

Design of Hybrid control for Isothermal Continuous stirred tank Reactor

N. Sivaramakrishnan^{1,*}, P.R. Hemavathy², G.Anitha³

^{1,2,3}Assistant professor, Electronics and Instrumentation Engineering,
B S Abdur Rahman Crescent University, Chennai-48.

*sivaramakrishnaneie@bsauniv.ac.in

Abstract:

The Continuous stirred tank reactor plays a vital role in almost all the chemical industries. It is a highly nonlinear system with complex dynamic behavior dominated by its system parameters heavily. The product concentration has to be controlled by manipulating the feed flow rate effectively. The objective of this paper is to design a hybrid controller by combining the IMC based PID and Sliding Mode Control (SMC) for the continuous stirred tank reactor to improve the product concentration irrespective of the disturbances. In this paper, the hybrid controller is implemented to control the concentration of the Isothermal Chemical Reactor by manipulation of the reactant flow. Simulation results show that hybrid IMC based PID plus sliding mode control is better as compared to the PID controller

Keywords: Linearization, Mathematical model, State space model, IMC based PID, SMC control.

1. Introduction

An Isothermal process is a process in which the temperature remains constant i.e, $\Delta T = 0$, Since the CSTR shows highly non-linear characteristics, so it is very difficult to control it. The Isothermal chemical reaction. For designing the reactor following factors to be considered.

1. Size of the Reactor.
2. Products coming out from reactor.
3. Temperature inside the chemical reactor.
4. Pressure inside the chemical reactor.
5. Rate of chemical Reaction.
6. Stability and Controllability of the chemical reactor.

The van de vusse reaction is given (1) is under consideration and the desired product is the component B.



The Desired product coming out is the component B. and the intermediate component in the reaction. Here we find interesting steady-state and dynamic behaviour that can occur with this reaction scheme. Klatt and Engell (1998) note that the production of cyclopentenol from cyclopentadiene is based on such a reaction scheme (where A = cyclopentadiene, B = cyclopentenol , C = Cylopentanediol , D = dicyclopentadiene). The schematic diagram of the reactor is shown in the figure 1.

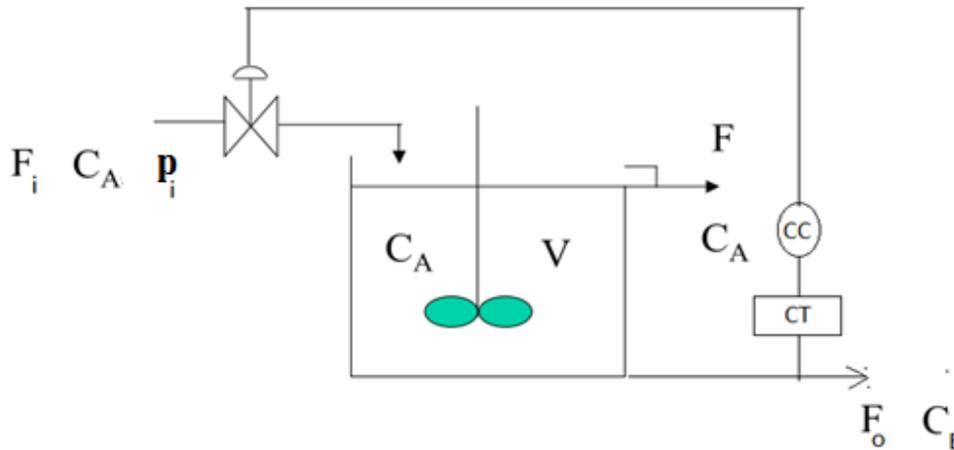


Figure 1 Isothermal CSTR

For reactor model overall mass balance equation is given by

$$\frac{d(pV)}{dt} = piFi - pFo \tag{2}$$

Where v is the volume in litre, Fi is the feed flow rate and Fo is output flow rate in litre/min, and p s are the feed flow density and output flow density respectively.

