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VAGUE IDEAL OF A NEAR-RING

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Abstract. The main motivation of this paper to introduce the notion of Vague sub near-ring and Vague ideals of a near-ring. Based on these concepts, we analyzed some properties and results for development of the-orems illustrated with examples.

Keywords: vague sub near-ring and Vague ideal of a near-ring.
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1 Introduction

In mathematics, fuzzy sets are sets whose elements have degrees of member-ship. The advantage of fuzzy sets and fuzzy subsets were firstly introduced by L.A.Zadeh [12] in 1965. W. Liu [8] has studied fuzzy ideal of rings and many authors [5,6] are extending the concepts. The notion of fuzzy ideals are used different areas, like semi groups[7], near-rings[1,10] etc. To increase the study of vague sets, many authors have considered several ex-tension works in fuzzy [13] sets. W.L.Gau et al [4] was the first to study the notion of vague sets. Also pointed out two important membership func-tions. First one is that, a true membership function and second one is false membership function, which named as interval membership function, as opposed to point membership in the context of fuzzy sets. R.Biswas [2] initiated the study of vague groups etc. T.Eswarlal[3] was introduced the notion of vague field and vague vector space. In [11] Seung Dong Kim and Hee Sik Kim has studied the notion of fuzzy sub near-ring and fuzzy ideal of near-ring and P.Narsimha Swamy [9] studied sum of fuzzy ideal of near-ring In this direction, we proposed the new concepts vague sub near-ring and vague ideals of a near-ring. And also, we have studied some properties and their results discussed lucid manner.

2 Preliminaries

For the sake of continuity we recall some basic definitions.

Definition 1. [4] A vague set A in the universe of discourse U is a pair (t_A, f_A), where $t_A : U \to [0,1]$, $f_A : U \to [0,1]$ are mappings such that $t_A(u) + f_A(u) \leq 1$ for all u∈U. The functions t_A and f_A are called true membership function and false membership function in [0,1] respectively.

Definition 2. [3] The interval $[t_A(u), 1-f_A(u)]$ is called the vague value of u in A and it is denoted by $A(u)$, i.e. $A(u) = [t_A(u), 1-f_A(u)]$. Where A is a vague set and u∈U is the universal of discourse or classical objects.

Definition 3. [2] Let (G,*) be a group. A vague set A of G is called vague group of G if for all x,y in G, $A(xy) \geq \min\{A(x), A(y)\}$ and $A(x^{-1}) \geq A(x)$ for all x in G i,e. $t_A(xy) \geq \min\{t_A(x), t_A(y)\}$ and $f_A(xy) \leq \max\{f_A(x), f_A(y)\}$ and $t_A(x^{-1}) \geq t_A(x), f_A(x^{-1}) \leq f_A(x)$, Here the element xy stands for x*y

Notation 1. [3] Let I[0, 1] denotes the family of all closed subinterval of[0, 1]. If $I_1 = [a_1, b_1]$ and $I_2 = [a_2, b_2]$ be two elements of I[0, 1], we call $I_1 \geq I_2$ if $a_1 \geq a_2$ and $b_1 \geq b_2$ with the order in I[0, 1] is a lattice with operations min or inf and max or sup given by

$$\min\{I_1, I_2\} = [\min(a_1, a_2), \min(b_1, b_2)]$$
$$\max\{I_1, I_2\} = [\max(a_1, a_2), \max(b_1, b_2)]$$

Definition 4. [11] Let R and S be near-ring. A map $\phi : R \to S$ is called near-ring homomorphism if $\phi(x+y) = \phi(x)+\phi(y)$ and $\phi(xy) = \phi(x)+\phi(y)$ for all x, y∈ R.

3 Vague ideal of a near-ring

Definition 5. Let A be a vague set of a near-ring N. Then A is called vague sub near-ring of N , if it satisfies the following conditions:
(i) $V_A(x+y) \geq \min(V_A(x), V_A(y))$, (ii) $V_A(-x) = V_A(x)$,
(iii) $V_A(xy) \geq \min(V_A(x), V_A(y))$ for every $x, y \in N$.

