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ROUGH SOFT CLOSURE SPACES

M.Gilbert Rani[1], R.Rajeswari[2]

[1]Assistant Professor, Arul Anandar College, Karumathur.

[2]Assistant Professor, Fatima College, Madurai.

ABSTRACT

The Purpose of this paper is to demonstrate *Čech* rough soft closure spaces and also study the properties of the *Čech* rough soft closure spaces.

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Keywords: *Čech* rough soft closure spaces, rough soft topological space, rough soft sets.

1. INTRODUCTION

In a ringing of Mathematical applications, rough set and soft set play a vital role today. To overcome vagueness, uncertainty and delay many researches handle those sets as a powerful tool.

Theory of rough sets was introduced by Pawlak[3]. It studies the data with incomplete information. Two approximation namely lower approximation and upper approximation lead a rough set. Theory of soft set was introduced by Molodtsov[5] and Theory of rough soft set was introduced by M.I.Ali[6].

Čech closure space is the generalization of a topological space and it seems as a conceptual framework for the study of structural configuration [11] in Biology, Chemistry etc.

The concept of *Čech* closure space was characterized by *Čech* E [1, 2]. In this paper, we introduced *Čech* rough soft closure spaces over the rough soft sets on a non-empty set X and we exhibit some results related to these concepts.

2. Preliminaries.

In this section we recall some properties of basic concepts which are useful in the sequel.

Definitions: 2.1[1]

A function C: P(X)$\rightarrow$P(X), P(X) is a power set of a set X, is called a $\check{C}ech$ closure operator for X provided that the following conditions are satisfied:

(i) C($\emptyset$) =$\emptyset$.

(ii) A $\subset C(A)$ *for each* $A \subset X$.

(iii) C(AUB) = C(A) U C(B) for all A, B $\subset$X.

Then C, together with the underlying set X, is called a $\check{C}ech$ closure space and is denoted by (X, C). The $\check{C}ech$ closure space (X, C) is said to be topological space if C(C(A)) = C(A) *for all* $A \subset X$

Definition 2.2:

A soft binary relation (σ, A) over a set U, is called a soft equivalence relation over U, if $\sigma(\alpha_i) \neq \phi$ is an equivalence relation on U for all $\alpha_i \in$A.

Definition 2.3:

Any subset X of the universal set U can be approximated by the equivalence relation $\sigma(\alpha_i)$. The equivalence class containing an element $x \in$U determined by the relation $\sigma(\alpha_i)$ is denoted by $[x]_{\sigma(\alpha i)}$.

The parameterized collection of subsets denoted by ($\underline{\sigma}^X$, A) defined as

$$\underline{\sigma}^X(\alpha_i) = \bigcup_{x \in X} \{[x]_{\sigma(\alpha_i)} : [x]_{\sigma(\alpha_i)} \subseteq X\}$$

for all $\alpha_i \in$A, is called soft lower approximation of X with respect to soft equivalence relations(σ, A).

The parameterized collection of subsets denoted by ($\overline{\sigma}^X$, A) defined as

$$\overline{\sigma}^X(\alpha_i) = \bigcup_{x \in X} \{[x]_{\sigma(\alpha_i)} : [x]_{\sigma(\alpha_i)} \cap X \neq \phi\}$$

for all $\alpha_i \in$A, is called soft upper approximation of X with respect to soft equivalence relations(σ, A).

The soft set (Bσ^X, A) defined by

$$(B\sigma^X, A) = \overline{\sigma}^X(\alpha_i) - \underline{\sigma}^X(\alpha_i)$$

for all $\alpha_i \in A$, is called soft boundary region of X, with respect to soft equivalence relation (σ, A).

A subset X of U is called totally rough with respect to soft equivalent relation (σ, A) if $B\sigma^X(\alpha_i) \neq \phi$ for all $\alpha_i \in A$.

A subset X of U is called partly rough or partly definable with respect to soft equivalent relation (σ, A) if $B\sigma^X(\alpha_i) = \phi$ for some $\alpha_i \in A$.