Assuming constant density i.e. pi=p then, equation (2) reduces to

$$\frac{dV}{dT} = Fi - Fo \tag{3}$$

The component material balance of A is given by,

$$\frac{d(CAV)}{dt} = CAiFi - CAFo + rAV \tag{4}$$

Where CA is concentration of component A in g mol/liter, CAi is correction of component A in g mil.liter, CAi is the concentration of component A in feed flow in g mol/liter and rA represents generation of species of A per unit volume. It is given by the equation

$$r_A = -K_1C_A - K_3C_A^2 \quad (5)$$

Where K_1 and K_3 are the reaction rate constants of equation(1),

From the equations (2),

$$\frac{d(C_A V)}{dT} = V \frac{dC_A}{dT} + C_A \frac{dV}{dT} \quad (6)$$

Hence equation (4) can be written as

$$\frac{dC_A}{dT} = \frac{F_i}{V} (C_{Ai} - C_A) - K_1C_A - K_3C_A^2 \quad (7)$$

Then, component material balance for B is given by

$$\frac{d(C_B V)}{dT} = -C_B F + r_B V \quad (8)$$

Where C_B is the concentration of component B in g mol/liter and r_B is generation of species of B per unit volume, which is given by

$$r_B = K_1C_A - K_2C_B \quad (9)$$

Where K_2 is reaction rate constant for the equation (1),(2) equation (6) can be written as,

$$\frac{dC_B}{dT} = \frac{F_i}{V} C_B + K_1C_A - K_2C_B \quad (10)$$

Thus model consists of two differential equations therefore two state variables. Often other simplifying techniques are made to reduce the number of differential equations to make them easier to analyze and faster to solve. Assuming constant volume, resulting differential equations governing the isothermal chemical reaction are given by following equations,

$$\frac{dC_A}{dT} = \frac{F_i}{V} (C_{Ai} - C_A) - K_1C_A - K_3C_A^2 \quad (11)$$

$$\frac{dC_B}{dT} = \frac{F_i}{V} C_B + K_1C_A - K_2C_B \quad (12)$$

Here we consider $F/V=D$ as the manipulated variable/input. C_A and C_B as state variables, C_{Ai} as disturbance input and C_B as output variables. For one particular situation, $C_{Ai} = 3$ g mol/liter, $F_s/V=D_s=0.5714$ min⁻¹, $C_Bs=1.117$ gmol/lier, $K_1=5/6$ min⁻¹, $K_2 =5/3$ min⁻¹ and $K_3=1/6$ min⁻¹

2. Linear Analysis

The linear state model is,

$$\dot{X} = AX + BU \quad (13)$$

$$Y = CX + DU \quad (14)$$

Where the states, inputs and output are in deviation variable form. The first input(dilution rate) is manipulated and second (feed concentration of A) is a disturbance input.

$$X = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} = \begin{bmatrix} C_A & -C_{AS} \\ C_B & -C_{BS} \end{bmatrix} \quad (15)$$

$$U = \begin{bmatrix} \frac{F}{V} & \frac{-F_S}{V} \\ C_{Af} & -C_{Afs} \end{bmatrix} \quad (16)$$

$$y = X_2 = [C_B - C_{BS}] \quad (17)$$

Now after linearizing the two modeling equation at steady-state solution to find the following state space matrices:

$$A_{11} = \frac{\partial F_1}{\partial C_A} - \frac{F_S}{V} - K_1 - 2K_3C_{AS}; A_{12} = \frac{\partial F_1}{\partial C_B} = 0; A_{21} = \frac{\partial F_2}{\partial C_A} = 0; A_{22} = \frac{\partial F_2}{\partial C_B} - \frac{F_S}{V} - K_2$$

$$B_{11} = C_{Afs} - C_{As}; B_{12} = \frac{F_S}{V}; B_{13} = -C_{BS}; B_{14} = 0$$

Therefore,

$$A = \begin{bmatrix} \frac{-F_S}{V} - K_1 - 2K_3C_{AS} & 0 \\ K_1 & \frac{-F_S}{V} - K_2 \end{bmatrix}; B = \begin{bmatrix} C_{Afs} - C_{As} & \frac{F_S}{V} \\ -C_{BS} & 0 \end{bmatrix}; C = [0 \ 1]; D = [0 \ 0]$$

Based on the steady state operating point of $C_{AS}=3$ gmol/litre, $C_B=1.117$ g mol/litre, and $F_S/V=0.5714$ /min, the steady space model is

$$A = \begin{bmatrix} -2.4048 & 0 \\ 0.83333 & -2.2381 \end{bmatrix}; B = \begin{bmatrix} 7 & 0.5714 \\ -1.117 & 0 \end{bmatrix}; C = [0 \ 1]; D = [0 \ 0]$$

