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Multiple Domination in Bipolar Fuzzy Graphs

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Abstract.

In this paper we introduce the concept of multiple dominating set in bipolar fuzzy graph. Multiple domination number γ_k (G) for several classes of bipolar fuzzy graphs have been determined. The definition of k- dominating set and its domination number in bipolar fuzzy graphs are defined and some properties are analyzed with suitable examples. Also, we obtain the bounds of the multiple domination number in operations on bipolar fuzzy graphs like join, Cartesian product, composition.

Keywords: Bipolar fuzzy graphs, multiple dominating set and multiple domination number

1. Introduction

Rosenfeld introduce the fuzzy graph and define several graph theoretic concepts as path, cycle and connectedness. Mordeson introduced the concept of fuzzy line and developed its basic properties. Bhattacharya and Bhutain investigated the concept of fuzzy automorphism groups. The concept of domination in fuzzy graph was introduced by Somasundaram. He also investigates the concept of independent domination, total domination and connected domination in fuzzy graphs. In this paper we investigate some bounds of multiple domination and we find some results of multiple domination in operations on fuzzy graphs.

2. Basic definitions

A fuzzy graph with V as the underlying set is a pair G: (σ, μ) where σ : V□[0,1] is a fuzzy subset, μ :V×V□[0,1] is a fuzzy relation on the fuzzy subset σ ,such that μ(x ,y) □ σ (x) σ (y) for all x, y ε V. Two nodes x and y are said to be neighbor if μ(x, y)>0.

The underlying crisp graph of the fuzzy graph G:(σ,μ) is denoted as G*:(σ*,μ*) where σ* = {u ε V / σ(u)>0} and μ* ={(u v) є V×V /μ(u ,v)>0}.

A fuzzy graph G: (σ,μ) with the underlying set V, the order of G is defined and denoted by O(G)= $_{x \in V}$ σ(x) and size of G is define and denoted by S(G)= $_{x, y \in V}$ μ(x, y)

Let G: (σ,μ) be a fuzzy graph. The degree of a node is defined as d(u)= $_{v\, u, v \in V}$ μ(u, v). An edge in a fuzzy graph G: (σ,μ) is said to be an effective edge if μ(x, y)=σ(x) σ(y).

A fuzzy graph G: (σ,μ) is said to be a complete fuzzy graph if μ(x, y)=σ(x) σ(y) for all x, y in σ*.A fuzzy graph G: (σ,μ) is said to be a strong fuzzy graph if μ(x, y)=σ(x) σ(y) for all (x, y) in μ*.

Let G: (σ,μ) be a fuzzy graph and u be a node in G then there exist a node v such that (u, v) is a strong arc then we say that u dominates v. Let G: (σ,μ) be a fuzzy graph. A set D of V is said to be fuzzy dominating set of G if every v ε V-D there exit u ε D such that u dominates v. A fuzzy dominating set D of a fuzzy graph G is called minimal fuzzy dominating set of G, if every node v ε D, D-{v} is not a fuzzy dominating set.The fuzzy dominating number (G) of the fuzzy graph G is the minimum cardinality taken over all minimal fuzzy dominating set o f G.

The nodes in s fuzzy graph G are said to be fuzzy independent if there is no strong arc between them. A subset S of V is said to be fuzzy independent set of G if every two nodes of S G: (σ,μ) are fuzzy independent.

3. Multiple domination

In this session we give the definition of k- multiple domination in fuzzy graphs.

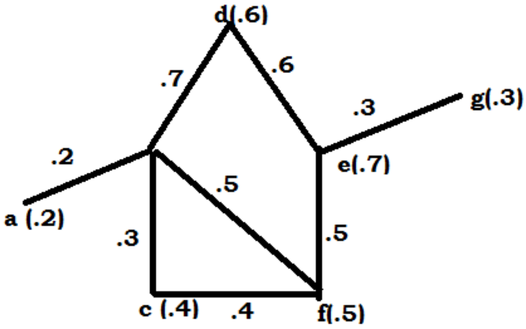
3.1. K- multiple domination in fuzzy graphs

Definition3.1.1.

Let G = (σ, μ) be a fuzzy graph on V. A subset D of V is said to be K dominating if for every vertex $u \in V - D$ is dominated by at least K vertices in D.

