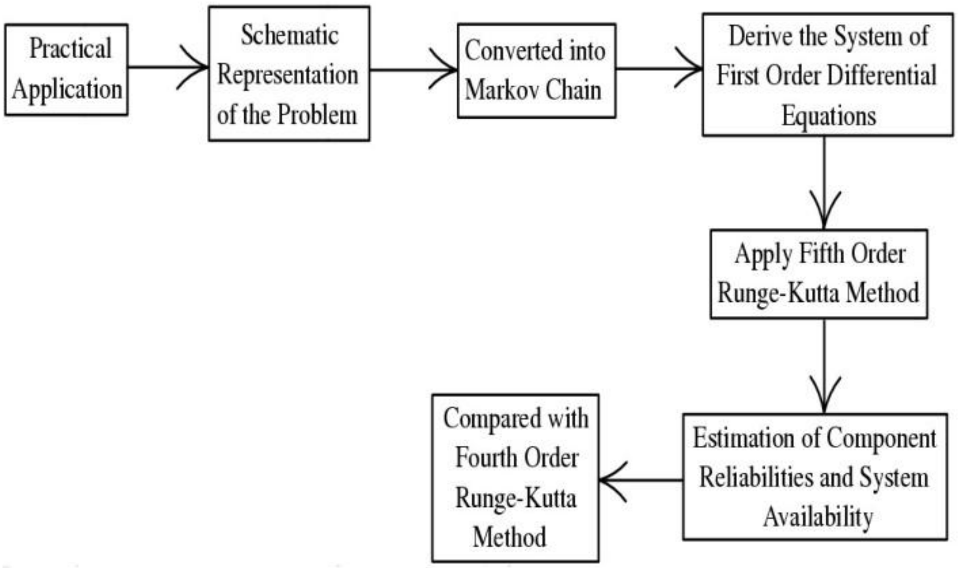

Figure 1: Schematic Representation for the Detailed Work-Flow for System Availability Evaluation

The main aspect of this proposed method is to estimate the system availability based on the rate transition matrix and its corresponding initial probability vectors. As the size of the system increases the number of differential equations to be solved also increases. In such situations application of the conventional Laplace transform technique is tedious and time consuming to compute the system availability as compared to proposed method. Here, the fifth order R-K method has more accuracy when compared to 4th order R-K method. In 5th R-K method, the error per step is on the order of h^6 while the total accumulated error has order h^4.The proposed schematic representation shown in Figure. 1, investigates the component reliabilities along with system availability quantification using fifth order R-K method and these solutions are compared with 4th order R-K method. The computations of the vector version of 5th order R-K method is solved by MATLAB.

5. The Proposed 5th Order R-K Method

In this section, the general 5th order R-K method, vector verson and Matlab pseudo-code representation for evaluating the system availability is presented.

General 5th order R-K Method

The 5th order R-K method is most widely used to solve the system of first order differential equation of the form $\frac{dY}{dx} = f(x, Y)$ with initial condition $Y(X_0) = Y_0$.

Then, the R–K method of order 5 is given by

$$Y_{n+1} = Y_n + \left(\frac{7K_1+32K_3+12K_4+32K_5+7K_6}{90}\right) \quad (18)$$

where

$$K_1 = h \times f(x_0, Y_0) \quad (19)$$

$$K_2 = h \times f\left(x_n + \frac{h}{2}, Y_n + \frac{K_1}{2}\right) \quad (20)$$

$$K_3 = h \times f\left(x_n + \frac{h}{4}, Y_n + \frac{3K_1+K_2}{16}\right) \quad (21)$$

$$K_4 = h \times f\left(x_n + \frac{h}{2}, Y_n + \frac{K_3}{2}\right) \quad (22)$$

$$K_5 = h \times f\left(x_n + \frac{3h}{4}, Y_n + \frac{-3K_2+6K_3+9K_4}{2}\right) \quad (23)$$

$$K_6 = h \times f\left(x_n + h, Y_n + \frac{K_1+4K_2+6K_3-12K_4+8K_5}{7}\right) \quad (24)$$

The vector version of the 5th order R–K method can be revised in the next section based on the rate transition matrix and initial probability transition vector. This method is computationally more efficient, less time consuming with high precision and has good rate of convergence [16].