A. Converting State space to Transfer function Model:

Formula,

$$G(s) = C(SI - A)^{-1}B + D \quad (18)$$

After simplification in the above equation (13) the transfer function is obtained as,

$$G_p(s) = \frac{-1.170s + 3.1472}{s^2 + 4.643s + 5.382} \tag{19}$$

$$G_p(s) = \frac{0.5848(-0.3549s + 1)}{0.1828s^2 + 0.8627s + 1} \tag{20}$$

3. Internal Model Control (IMC)

is a technique that provides a transparent method for the design and tuning of various types of control. The ability of proportional, proportional –integral and proportional –integral-derivative controllers to meet most of the control objectives has led to their widespread acceptance in the control industry. The Internal Model Control (IMC) based approach for controller design is one of them using IMC and its equivalent IMC based PID to be used in control applications in industries. Also the IMC-PID controller allows good set-point tracking but insensitive disturbance response especially for the process with the time-delay ratio. But, for many process disturbance rejection for the unstable processes is more important than set point tracking. Hence, controller which emphasizes on disturbance rejection rather than set point tracking is a more important design problem that has to be taken in to consideration.

This modification in the design procedure of IMC is developed to improve the input disturbance rejection. The IMC based PID structure which uses a standard feedback structure in which the process model is used in an implicit manner. The PID tuning parameter is adjusted based upon the transfer function. In calculating the PID parameter the term λ filter tuning factor is very important. Based on the λ value the value of the term K_c will be determined.

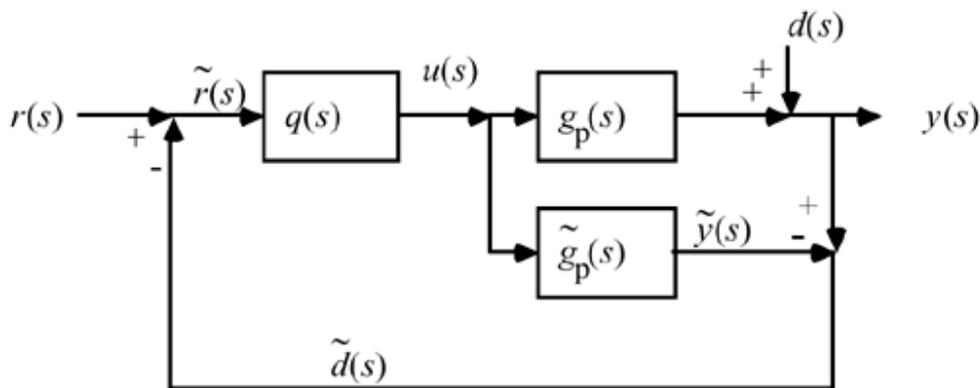


Figure 2 IMC structure

IMC based PID controller for second order process:

Approximated Second order process

$$G_S = \frac{q(s)}{1-g_p(s)q(s)} \tag{21}$$

$$\widetilde{g_p}(s) = \frac{K_p(-\beta s+1)}{\tau^2 s^2+2\epsilon\tau s+1} \tag{22}$$

$$g_p(s) = \frac{-1.170s+3.1472}{s^2+4.6429s+1} \tag{23}$$

The g_p can be rearranged in to the following form,

$$\widetilde{g_p}(s) = \frac{0.5848(-0.3549s+1)}{0.1828s^2+0.8627s+1} \tag{24}$$

$k_p=0.5848, \beta=0.3549; \tau = 0.4275, \epsilon=1.009$

PID parameter is calculated by ,

For $\lambda=0.5; K_c = 1.7258; K_i = 2.00035; K_d=0.3654$

4. Sliding Mode Control

The sliding mode control (SMC) is control action which is used because of its robustness against the disturbances. The methodology behind the sliding mode control is to force the system to reach toward a selected surface. The idea behind SMC is to choose a sliding surface along which the system can slide to its desired final value. The main disadvantage of this robust control is the chattering phenomenon