The minimum fuzzy cardinality among the K-multiple dominating sets in G is called K-multiple domination number of G and is denoted by $\gamma_k(G)$.

Example 3.1.1



In the fuzzy graph G, $D = \{a, b, c, d, g\}$ is the 2-multiple dominating set and minimal 2-multiple dominating number $\gamma_2(G) = 2.3$

Theorem.3.1.1.

For a fuzzy graph of G of order p, $\gamma(G) \le \gamma_k(G) \le p - \sigma(u)$, Where 'u' be the vertex in a fuzzy graph G having the highest degree.

Proof: Since every K-multiple dominating set is a dominating set of G, $\gamma(G) \le \gamma_k(G)$. Let $u \in V$ if $d_N(u) = \Delta_N(G)$. Then clearly V-u is a K-multiple dominating set of G. K is the number of strong arc between u and V-u. Therefore clearly $\gamma_k(G) \le |V - u|_f$ .i.e., $\gamma_k(G) \le p - \sigma(u)$.

Definition.3.1.2.

Let $G_1(\sigma_1, \mu_1)$ and $G_2(\sigma_2, \mu_2)$ be two fuzzy graphs on V_1 and V_2 respectively with $V_1 \cap V_2 = \phi$. The union of G_1 and G_2 is the fuzzy graph G on $V_1 \cup V_2$ defined by $G = (G_1 \cup G_2) = (\sigma_1 \cup \sigma_2, \mu_1 \cup \mu_2)$ where

$$(\sigma_1 \cup \sigma_2)(\mathrm{u}) = \begin{Bmatrix} \sigma_1(u) \text{ if } \mathrm{u} \in \mathrm{V}_1 \\ \sigma_2(u)\, if\ u \in V_2 \end{Bmatrix} \text{ and}$$

$$(\mu_1 \cup \mu_2)(\mathrm{uv}) = \begin{Bmatrix} \mu_1(uv) \text{ if } \mathrm{u,v} \in \mathrm{V}_1 \\ \mu_2(uv)\, if \text{ u,v} \in \mathrm{V}_2 \\ 0 \quad otherwise \end{Bmatrix}$$

Theorem.3.1.2.

Let D_1 and D_2 be the K_1 and K_2 minimum dominating sets of a fuzzy graph G_1 and G_2 respectively, if $K_1 < K_2$, then $D_1 \cup D_2$ is the K_1 dominating set of $G_1 \cup G_2$.

Proof: If $K_1 < K_2$ let D_1 is a minimum K_1 dominating set of a fuzzy graph G_1, therefore every vertex $x \in V_1 - D_1$ is dominated by at least K_1 vertices in D_1.Similarly every vertex $y \in V_2 - D_2$ is dominated by at least K_2 vertices in D_2. Clearly by the definition of $G_1 \cup G_2$, $x \in (V_1 - D_1) \cup (V_2 - D_2)$ if $x \in (V_1 - D_1)$ is dominated by at least K_1 vertices in D_1, if

$x \in (V_2 - D_2)$ is dominated by at least K_2 vertices in D_2. Since $D_1 \cup D_2$ is the K_1 dominating set of $G_1 \cup G_2$.

Definition.3.1.3.

Let $G_1(\sigma_1, \mu_1)$ and $G_2(\sigma_2, \mu_2)$ is two fuzzy graphs on V_1 and V_2 respectively with $V_1 \cap V_2 = \phi$. The join of G_1 and G_2, denoted by $G_1 + G_2$, is the fuzzy graph on $V_1 \cap V_2$ defined as follows.

$$G_1 + G_2 = (\sigma_1 + \sigma_2, \mu_1 + \mu_2) \text{ Where}$$

$$(\sigma_1 + \sigma_2)(u) = \begin{cases} \sigma_1(u) if\ u \in V_1 \\ \sigma_2(u) if\ u \in V_2 \end{cases} \text{ And}$$

$$(\mu_1 + \mu_2)(uv) = \begin{cases} \mu_1(uv)\, if\ u, v \in V_1 \\ \mu_2(uv)\, if\ u, v \in V_2 \\ \sigma_1(u) \Lambda \sigma_2(v) \text{ if } u \in V_1\ and\ v \in V_2 \end{cases}$$

Theorem.3.1.3.