The Vector Version of 5th order R-K Method

Consider $\frac{dY}{dx} = f(x, Y)$, and $\frac{dP(t)}{dt} = QP(t)$. Comparing these two forms, $f(x, Y) = QP(t)$. Therefore, the vector version of the fifth order R-K method is as follows:

$$P_{n+1} = P_n + \left(\frac{7K_1 + 32K_3 + 12K_4 + 32K_5 + 7K_6}{90}\right) \quad (25)$$

where

$$K_1 = h \times P_n \times Q \quad (26)$$

$$K_2 = h \times \left(P_n + \frac{K_1}{2}\right) \times Q \quad (27)$$

$$K_3 = h \times \left(P_n + \frac{3K_1 + K_2}{16}\right) \times Q \quad (28)$$

$$K_4 = h \times \left(P_n + \frac{K_3}{2}\right) \times Q \quad (29)$$

$$K_5 = h \times \left(P_n + \frac{-3K_2 + 6K_3 + 9K_4}{2}\right) \times Q \quad (30)$$

$$K_6 = h \times \left[P_n + \left(\frac{K_1 + 4K_2 + 6K_3 - 12K_4 + 8K_5}{7}\right)\right] \times Q(31)$$

Here, P_n is the initial probability vector, h is the step size and Q is the rate transition matrix.

Pseudo-Code Representation of the 5th Order R-K Method

From the above vector version, the pseudo-code representation of the 5th order R-K method is applied to quantify the system availability through MATLAB based on initial probability vector P_n is considered as P and the rate transition matrix as

$select\ P_0; P \leftarrow P_0; t \leftarrow 0; i \leftarrow 0; select\ h;$

while $t < T$

$K_1 \leftarrow h * P_n * Q$

$K_2 \leftarrow h * \left(P_n + \frac{K_1}{2}\right) * Q$

$K_3 \leftarrow h * \left(P_n + \frac{3K_1 + K_2}{16}\right) * Q$

$K_4 \leftarrow h\left(P_n + \frac{K_3}{2}\right) * Q$

$K_5 \leftarrow h\left(P_n + \frac{-3K_2 + 6K_3 + 9K_4}{2}\right) * Q$

$K_6 \leftarrow h$

$\left[P_n + \left(\frac{K_1 + 4K_2 + 6K_3 - 12K_4 + 8K_5}{7}\right)\right] * Q$

$P \leftarrow P + \left(\frac{7K_1 + 32K_3 + 12K_4 + 32K_5 + 7K_6}{90}\right)$

$t \leftarrow t + h; i \leftarrow i + 1;$

Here, the notation h, i and T represents the step size, iteration count and system availability time (algorithm stopping criteria) respectively. $O(h^5)$ is the order of truncation error.

For solving first order differential equations, R-K method of order 4 is a well known approach and most commonly preferred in science and engineering. Here, 5th order R-K method is used to compute system availability. In 4th order R-K method, the error per step is on the order of h^5 while the total accumulated error is of order h^4 .

Whereas, in 5th order R-K method, the error per step is on the order of h^6 while the total accumulated error has order h^4. Due to this high level of accuracy is achieved with fast convergence. This is especially needed in safety critical systems.

6. Case Study – Evaluation of System Availability of Furnace Draft Air Cycle in TPP

In this case study, the availability of furnace draft air cycle in TPP which consists of several subsystems connected in parallel and series combination for air supply to furnace is estimated. The optimization of each sub-system in relation to one another is required to make the power plant more profitable and viable for operation. There are several subsystems in the furnace draft air cycle: (i.) primary air fan; (ii.) furnace; (iii.) Induced draft fan.