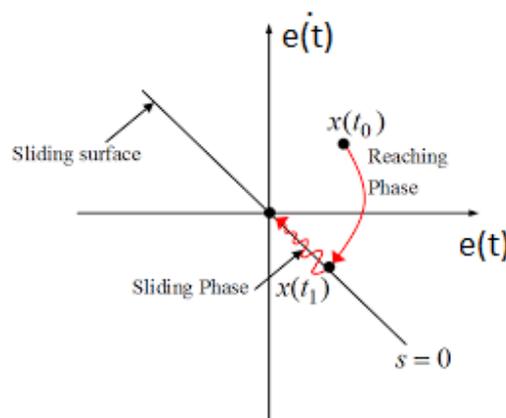


Figure 3. Principle of Sliding Mode Control

The structure of SMC is deliberately changed when the system state trajectory crosses the sliding surface in accordance with a given control law. Hence first of all a sliding surface is selected for the designing of SMC. By designing the SMC, a sliding surface has been selected at first, and then a suitable control law is designed so that the control variable is being driven to its reference value. The structure of SMC law $U(t)$ is based on two main parts; a continuous part $U_c(t)$ and a discontinuous part $U_d(t)$. That is

$$U(t) = U_c(t) + U_d(t) \quad (25)$$

$U_c(t) = U_{eq}(t)$ is the dominated equivalent control, represents the continuous part of the controller that maintains the output of the system restricted to the sliding surface. The continuous part of SMC is given by,

$$U_c(t) = f[R(t), Y(t)] \quad (26)$$

It is a function of the reference value $R(t)$ and controlled variable $Y(t)$. The part U_d (discontinuous) of SMC comprises a non-linear element that contains the switching element of the control law. This part U_d of the controller is discontinuous across the sliding surface.

5. SMC Controller design

In SMC, the objective is to make the error and derivative of error equal to zero. The equation for the n th order sliding function is given by

$$S(t) = \left(\frac{d}{dt} + c\right)^{n-1} e \quad (27)$$

The 2nd order sliding function ($n=2$) can be written as

$$S = \dot{e} + ce \quad (28)$$

Where $c > 0$ is the slope of sliding surface. The basic discontinuous control law of SMC is given by

$$U_d(t) = -K \operatorname{sgn}(s) \quad (29)$$

Where the parameter K is the constant manual tuning parameter and is responsible for the reaching mode. The main disadvantage of SMC is the chattering phenomena. Chattering is a high frequency oscillation around the desired equilibrium point. The chattering problem could be solved satisfactorily if we use the sgn function. Here the K & e values are found by trial and error and c is found to be 0.1 and K is found to be also 0.1. The chattering problem could be solved satisfactorily if we use the signum function. For the continuous control IMC based PID controller is designed here and it is given to the CSTR transfer function in addition with the discontinuous control.

6. PID controller

The expression for PID controller is given by

$$U(t) = K_e \left(e(t) + \frac{1}{T_i} \int_0^t e(t)dt + T_d \frac{de(t)}{dt} \right) \tag{30}$$

The PID controller is tuned using Zeigler-Nichols closed loop tuning method. The values of proportional gain, integral gain and derivative gain of the PID controller are 0.20, 0.95 and 0.23 respectively.

7. Hybrid Controller

The objective of the Hybrid control is that since the IMC based PID control provides time delay compensation and disturbance rejection and SMC is insensitive to the parameter uncertainties and external disturbances which provides the better response. The values of PID parameter i.e. are proportional gain, integral gain, and derivative gains are obtained by the IMC method as formula is given in equation (19).