Let D_1 and D_2 be the K_1 and K_2 minimum dominating sets of a fuzzy graph G_1 and G_2 respectively, and then D_1, D_2 is the K_1 & K_2 dominating set of $G_1 + G_2$ respectively.

Proof: let D_1 be the K_1 dominating set of G_1, therefore every vertex $x \in V - D_1$ is dominated by at least K_1 vertices in D_1. Clearly from the definition of $G_1 + G_2$ every vertex in G_2 dominated by G_1, such that every vertex in $x \in G_1$ and $x \in G_2$ is dominated by K_1 vertices in D_1.

Hence proved.

Definition.3.1.4.

Let $G_1 = (\sigma_1, \mu_1)$ and $G_2(\sigma_2, \mu_2)$ is two fuzzy graphs on V_1 and V_2 respectively. Then the Cartesian product of G_1 and G_2, denoted by $G_1 \times G_2$, is the fuzzy graph on $V_1 \times V_2$ defined as follows

$$G_1 \times G_2 = (\sigma_1 \times \sigma_2, \mu_1 \times \mu_2)\, where$$

$$(\sigma_1 \times \sigma_2)(u_1, u_2) = \sigma_1(u_1) \wedge \sigma_2(u_2)\, and$$

$$(\mu_1 \times \mu_2)((u_1, u_2), (v_1, v_2)) = \begin{cases} \sigma_1(u_1) \wedge \mu_2(u_2 v_2) & if\ u_1 = v_1 \\ \sigma_2(u_2) \wedge \mu_1(u_1 v_1) & if\ u_2 = v_2 \\ 0\ otherwise \end{cases}$$

Theorem.3.1.4.

Let D_1 and D_2 be the K_1 and K_2 minimum dominating sets of a fuzzy graph G_1 and G_2 respectively, then $D_1 \times V_2$ and $V_1 \times D_2$ is the K_1 & K_2 dominating set of $G_1 \times G_2$ respectively.

Proof: let D_1 be the K_1 dominating set of a fuzzy graph G_1. There exist every $x \in V_1 - D_1$ is dominated by K_1 vertices in D_1. there is K_1 strong arc between x and vertices in D_1. We prove $(V_1 - D_1) \times V_2$ is dominated by K_1 vertices in D_1. Let $x \in V_1 - D_1$, clearly x is dominated by

K_1 vertices in D_1. Let $x \in (V_1 - D_1) \times V_2$, there exist a vertex $y \in D_1$ such that $\mu_1(xy) = \sigma_1(x) \Lambda \sigma_1(y)$ .Therefore

$$\begin{aligned}(\mu_1 \times \mu_2)((xv_2),(yv_2)) &= \mu_1(xy)\Lambda\sigma_2(v_2)\\ &= \sigma_1(x)\Lambda\sigma_1(y)\Lambda\sigma_2(v_2)\\ &= (\sigma_1(x)\Lambda\sigma_1(v_2))\Lambda(\sigma_1(y)\Lambda\sigma_2(v_2))\\ &= (\mu_1 \times \mu_2)(xv_2)\Lambda(\mu_1 \times \mu_2)(yv_2)\end{aligned}$$

This implies the vertex (x, v_2) is dominated by vertex (y, v_2) in $G_1 \times G_2$. Since $D_1 \times V_2$ is the K_1 dominating set of $G_1 \times G_2$.

Similarly we can prove $V_1 \times D_2$ is the K_2 dominating set of $G_1 \times G_2$.

Definition.3.1.5. Let $G_1 = (\sigma_1, \mu_1)$ and $G_2(\sigma_2, \mu_2)$ is two fuzzy graphs on V_1 and V_2 respectively. Then the composition of G_1 and G_2, denoted by $(\sigma_1 \circ \sigma_2)(u_1, u_2) = \sigma_1(u_1) \wedge \sigma_2(u_2)$, is the fuzzy graph on $V_1 \times V_2$ defined as follows