A subset X of U is called totally definable with respect to soft equivalent relation (σ, A) if $B\sigma^X(\alpha_i) = \phi$ for all $\alpha_i \in A$.

3. $\breve{C}ECH$ ROUGH SOFT CLOSURE SPACES

Definition 3.1

Let P(X) be the power set of a rough soft set X in the approximation space (U,R). A function $\breve{C}$:P(X) $\rightarrow$P(X) ia called $\breve{C}ech$ rough soft closure operator for X if it satisfy the following conditions:

i. $\breve{C}(\phi) = \phi$
ii. $A \subset \breve{C}(A)$ for each $A \subset X$
iii. $\breve{C}(A \cup B) = \breve{C}(A) \cup \breve{C}(B)$ for each $A, B \subset X$

Then $\breve{C}$ together with the rough soft set X, is called a $\breve{C}ech$ rough soft closure space (simply RS-closure space) and it is denoted by (X, $\breve{C}$).

Definition 3.2

$(X, \breve{C})$ is a rough soft topological space if $\breve{C}$ satisfies $\breve{C}(\breve{C}(A)) = \breve{C}(A)$ for each $A \subset X$.

Definition 3.3

Let $\breve{C}_1$ and $\breve{C}_2$ be two $\breve{C}ech$ rough soft closure operators on a set X, $\breve{C}_1$ is said be coarse than $\breve{C}_2$, or equivalently $\breve{C}_2$ is finer than $\breve{C}_1$, if $\breve{C}_2(A) \subset \breve{C}_1(A)$, for each rough soft set A of X.

Definition 3.4

Let $(X, \breve{C})$ be a $\breve{C}ech$ rough soft closure space. A rough soft subset A of X is called rough soft closed provided $A = \breve{C}(A)$. A rough soft subset A of X is called rough soft o,mnn.pen provided its rough soft complement X- A is rough soft closed.

Remark 3.5

For each $\breve{C}ech$ rough soft closure space , there exists an underlying topological space that can be defined in a natural way. If $(X, \breve{C})$ is a $\breve{C}ech$ rough soft closure space , we denote the associated topology on X by $\breve{\tau}(\breve{C})$.

That is $\breve{\tau}(\breve{C}) = \{X - A : \breve{C} = \breve{C}(A)\}$ where X-A denotes the complement of A with respect to X. Members of $\breve{\tau}(\breve{C})$ are the rough soft open sets of $(X, \breve{C})$ and the complements are the rough soft closed sets.

EXAMPLE 3.6

Let U={C_1,C_2,C_3,C_4,C_5,C_6} be the collection of some engineering colleges. Let E={e1,e2,e3,e4,e5}be a set of some attributes which help to identify the quality institution to the students community and A={e1,e2,e3,e4} be a subset of E.

Let e_1 stands for approval status,

e_2 stands for infrastructure facility,

e_3 stands for qualified faculties,

e_4 stands for quality education,

e_5 stands for transportation.

Let (F,A) be the soft set to categorize the engineering colleges with respect to parameters given by A, such that

	C1	C2	C3	C4	C5	C6
e1	0	0	1	0	0	1
e2	0	1	1	0	0	1
e3	1	0	1	1	0	1
e4	0	0	1	0	1	1

Now it is easy to see from the above table that each of the parameter ei:i=1,2,3,4 induces an equivalence relation on U. So we have a soft equivalence relation say (σ, A) over U. Hence we get the following equivalence relation as follows for $\sigma(e_1)$ the equivalence classes are $\{C_3,C_4\},\{C_1,C_2,C_5,C_6\}$

$\sigma(e_2)$ the equivalence classes are $\{C_1,C_3,C_4\},\{C_2,C_5,C_6\}$

$\sigma(e_3)$ the equivalence classes are $\{C_2,C_3,C_4,C_5\},\{C_1,C_6\}$

$\sigma(e_4)$ the equivalence classes are $\{C_3,C_4,C_6\},\{C_1,C_2,C_5\}$

The partition of U obtained by indiscernibility relation IND (F, A) is $\{C_1\},\{C_2,C_5,\},\{C_3,C_4\},\{C_6\}$

We have 44 rough soft sets from above example.