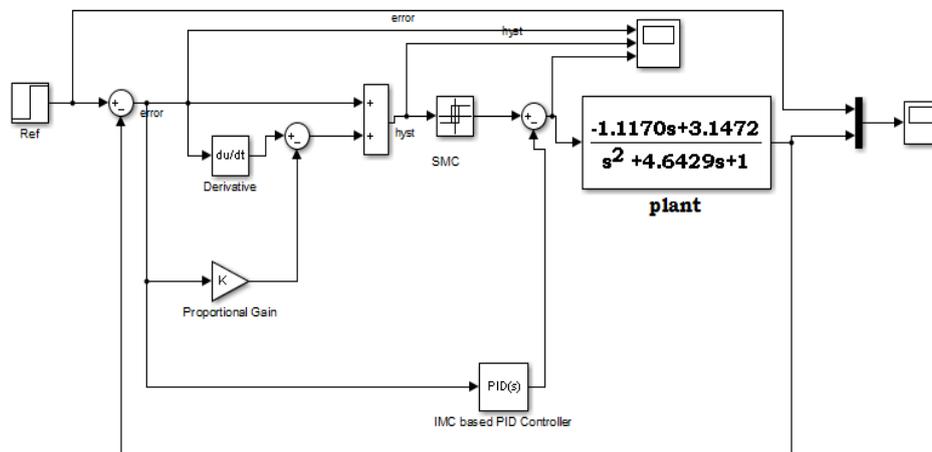


Figure 4. Simulation of Isothermal CSTR using Hybrid controller

The IMC based PID controller is tuned as $K_c=1.17258$; $K_i=2.00035$; $K_d=0.3654$ for $\lambda=0.5$ as it gives best response. And the IMC based PID is used as continuous controller in the SMC controller.

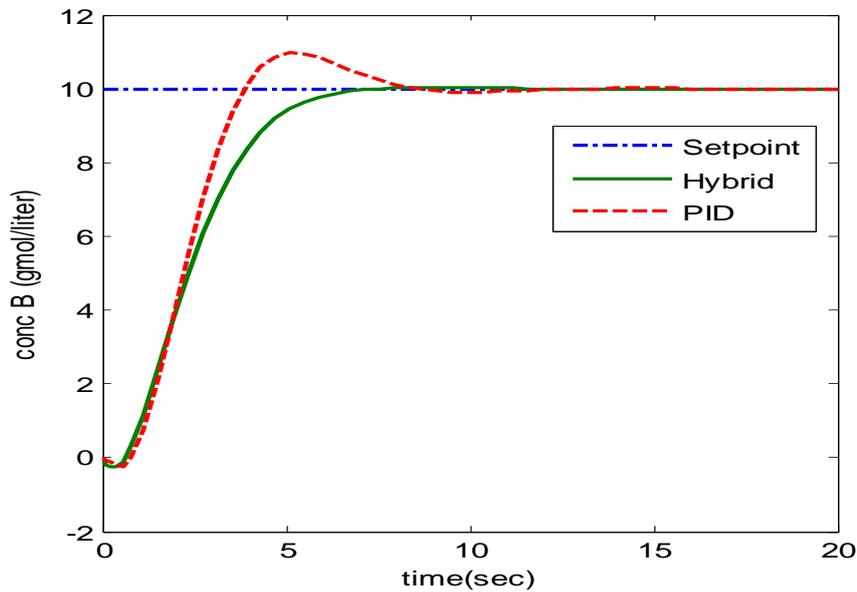


Figure 5 Response of PID and Hybrid controller for Isothermal Continuous Stirrer Tank Process

In Figure 5 shows the response of PID and Hybrid controller for the process. The PID controller having the maximum peak overshoot and high delay time and settling time whereas in hybrid controller response having less settling time and delay time compare with PID controller and no peak overshoot

Table 1: Transient Response

Parameter/C ontroller	Rise time(sec)	Settling time(sec)	Peak overshoot (%)	Peak time(sec)
PID	2.92	7.42	7.43	1.07
HYBRID	1.08	2	0	0.997

Table 1 shows the comparison of controller performance of PID and Hybrid. From the simulation results, it is observed that hybrid controller (IMC based PID +SMC controller) has a better servo tracking compared with PID. Table 2 shows the performance criteria of PID and Hybrid controller. The parameters Integral Absolute Error, Integral Square Error and Integral Time Absolute Error of the system are drastically reduced while using Hybrid controller. This Hybrid controller proves that it has better performance compare with PID controller

Table 2: Performance Criteria of Controllers

Parameter/Controller	IAE	ISE	ITAE
PID	2.141	1.465	3.179
HYBRID	0.9549	0.5198	2.325

8. Conclusion

In this paper Internal Model Control (IMC) based PID with Sliding Mode Control (SMC) is presented and applied to a Isothermal Continuous Stirrer Tank Reactor (CSTR) which has highly non-linear characteristics. The application of the proposed controller eliminates the chattering problem. After time response analysis and error criteria it is observed that hybrid controller provides a satisfactory control performance.

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