$G_1 \circ G_2 = (\sigma_1 \circ \sigma_2, \mu_1 \circ \mu_2)$ Where

$(\sigma_1 \circ \sigma_2)(u_1, u_2) = \sigma_1(u_1) \wedge \sigma_2(u_2)$

and $$(\mu_1 \circ \mu_2)((u_1,u_2)(v_1,v_2)) = \begin{cases} \sigma_1(u_1) \wedge \mu_2(u_2,v_2)\ if\ u_1 = v_1\ and\ u_2 \neq v_2 \\ \sigma_2(u_2) \wedge \mu_1(u_1,v_1)\ if\ u_1 \neq v_1\ and\ u_2 = v_2 \\ \sigma_2(u_2) \wedge \sigma_2(v_2) \wedge \mu_1(u_1,v_1)\ otherwise \end{cases}$$

4. Bipolar fuzzy graphs introduction

In this section, some basic definitions related to bipolar fuzzy are given.

A bipolar fuzzy graph (BFG) is of the form G = (V, E) where $V = \{v_1, v_2, ... v_n\}$ such that $\mu_1^+ : X \to [0, 1]$ and $\mu_1^- : X \to [-1, 0]$ and $E \subset V \times V$ where $\mu_2^+ : V \times V \to [0, 1]$ and $\mu_2^- : V \times V \to [-1, 0]$ Such that $\mu_{2_{ij}}{}^+ = \mu_2^+(v_i v_j) \le \min(\mu_1^+(v_i), \mu_1^+(v_j))$ and $\mu_{2_{ij}}{}^- = \mu_2^-(v_i v_j) \le \max(\mu_1^-(v_i), \mu_1^-(v_j))$ for all $(v_i, v_j) \in E$.

A bipolar fuzzy graph (BFG) G = (V, E) is called strong if $\mu_2^+(v_i, v_j) = \max(\mu_1^+(v_i), \mu_1^+(v_j))$ and $\mu_2^-(v_i, v_j) = \min(\mu_1^-(v_i), \mu_1^-(v_j))$ for all $(v_i, v_j) \in E$.

A bipolar fuzzy graph (BFG) G = (V, E) is called complete if $\mu_2^+(v_i, v_j) = \min(\mu_1^+(v_i), \mu_1^+(v_j))$ and $\mu_2^-(v_i, v_j) = \max(\mu_1^-(v_i), \mu_1^-(v_j))$ for all $(v_i, v_j) \in V$.

Let G = (V, E) be a bipolar fuzzy graph. Then the cardinality of G is defined to be

$$|G| = \sum_{v_i \in V} \left(\frac{1 + \mu_1^+(v_i) + \mu_1^-(v_i)}{2}\right) + \sum_{(v_i, v_j) \in E} \left(\frac{1 + \mu_2^+(v_i v_j) + \mu_2^-(v_i v_j)}{2}\right)$$

Let G = (V, E) be a bipolar fuzzy graph. Then the vertex cardinality of G is defined by

$|V| = \sum_{v_i \in V} (\frac{1+\mu_1^+(v_i)+\mu_2^-(v_j)}{2})$ for all $v_j \in V$.

The number of vertices (the cardinality of V) is called the order of a BFG and is defined by O(G) = $\sum_{v_i \in V} (\frac{1+\mu_1^+(v_i)+\mu_1^-(v_i)}{2})$

Let G = (V, E) be a bipolar fuzzy graph. Then the edge cardinality of G is defined by $|E| = \sum_{(v_i,v_j)\in E} (\frac{1+\mu_2^+(v_iv_j)+\mu_2^-(v_iv_j)}{2})$.

The number of edges (the cardinality of E) is called the size of a BFG and is defined by $S(G) = \sum_{(v_i,v_j)\in E} (\frac{1+\mu_2^+(v_iv_j)+\mu_2^-(v_iv_j)}{2})$ for all $(v_iv_j) \in E$.

The degree of a vertex v in a BFG, G(V, E) is defined to be sum of the cardinality of strong arcs incident at v .It is denoted by $d_G(v)$.The minimum degree of G is $\delta(G) = \min\{d_G(v)/v \in V\}$ The maximum degree of g is $\Delta(G) = \max\{d_G(v)/v \in V\}$.