Here X = $\{C_2,C_3,C_6\} \subseteq$ U Iis a rough soft set on U with respect to IND (F, A).

Define $\breve{C}: P(X) \to P(X)$ by

$\breve{C}(\phi) = \phi, \breve{C}(\{C_2\}) = \{C_2, C_6\}, \breve{C}(\{C_3\}) = \breve{C}(\{C_2, C_3\}) = \{C_2, C_3\}, \breve{C}(\{C_3, C_6\}) = \breve{C}(\{C_2, C_6\}) = \breve{C}(X) = X$

Then the operator $\breve{C}$ is the $\breve{C}ech$ rough soft closure operator and $(X, \breve{C})$ is a rough soft closure space and the collection $\tau(\breve{C}) = \{\phi, X, \{C_6\}\}$ is the set of all rough soft open sets in $(X, \breve{C})$. Note that $(X, \breve{C})$ is not a rough soft topological space since $X = \{C_6\}$ is not rough soft.

THEOREM 3.7

Let $(X, \breve{C})$ be a rough soft closure space and $A, B \subset X$. Then the following statements are true.

i. If $A \subset B$ then $\breve{C}(A) \subset \breve{C}(B)$

ii. $\breve{C}(A \cap B) \subset \breve{C}(A) \cap \breve{C}(B)$

Proof:

i. $\breve{C}(A) \subset \breve{C}(A) \cup \breve{C}(B) = \breve{C}(A \cup B) = \breve{C}(B)$ since $A \cup B = B$.

ii. Since $A \cap B \subset A$ and $A \cap B \subset B$ then $\breve{C}(A \cap B) \subset \breve{C}(A) \cap \breve{C}(B)$.

THEOREM 3.8

Let $(X,\breve{C})$ be rough soft closure space and $A,B \subset X$. Then the collection of all rough soft closed sets of a rough soft closure space $(X,\breve{C})$ is rough soft closed under finite unions and arbitrary intersections.

Proof:

Follows from (iii) in definition 3.1 and theorem 3.7 (ii).

THEOREM 3.9

Let $(X,\breve{C})$ be a $\breve{C}ech$ rough soft closure space and A be a rough soft set of X. If $\breve{C}(A)$ is contained in A, then A is rough soft closed.

Proof:

Follows from the definition 3.1(ii).

THEOREM 3.10

Let $(X,\breve{C})$ be a $\breve{C}ech$ rough soft closure space. Then $\breve{\tau}(\breve{C})$ is called the underlying rough soft topology of $(X,\breve{C})$.

Proof:

Clearly, X and ϕ are members of $\breve{\tau}(\breve{C})$. Suppose A and B are members of $\breve{\tau}(\breve{C})$. Then $X-(A\cap B)=(X-A)\cup(X-B)=\breve{C}(X-A)\cup\breve{C}(X-B)=\breve{C}((X-A)\cup(X-B))=\breve{C}(X-(A\cap B))$. Now consider an arbitrary collection of rough soft sets $\{A_i : i\in J\}$ each a member of $\breve{\tau}(\breve{C})$ .For each $i\in J$, X-Ai is rough soft closed and $\bigcap\{X-Ai : i\in J\}$ is contained in X-Ai. Remark 3.9, then implies that $\breve{C}(\bigcap\{X-Ai : i\in J\})$ is contained in $\breve{C}(X-Ai)=X-Ai$ for every $i\in J$. Hence $\breve{C}(\bigcap\{X-Ai : i\in J\})$ is contained in $\bigcap\{X-Ai : i\in J\}$ and by theorem 3.7 , $\bigcap\{X-Ai : i\in J\}=X-\bigcup\{Ai : i\in J\}$ is rough soft closed.