Definition 6. Let A be a vague set of a near-ring N . Then A is said to be a vague ideal of N , if it satisfies the following conditions:
(i) $V_A(x+y) \geq \min(V_A(x), V_A(y))$ (ii) $V_A(-x) = V_A(x)$,

(iii)$V_A(z + x - z) \geq V_A(x)$,
(iv) $V_A(xy) \geq V_A(x)$,
(v) $V_A(x(y + i) - xy) \geq V_A(i)$(equivalently $V_A(xz - xy) \geq V_A(z - y)$) for every x, y, z, i $\in$ N.
A is a vague right ideal of N if it satisfies (i), (ii), (iii) and (iv). A is a vague left ideal of N if it satisfies (i), (ii), (iii) and (v).

Note: (a) In the above definition the conditions (i) and (ii) together can be written as $V_A(x - y) \geq \min(V_A(x), V_A(y))$.
(b) If A is a vague ideal of N , then $V_A(x+y) = V_A(y+x)$ for every x, y $\in$ N. (c) If A is a vague ideal of N , then $V_A(0) \geq V_A(x)$ for every x $\in$ N.
Example 1. Let $N_1 = (Z_3 = \{0, 1, 2\})$ be a near-ring under residue classes of addition and multiplication modulo-3.

A vague set $A = (t_A, f_A)$ of N_1 defined as

$t_A : N_1 \to [0, 1]$ and $f_F : N_1 \to [0, 1]$ by

$$t_A(x) = \begin{cases} 0.5 & \text{if } x = 0, \\ 0.5 & \text{if } x = 1, 2. \end{cases}$$

$$f_A(x) = \begin{cases} 0.5 & \text{if } x = 0, \\ 0.5 & \text{if } x = 1, 2. \end{cases}$$

It is clear that, A is a vague ideal of N_1 for every x,y $\in$ N_1.
Example 2. Let $N_2 = Z_6 = \{0, 1, 2, 3, 4, 5\}$ be a near-ring under residue classes of addition and multiplication modulo-6.

A vague set $A = (t_A, f_A)$ of N_2 defined as

$t_A : N_2 \to [0, 1]$ and $f_A : N_2 \to [0, 1]$ by

$$f_A(x) = \begin{cases} 0.3 & \text{if } x = 0, 1, \\ 0.3 & \text{if } x = 2, 3, \\ 0.3 & \text{if } x = 4, 5. \end{cases}$$

$$t_A(x) = \begin{cases} 0.3 & \text{if } x = 0, 1, \\ 0.3 & \text{if } x = 2, 3 \\ 0.3 & \text{if } x = 4, 5. \end{cases}$$

It is clear that, A ie a vague ideal of N_2 for every x,y $\in$ N_2.
Remark 1. Let A be a vague ideal of N , then the condition $V_A(xz - xy) \geq V_A(z - y)$ is equivalent to the condition $V_A(x(y + i) - xy) \geq V_A(i)$.

Proof. Suppose that $V_A(xz - xy) \geq V_A(z - y)$. Then
$V_A(x(y + i) - xy) \geq V_A(y + i - y) = V_A(i)$.
Conversely, suppose that $V_A(x(y + i) - xy) \geq V_A(i)$.
Then $xz - xy = x(y - y + z) - xy = x(y + i) - xy$ where $i = -y + z$.

$$
\begin{aligned}
\text{T hus } V_A(xz - xy) &= V_A(x(y + i) - xy) \geq \\
&V_A(i) \\
&= V_A(-y + z) \\
&= V_A(z - y).
\end{aligned}
$$

□

Lemma 1. Let N be a near-ring and A be a vague set of N satisfies the condition $V_A(x - y) \geq \min(V_A(x), V_A(y))$ then (i) $V_A(0) \geq V_A(x)$ (ii) $V_A(-x) \geq V_A(x)$