The two vertices v_i and v_j are said to be effective neighbours in BFG if either one of the following conditions holds

i) $\mu_2^+(v_iv_j) > 0$ *and* $\mu_2^-(v_iv_j) = 0$

ii) $\mu_2^+(v_iv_j) = 0$ *and* $\mu_2^-(v_iv_j) < 0$

iii) $\mu_2^+(v_iv_j) > 0$ *and* $\mu_2^-(v_iv_j) < 0, (v_i, v_j) \in E$

The complement of a BFG, G = (V, E) is a BFG $\overline{G} = (\overline{V}, \overline{E})$, where

i) $\overline{V} = V$

ii) $\overline{\mu_{1i}^+} = \mu_{1i}^+$ and $\overline{\mu_{1i}^-} = \mu_{1i}^-$ for all i = 1, 2, ...n.

iii) $\overline{\mu_{2_{ij}}^+} = \min(\mu_{1_i}^+, \mu_{1_j}^+) - \mu_{2_{ij}}^+$ and

$\overline{\mu_{2_{ij}}^-} = \max(\mu_{1i}^-, \mu_{1j}^-) - \mu_{2ij}^-$ for all i, j = 1, 2, ..n.

An edge (u, v) is said to be strong edge in BFG, G = (V, E) if $\mu_2^+(u,v) \geq (\mu_2^+)^\infty(u,v), \mu_2^-(u,v) \leq (\mu_2^-)^\infty(u,v)$

Where $\mu_2^+(u,v) = \max\{(\mu_2^+)^k(u,v)/k = 1,2,3,...n\}$ *and*
$\mu_2^-(u,v) = \min\{(\mu_2^-)^k(u,v)/k = 1,2,3,...n\}$

A vertex u be a vertex in BFG, G = (V, E) then N(u) = {v, v∈V and (u, v) is a strong arc in G} is called neighbourhood of u in G. A vertex u∈V of a BFG, G = (V, E) is said to be isolated vertex if $\mu_2^+(u,v) = 0$ *and* $\mu_2^-(u,v) = 0$ *for all* $v \in V, u \neq v$ That is N(u) = 0. Thus an isolated vertex does not dominate any other vertex of G.Let G = (V, E) be a BFG on V. Let u, v∈V , we say that u dominates v in G if there exists a strong edge between them.

Note :

i) For any u, v∈V of u dominates v then v dominates u and hence domination is a symmetric relation on V.

ii) For any v∈V, N(v) is precisely the set of all vertices in V which are dominated by v.

iii) If $\mu_2^+(u,v) < (\mu_2^+)^\infty(u,v)$ and $\mu_2^-(u,v) > (\mu_2^-)^\infty(u,v)$ for all u, v∈V. Then the dominating set of G is V.

A subset S of V is called dominating set in G if for every v∈V- S, there exist u∈S such that u dominates v. A dominating set S of a BFG is said to be minimal dominating set if no proper subset of S is a dominating set. The Minimum cardinality among all minimal dominating set is called the lower domination number of G and is denoted by γ_{bf}(G). The Maximum cardinality among all minimal dominating set is called the upper domination number of G and is denoted by Γ_{bf}(G).

5. Multiple domination in bipolar fuzzy graphs

Definition.5.1.

Let G = (σ,μ) be a fuzzy graph on V. A subset D of V is said to be K dominating if for every vertex $u \in V - D$ is dominated by at least K vertices in D. The minimum fuzzy cardinality among the K-multiple dominating sets in G is called K-multiple domination number of G and is denoted by $\gamma_{bfk}(G)$.

Example.5.1

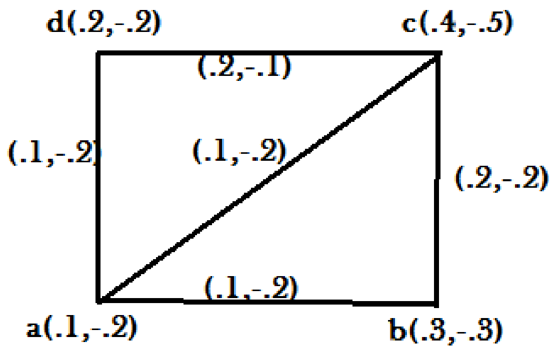


Fig 5.1

From the above example the dominating set of the bipolar graph G is D={a} and the dominating number is .45

Remark

Let G be a complete bipolar fuzzy graph of K vertices and D be a minimum dominating set of G. Then V-D be a minimum (K-1) multiple dominating set of G.