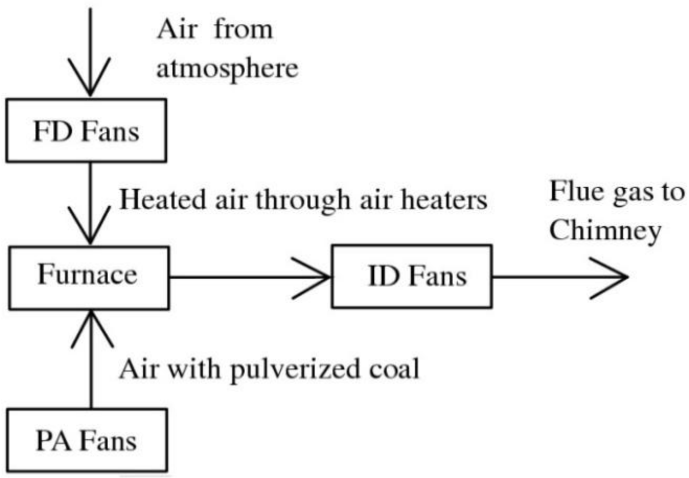

Figure 2: Furnace Draft air Cycle in TPP

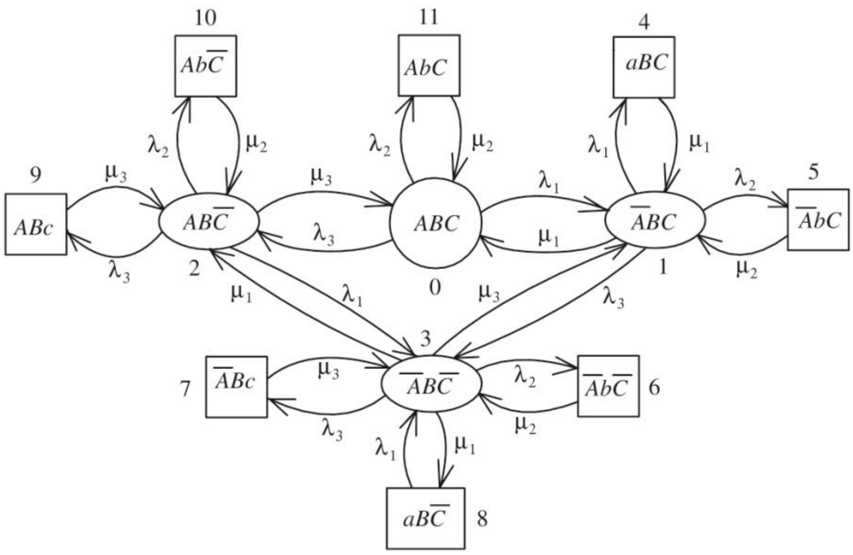

Figure 3: Markov Transition Diagram for System Availability of Furnace Draft air Cycle in TPP

Furthermore, the hot air from primary air fan enters into primary air distribution headers and coal is transported from primary air feeders to coal bunkers with the help of air medium in the furnace. Flue gases from furnace are passed to chimney through the induced draft fans.

The descriptions of these subsystems are discussed as follows: The primary air fan consists of two units wherein the reduced capacity state occurs when one unit fails, the system failure occurs when both units fail. The furnace has single unit, the failure of that unit yields the system failure. In induced draft fan, there are three units; failure of one or two fan(s) reduces the capacity of system and the failure of three fans results into the system failure.

The system description of the case study of the furnace draft air cycle in TPP is shown in Figure. 2. For constructing the Markov system, we use the following assumptions

1. In this probabilistic model, the failure and repair rates for each subsystem are constant and statistically independent. Only one failure occurs at a time.
2. A repaired unit is as good as new, performance wise.
3. The standby units are of the same nature and capacity as the active units.

In order to find the availability of the system, the problem formulation is developed using Markov birth-death process based upon probabilistic approach. The failure and repair rates for all the subsystems have been taken from maintenance history sheets of TPP [22]. The Markov transition diagram represents the interrelationship between the states that are completely working, working with reduced capacity and failed states of the furnace draft air cycle. The Markov transition diagram for the furnace draft air cycle in TPP is shown in Figure. 3.

In the above Markov transition diagram, the three subsystems in a furnace draft air cycle are primary air fan, furnace and induces draft fan denoted by $A, B, and\ C$ respectively. The state 0 is completely working and states 1, 2 and 3 are working in reduced capacity. The sequences of states from 4 to 11 represent the failed states of the Markov system. Now, we describe the notations for each state in the Markov transition diagram as follows: Let A, and C denotes the subsystems are working with full capacity, the symbols a, b and c indicates the failure of the subsystems respectively. Similarly, the notations $\overline{A}$ and $\overline{C}$ represent that the subsystems A and C are working with reduced capacity.