4. CONTINUITY ON ROUGH SOFT

DEFINITION 4.1

Let (X, $\breve{C}_1$) and (Y, $\breve{C}_2$) be two $\breve{C}ech$ rough soft closure spaces. A rough soft mapping $f:(X,\breve{C}_1)\to(Y,\breve{C}_2)$ is said to be rough soft continuity if $f(\breve{C}_1(A))\subseteq\breve{C}_2(f(A))$ for every rough soft subset A$\subseteq$X.

THEOREM 4.2

Let (X, $\breve{C}_1$) and (Y, $\breve{C}_2$) be two $\breve{C}ech$ rough soft closure spaces. If a rough soft mapping $f:(X,\breve{C}_1)\rightarrow(Y,\breve{C}_2)$ is rough soft continuity, then $\breve{C}_1(f^{-1}(B))\subseteq f^{-1}(\breve{C}_2(B))$ for every rough soft subset B$\subseteq$Y.

Proof:

Let B$\subseteq$Y. Then f^{-1}(B) $\subseteq$X. Since f is rough soft continuity, we have $f(\breve{C}_1(f^{-1}(B))\subseteq\breve{C}_2(f(f^{-1}(B)))\subseteq\breve{C}_2(B)$ .Therefore, $f^{-1}(f(\breve{C}_1(f^{-1}(B))))\subseteq f^{-1}(\breve{C}_2(B))$.
Hence $\breve{C}_1(f^{-1}(B))\subseteq f^{-1}(\breve{C}_2(B))$.

Clearly $f:(X,\breve{C}_1)\rightarrow(Y,\breve{C}_2)$ is rough soft continuity, then $f^{-1}(F)$ is closed a rough soft closed subset of (X,$\breve{C}_1$) for every rough soft closed subset F of (Y, $\breve{C}_2$). The following statement is evident.

THEOREM 4.3

Let (X, $\breve{C}_1$) and (Y, $\breve{C}_2$) be two $\breve{C}ech$ rough soft closure spaces. If $f:(X,\breve{C}_1)\rightarrow(Y,\breve{C}_2)$ is rough soft continuity, then $f^{-1}(G)$ is a rough soft open subset of (X, $\breve{C}_1$) for every rough soft open subset G of (Y, $\breve{C}_2$).

THEOREM 4.4

Let (X, $\breve{C}_1$), (Y, $\breve{C}_2$)and (Z, $\breve{C}_3$) be two $\breve{C}ech$ rough soft closure spaces. If a rough soft mapping $f:(X,\breve{C}_1)\rightarrow(Y,\breve{C}_2)$ and $g:(Y,\breve{C}_2)\rightarrow(Z,\breve{C}_3)$ are rough soft continuity, then $g\circ f:(X,\breve{C}_1)\rightarrow(Z,\breve{C}_3)$ is rough soft continuity.

Proof:

Let A$\subseteq$X. Since gof($\breve{C}_1$(A)) = g(f($\breve{C}_1$(A)) and f is rough soft continuity, $g(f(\breve{C}_1(A)))\subseteq g(\breve{C}_2(f(A)))$ .As g is rough soft continuity, we get $g(\breve{C}_2(f(A)))\subseteq\breve{C}_3(g(f(A))))$. Consequently $g(f(\breve{C}_1(A)))\subseteq\breve{C}_3(g(f(A))))$. Hence *gof* is rough soft continuity.

DEFINITION 4.5:

Let (X,$\breve{C}_1$) and (Y,$\breve{C}_2$) be two $\breve{C}ech$ rough soft closure spaces. A rough soft mapping f from (X,$\breve{C}_1$) to (Y,$\breve{C}_2$) is said to be rough soft closed (resp. rough soft open) if *f*(F) is a rough soft closed (resp. rough soft open) subset of (Y, $\breve{C}_2$) whenever F is rough soft closed (resp. rough soft open) subset of (X, $\breve{C}_1$).

THEOREM 4.6

A rough soft mapping f is rough soft closed if and only if for each subset B of Y and each rough soft open subset G of (X, $\breve{C}_1$) containing f^{-1}(B), there is a rough soft open subset U of (Y, $\breve{C}_2$) such that U $\subseteq$ B and f^{-1}(U) $\subseteq$ G.