Theorem.5.2

Let G be a strong bipolar fuzzy graph of K vertices and D be a minimum dominating set of G. Then V-D be a K-multiple dominating set of G, where $K = \min_{v_i \in D}\left\{\left\lceil \frac{d(v_i)}{\bigwedge_{u \in N(v_i)} |u|_f} \right\rceil\right\}$.

Proof: Let $D \subset V$ is a minimum dominating set of a bipolar fuzzy graph. Therefore every vertex $v \in V - D$ is dominated by a vertex in D. clearly $|D|_f \leq |V-D|_f$, since D is a minimum dominating set of G. therefore we get every vertex in D is adjacent to more than a vertex in V-D. such that V-D is a K-multiple dominating set of D.

Let $v_i \in D \;\&\; u \in N(v_i)$ such that the number of vertices adjacent to v_i is $\left\lceil \frac{d(v_i)}{\bigwedge_{u \in N(v_i)} |u|_f} \right\rceil$.

Therefore $K = \min_{v_i \in D}\left\{\left\lceil \frac{d(v_i)}{\bigwedge_{u \in N(v_i)} |u|_f} \right\rceil\right\}$. Hence proved.

Example 5.2

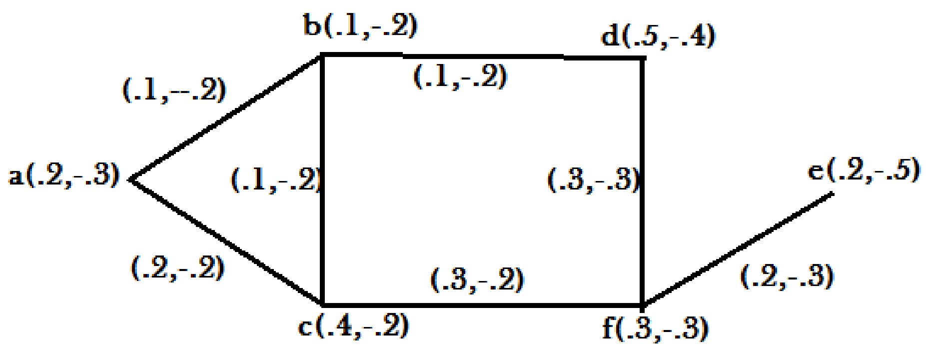


Fig 5.2

From the above example D={a,,f} is a minimum dominating set and V-D is a 2-dominating set.

$$d(a)=1.05, d(f)=1.5, \bigwedge_{u\in N(a_i)} |u|_f = 0.45, \bigwedge_{v\in N(f_i)} |v|_f = 0.6,, K = \min_{v_i\in D}\left\{\left\lceil \frac{d(v_i)}{\bigwedge_{u\in N(v_i)} |u|_f} \right\rceil\right\},$$

$$K = \min_{v_i\in D}\left\{\left\lceil \frac{1.05}{0.45} \right\rceil, \left\lceil \frac{1.5}{0.6} \right\rceil\right\} = 2.$$

Definition.5.2.

The Cartesian product G1 ×G2 is the pair (A,B) of bipolar fuzzy sets defined on the Cartesian product $G_1^* \times G_2^*$ such that

$$\begin{cases} \mu_A^+(x_1,x_2) = \min(\mu_{A_1}^+(x_1), \mu_{A_2}^+(x_2)) \\ \mu_A^-(x_1,x_2) = \max(\mu_{A_1}^-(x_1), \mu_{A_2}^-(x_2)) \quad for\ all\ (x_1,x_2) \in V_1 \times V_2 \end{cases}$$

$$\begin{cases} \mu_{\mathbf{B}}^+(x\ ,x_2)(x,y_2) = \min(\mu_{A_1}^+(x\ ), \mu_{B_2}^+(x_2y_2)) \\ \mu_B^-(x\ ,x_2)(x,y_2) = \max(\mu_{A_1}^-(x\ ), \mu_{B_2}^-(x_2y_2)) \quad for\ all\ x_2y_2 \in E_2 \end{cases}$$

$$\begin{cases} \mu_{\mathbf{B}}^+(x_1,z)(y_1,z) = \min(\mu_{B_1}^+(x_1y_1), \mu_{A_2}^+(z)) \\ \mu_B^-(x_1,z)(y_1,z) = \max(\mu_{B_1}^-(x_1y_1), \mu_{A_2}^-(z)) \quad or\ all\ x_1y_1 \in E_1 \end{cases}$$

Theorem.5.3

Let D_1 and D_2 be the K_1 and K_2 minimum dominating sets of a bipolar fuzzy graphs G_1 and G_2 respectively, then $D_1 \times V_2$ and $V_1 \times D_2$ is the $K_1 \& K_2$ dominating set of $G_1 \times G_2$ respectively.