Let $\lambda_i, i = 1, 2, 3$ denotes the failure rates of the Markov states from the subsystems $A, B, C, \bar{A}$ and $\overline{C}$ to the corresponding subsystems $\bar{A}, b, \bar{C}, a$and c respectively. Also, let $\mu_i,\ i = 1, 2, 3$ indicates the repair rates between the Markov states from the subsystems $\overline{A}, b, \bar{C}, a$ and c to the subsystems $A, B, C, \bar{A}$ and $\overline{C}$ respectively. Let $P_i^{'}(t),\ i = 0, 1, 2, \dots, 11$ be the derivatives of the component probabilities with respect to time t of the each Markov states

$P_i^{'}(t), i = 0, 1, 2, \ldots, 11$ respectively. Let $R(t)$ and $A(t)$ be the system reliability and availability the system respectively. The general Markov transition rule for deriving the first order system of ordinary probability differential equation for each Markov states are as follows:

$$P_i^{'}(t) = (inflow\ to\ state\ i) - (outflow\ from\ state\ i)$$
$$= \sum_{j\neq i}(rate\ of\ transition\ from\ state\ j\ to\ state\ i) \times (probility\ of\ state\ j) - \sum_{j\neq i}(rate\ of\ transition\ from\ state\ j\ to\ state\ i) \times (probility\ of\ state\ j) \quad (32)$$

According to the general Markov transition rule.

According to the general Markov transition rule in Eq. (32), the set of differential equations for furnace draft air cycle in TPP can be written as,

$$P_0^{'}(t) = -(\lambda_1 + \lambda_2 + \lambda_3)P_0(t) + \mu_1 P_1(t) + \mu_3 P_2(t) + \mu_2 P_{11}(t) \quad (33)$$
$$P_1^{'}(t) = -(\lambda_1 + \lambda_2 + \lambda_3 + \mu_1)P_1(t) + \lambda_1 P_0(t) + \mu_1 P_4(t) + \mu_2 P_5(t) + \mu_3 P_3(t) \quad (34)$$
$$P_2^{'}(t) = -(\lambda_1 + \lambda_2 + \lambda_3 + \mu_1)P_2(t) + \lambda_3 P_0(t) + \mu_3 P_9(t) + \mu_2 P_{10}(t) + \mu_1 P_3(t) \quad (35)$$
$$P_3^{'}(t) = -(\lambda_1 + \lambda_2 + \lambda_3 + \mu_1 + \mu_3)P_3(t) + \lambda_1 P_2(t) + \lambda_3 P_1(t) + \mu_2 P_6(t) + P_7(t) + \mu_1 P_8(t) \quad (36)$$
$$P_4^{'}(t) = -\mu_1 P_4(t) + \lambda_1 P_4(t) \quad (37)$$
$$P_5^{'}(t) = -\mu_2 P_5(t) + \lambda_2 P_1(t) \quad (38)$$
$$P_6^{'}(t) = -\mu_2 P_6(t) + \lambda_2 P_3(t) \quad (39)$$
$$P_7^{'}(t) = -\mu_3 P_7(t) + \lambda_3 P_3(t) \quad (40)$$
$$P_8^{'}(t) = -\mu_1 P_8(t) + \lambda_1 P_3(t) \quad (41)$$
$$P_9^{'}(t) = -\mu_3 P_9(t) + \lambda_3 P_2(t) \quad (42)$$
$$P_{10}^{'}(t) = -\mu_2 P_{10}(t) + \lambda_2 P_2(t) \quad (43)$$
$$P_{11}^{'}(t) = -\mu_2 P_{11}(t) + \lambda_2 P_0(t) \quad (44)$$

Moreover, the system of differential equation in Eq. (33-44) is written in Eq. (46), with initial condition is shown in Eq. (47). Then,

$$\frac{dP(t)}{dt} = QP(t) \quad (45)$$
$$[P_0(0)\ \ P_1(0)\ \ P_2(0)\ \ \ldots\ \ P_{11}(0)] = [1\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0] \quad (46)$$

In this initial probability vector, the state 1 is full working capacity, the other states are zero. The rate transition matrix Q obtained from the system of first order differential equations presented in Eq. (33-44) is shown in Eq. (45) with initial condition shown in Eq. (46). The constant failure and repair rates for the furnace draft air cycle in TPP are $\lambda_1 = 0.001, \lambda_2 = 0.0006,\ \lambda_3 = 0.0001$ & $\mu_1 = 0.01,\ \mu_2 = 0.02,\ \mu_3 = 0.02$ respectively.