Proof:

Suppose that f is rough soft closed let B be a rough soft subset of Y and G be a rough soft open subset of (X, $\breve{C}_1$) such that f^{-1}(B)⊆G. Then f(X −G) is a rough soft closed subset of (Y, $\breve{C}_2$). Let U = Y − f(X −G). Then U is a rough soft open subset of (Y, $\breve{C}_2$) and f^{-1}(U) = f^{-1}(Y − f(X − G)) = X − f^{-1}(f(X − G))⊆ X − (X− G) = G. Therefore, U is a rough soft open subset of (Y, $\breve{C}_2$) containing B such that f^{-1}(U) ⊆ G. Conversely, suppose that F is a rough soft closed subset of (X, $\breve{C}_1$). Then f^{-1}(Y− f(F)) ⊆ X − F and X− F is a rough soft open subset of (X, $\breve{C}_1$) .By hypothesis, there is a rough soft open subset U of (Y, $\breve{C}_2$) such that Y−f(F)⊆U and f^{-1}(U)⊆X − F. Therefore F⊆ X − f^{-1}(U). Consequently, Y − U ⊆ f(F) ⊆ f(X − f^{-1} (U))⊆ Y − U, which implies that f(F)=Y − U . Thus, f(F) is a rough soft closed subset of (Y, $\breve{C}_2$). Hence f is rough soft closed.

THEOREM 4.7

Let (X, $\breve{C}_1$), (Y, $\breve{C}_2$)and (Z, $\breve{C}_3$) be $\breve{C}ech$ rough soft closure spaces. Let $f:(X,\breve{C}_1)\rightarrow(Y,\breve{C}_2)$ and $g:(Y,\breve{C}_2)\rightarrow(Z,\breve{C}_3)$ rough soft mappings. Then

1. If f and g are rough soft closed, then gof.
2. If gof is rough soft closed and f is rough soft continuous and surjection, then g is rough soft closed.
3. If gof is rough soft closed and g is rough soft continuous and injection, then f is rough soft closed.

Proof.

(i)Let F be a rough soft closed subset of (X, $\breve{C}_1$). Since f is rough soft closed, f(F) is rough soft closed in (Y, $\breve{C}_2$). Hence $g(f$(G)) is rough soft closed in (Z, $\breve{C}_3$). Thus $g\circ f$ is rough soft closed .

(ii) Let F be a rough soft closed subset of $\breve{C}_1$. Since f is a rough soft continuous map, f^{-1} (F) is rough soft closed in (X, $\breve{C}_1$). Since $g\circ f$ is rough soft closed , $g\circ f(f^{-1}(G))$ = g(f(f^{-1} (G))) is rough soft closed in (Z, $\breve{C}_3$). But f is surjection, so that $g\circ f(f^{-1}(G)) = g(f(f^{-1}(G))) = g(G)$. Hence, g(G) rough soft closed in (Z, $\breve{C}_3$). Therefore g is rough soft closed. (iii) Let F be a rough soft closed subset of (X, $\breve{C}_1$). Since $g\circ f$ is rough soft closed, $g(f$(F) is rough soft closed in (Z, $\breve{C}_3$). As g is rough soft continuity, $g^{-1}(g(f(F)))$ is rough soft closed in (Y, $\breve{C}_2$). But g is injective, so that $g^{-1}(g(f(F))) = f(F)$ is rough soft closed in (Y, $\breve{C}_2$). Therefore, f is rough soft closed.

CONCLUSION 4.9

In this paper, we have studied *Čech* rough soft closure operators which are defined in the set of all rough soft sets over a non-empty set and a fixed set of parameters. The notions of rough soft closed set, rough soft open set are also studied. Then we have studied for each *Čech* rough soft closure space there exists an underlying rough soft topological space that can be defined in a natural way. Finally, we have defined *Čech* rough soft continuous function and studied some of its properties.

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