Proof: Let D_1 be the K_1 multiple dominating set of a bipolar fuzzy graph G_1 . Therefore every $x \in (V_1 - D_1)$ is dominated by K_1 vertices in D_1. Now we prove $(V_1 - D_1)\times V_2$ is dominated by atleast K_1 vertices in D_1. Let $x \times v_2 \in (V_1 - D_1)\times V_2$, there exist a vertex $y \in D_1$, such that

$$\mu_{B_1}^+(xy) = \mu_{A_1}^+(x) \wedge \mu_{A_1}^+(y) \ \ \&$$

$$\mu_{B_1}^-(xy) = \mu_{A_1}^-(x) \vee \mu_{A_1}^-(y)$$

Let $(x,v_2) \in (V_1 - D_1)\times V_2$ and $(y,v_2) \in D_1 \times V_2$, therefore we get

$$\begin{aligned}(\mu_{B_1}^+ \times \mu_{B_2}^+)((x,v_2)(y,v_2)) &= \mu_{B_1}^+(xy) \wedge \mu_{A_2}^+(v_2) \\ &= \mu_{A_1}^+(x) \wedge \mu_{A_1}^+(y) \wedge \mu_{A_2}^+(v_2) \\ &= \mu_{A_1}^+(x) \wedge \mu_{A_2}^+(v_2) \wedge \mu_{A_1}^+(y) \wedge \mu_{A_2}^+(v_2) \\ &= (\mu_{A_1}^+ \times \mu_{A_2}^+)(x,v_2) \wedge (\mu_{A_1}^+ \times \mu_{A_2}^+)(y,v_2))\end{aligned}$$

$$(\mu^-_{B_1} \times \mu^-_{B_2})((x,v_2)(y,v_2)) = \mu^-_{B_1}(xy) \vee \mu^-_{A_2}(v_2)$$

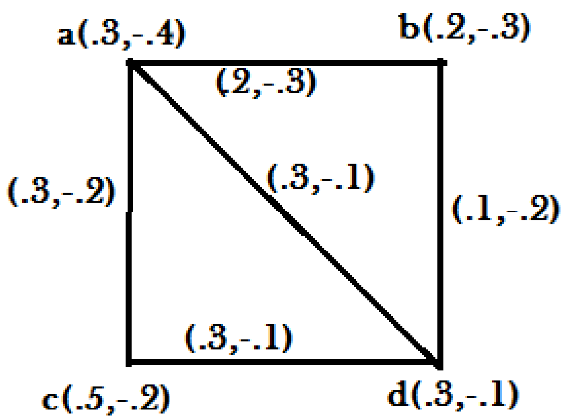
And
$$= \mu^-_{A_1}(x) \vee \mu^-_{A_1}(y) \vee \mu^-_{A_2}(v_2)$$
$$= \mu^-_{A_1}(x) \vee \mu^-_{A_2}(v_2) \vee \mu^-_{A_1}(y) \vee \mu^-_{A_2}(v_2)$$
$$= (\mu^-_{A_1} \times \mu^-_{A_2})(x,v_2) \vee (\mu^-_{A_1} \times \mu^-_{A_2})(y,v_2))$$

Hence (x,v_2) is dominated by (y,v_2). D_1 is a K_1 multiple dominating set of G_1, therefore $(x,v_2) \in (V_1 - D_1) \times V_2$ is dominated by K_1 vertices in $D_1 \times V_2$, since by the definition of $G_1 \times G_2$. Hence $D_1 \times V_2$ is a K_1 dominating set of $G_1 \times G_2$.