Substitute these values in the above transition matrix in Eq. (33-44), and then the revised rate-transition matrix Q is of order 12×12 and is given in Eq. (47), where *A*, *B*, *C* are sub matrices

$$Q = (A \quad B \quad C) \quad (47)$$

Where,

$$A = \begin{pmatrix} -0.0017 & 0.0100 & 0.02 \\ 0.0010 & -0.0117 & 0 \\ 0.0001 & 0 & -0.0217 \\ 0 & 0.0001 & 0.001 \\ 0 & 0.0010 & 0 \\ 0 & 0.0006 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0.0001 \\ 0 & 0 & 0.0006 \\ 0.0006 & 0 & 0 \end{pmatrix}$$

$$B = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0.02 & 0.01 & 0.02 & 0 \\ 0.01 & 0 & 0 & 0 \\ -0.0317 & 0 & 0 & 0.02 \\ 0 & -0.01 & 0 & 0 \\ 0 & 0 & -0.02 & 0 \\ 0.0006 & 0 & 0 & -0.02 \\ 0.0001 & 0 & 0 & 0 \\ 0.0010 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0.02 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.02 & 0.01 & 0 \\ 0.02 & 0.01 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -0.02 & 0 & 0 & 0 & 0 \\ 0 & -0.01 & 0 & 0 & 0 \\ 0 & 0 & -0.02 & 0 & 0 \\ 0 & 0 & 0 & -0.02 & 0 \\ 0 & 0 & 0 & 0 & -0.02 \end{pmatrix}$$

Using pseudo-code representation of 5th order R-K method presented in section 5.3, to solve the system of differential equations for computing the availability of the furnace draft air cycle in TPP is given in Eq. (48). The system availability is calculated as

$$P_0(t) + P_1(t) + P_2(t) + P_3(t) \qquad (48)$$

Similarly, the availability values are determined using 4th order R-K method by Eq. (13-17) which is closer to the 5th order R-K method. The availability values are depicted in Table 1 and shown in Fig. 4 respectively. Comparison of

Availability values are demonstrated in Fig. 5.

7. Results and Discussion

In this paper, the 5^{th} order R-K method is applied to estimate the component reliability and their system availability for furnace draft air cycle in TPP. A pseudo code representation of the 5^{th} order R-K method is derived thereby the solution of the differential equation is solved using MATLAB which is based on the initial probability vector and the rate transition matrix of furnace draft air cycle in TPP. The system availability is calculated based on the active states in furnace draft air cycle.

Therefore, system availability values decreases slowly and then becomes constant over the time hours. In addition, the 4^{th} order and 5^{th} order availability values are very closer and similar that has varies after 12 decimal places as shown in Table 1, Fig. 4 and Fig. 5 respectively. Finally, the long run availability of furnace draft air cycle in TPP is 0.962356663917299 over the longer period of 100000 hours.

Generally, the method is applied to the complex system having a huge number of sequences of Markov states. For each state, we derive a state probability differential equations and it helps to compute the long-run availability and reliability.

8. Conclusion

In the present study, combining Markov chain and 5^{th} order R-K method to calculate the system availability of furnace draft air cycle in TPP is discussed. In the case study, the furnace draft air cycle in TPP which consists of several subsystems connected in parallel and series combination for air supply to furnace is considered.

The furnace draft air cycle is one of the subsystems of the huge TPP. The mathematical formulation of the problem is developed using Markov birth-death process along with general Markov transition rule based on the probabilistic approach. The failure and repair rates for all the subsystems have been taken from the maintenance history sheets of TPP.