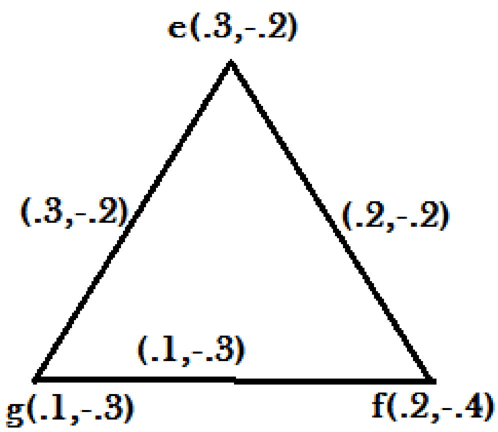
Next we prove $D_1 \times V_2$ is minimum. Assume $(D_1 \times V_2) - (x_1, v_2)$ is a minimum K_1 dominating set of $G_1 \times G_2$. x_1 is dominated by K_1 vertices in D_1, we get $(D_1 \times x_1)$ is K_1 dominating set of $G_1 \times G_2$. This is contradict to our assumption D_1 is minimum K_1 dominating set of G_1. Therefore $D_1 \times V_2$ is a minimum K_1 multiple dominating set of $G_1 \times G_2$.

Similarly we can prove $V_1 \times D_2$ is the K_2 dominating set of $G_1 \times G_2$. Hence proved.

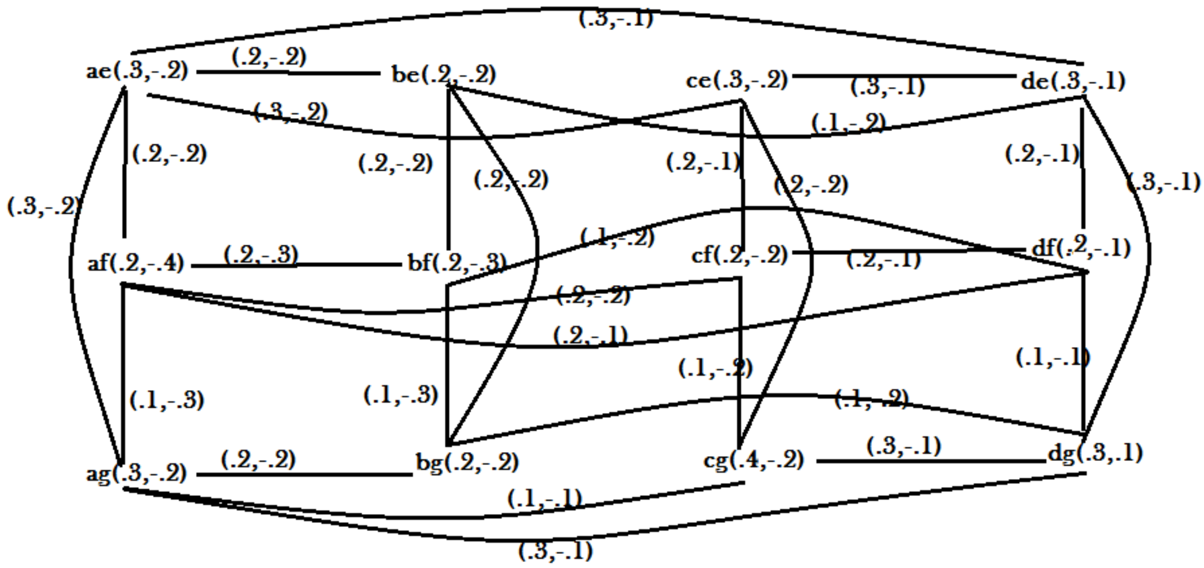
Example 5.3



G_1



G_2



G_1 x G_2

Figure 5.3

From the above figure minimum multiple dominating set of G_1 and G_2 is D_1 ={c,b,d} and D_2={f,g}. D_1 is 3-dominating set of G_1 and D_2 is 2-dominating set of G_2. In $G_1 \times G_2$ $\{D_1 \times V_2\}\&\{V_1 \times D_2\}$ are the 3-dominating se and 2- dominating set of $G_1 \times G_2$ respectively.

Conclusion

In this paper the concept of multiple domination and multiple domination number is introduced and investigate the multiple domination number in operations on bipolar fuzzy graph, we can extended the research work to other domination parameter.

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