For system having large number of components, the Laplace transform technique is tedious instead of using R-K method to estimate availability. This method yields more accurate availability values and is very useful for reliability engineers in TPP.

The computed availability values are given back to the plant from which the failure and repair rates are taken so that they can make efforts to run the plant efficiently. Furthermore, this technique is applicable for all components which obey the concepts of Markov Chain.

Table 1: System Availability Values of the Furnace Draft air Cycle in TPP

S. No.	Time(t) Hours	Fifth order R-K method	Fourth order R-K method
1	0	0.999701413962489	0.999701413962515
2	0.5	0.999405636798752	0.999405636798803
3	1	0.999112640152985	0.999112640153060
4	2.5	0.998250054130880	0.998250054131027
5	5	0.996865457993555	0.996865457993812
6	10	0.994283532704832	0.994283532705273
7	15	0.992156750558153	0.991931203919288
8	25	0.987829377244886	0.987829377245670
9	50	0.980290639593892	0.980290639594815
10	75	0.975389100894891	0.975389100895711
11	100	0.972111086928596	0.972111086929244
12	125	0.969848543579481	0.969848543579961
13	150	0.968234574082428	0.968234574082769
14	200	0.966144479111529	0.966144479111689
15	500	0.962737677836069	0.962737677836071
16	1000	0.962364980778418	0.962364980778419
17	2000	0.962356667701741	0.962356667701741
18	4000	0.962356663917299	0.962356663917299
19	6000	0.962356663917299	0.962356663917300
20	10000	0.962356663917299	0.962356663917300
21	25000	0.962356663917299	0.962356663917300
22	50000	0.962356663917299	0.962356663917300
23	100000	0.962356663917299	0.962356663917300

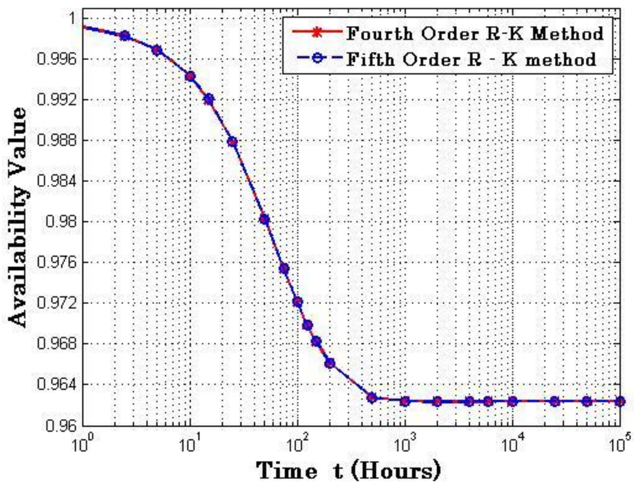

Figure 4: System Availability Values of the Furnace Draft Air Cycle in TPP by and 4th and 5th Order R-K Methods

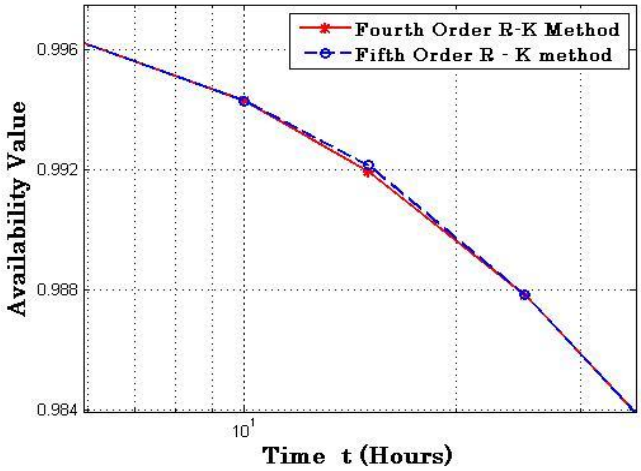

Figure 5: Comparison of the Availability Values of 4th and 5th Order R – K Methods

Also, in the future studies, the effect of the failure versus repair rates will be computed for various choices and then optimize and increase the system availability values. The system performance can also be verified using optimization technique involving more randomness which yields the optimum availability levels for different combinations of failure and repair rates for improving the performance system.

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