Proof.

$$
\begin{aligned}
\text{(i) } V_A(0) &= V_A(x - x) \\
&\geq \min(V_A(x), V_A(-x)) \\
&\geq \min(V_A(x), V_A(x)) \\
&= V_A(x) \text{ for every } x \in N \\
\text{(ii) } V_A(-x) &= V_A(0 - x) \\
&\geq \min(V_A(0), V_A(-x)) \\
&\geq \min(V_A(x), V_A(x)) = \\
&V_A(x)
\end{aligned}
$$

for all $x \in N$, finally we have that $V_A(-x) = V_A(x)$ □

Proposition 1. Let A be a vague ideal of near-ring N. If $V_A(x-y) = V_A(0)$ then $V_A(x) = V_A(y)$.

Proof. First suppose that $V_A(x - y) = V_A(0)$ for all $x,y \in N$

$$
\begin{aligned}
\text{then } V_A(x) &= V_A(x - y + y) \\
&\geq \min(V_A(x - y), V_A(y)) \\
&\geq \min(V_A(0), V_A(y)) \\
&= V_A(y)
\end{aligned}
$$

Similarly, using $V_A(y-x) = V_A(x-y) = V_A(O)$, we have $V_A(y) \geq V_A(x)$ □

Definition 7. Let A be a vague set of N, and g is a function defined on N, then the vague set h in g(N) define by $V_h(y) = \sup_{x \in g^{-1}(y)} V_A(x)$ for every $y \in g(N)$ is called the image of A under g.

Similarly, if B is a vague set in g(N), then the vague set $A = h \circ g$ in

N(i.e, the vague set defined as $V_A(x) = V_h(g(x))$ for every $x \in N$) is called the pre-image of h under g.

Theorem 1. A near-ring homomorphic pre-image of a vague left(right) ideal is a vague left(right) ideal.

Proof. Let $\psi : N \to S$ be a near-ring homomorphism, and h be a vague left ideal of S and A be the pre-image of h under ψ .

$$\begin{aligned} then\ V_A(x-y) &= V_h(\psi(x-y)) \\ &= V_h(\psi(x)-\psi(y)) \\ &\geq \min(V_h(\psi(x)), V_h(\psi(y))) \\ &= \min(V_A(x), V_A(y)) \end{aligned}$$

and

$$\begin{aligned} V_A(xy) &= V_h(\psi(xy)) \\ &= V_h(\psi(x)\psi(y)) \\ &\geq V_h(\psi(y)) \\ &= V_A(y)) \end{aligned}$$

and

$$\begin{aligned} V_A(y+x-y) &= V_h(\psi(y+x-y)) \\ &= V_h(\psi(y)+\psi(x)-\psi(y)) \\ &\geq V_h(\psi(x) \\ &= V_A(x)\ for\ every\ x, y \in N \end{aligned}$$

Now suppose that h is a vague right ideal of S, then

$$\begin{aligned} V_A((x+i)y-xy) &= V_h(\psi((x+i)y+xy)) \\ &= V_h((\psi(x)+\psi(i))\psi(y)+\psi(x)\psi(y)) \geq V_h(\psi(i)) \\ &= V_A(x)\ for\ every\ x, y, i \in N. \end{aligned}$$

proof is completed.

we say that a vague set A in N has the sup property if, for any subset T of N, $\exists\ t_0 \in T$ such that $V_A(t_0) = \sup_{t \in T} V_A(t)$ □

Theorem 2. A near-ring homomorphic Image of a vague left(right) ideal having the sup property is a vague left(right) ideal.

Proof. Let $\psi : N \to S$ be a near-ring homomorphism, and A be a vague left ideal of N with the sup property and h be the image of A under ψ Given $\psi(x), \psi(y) \in \psi(N)$ and

Let $x_0 \in \psi^{-1}(\psi(x))$, $y_0 \in \psi^{-1}(\psi(y))$ be such that $V_A(x_0) = \sup_{t\in\psi^{-1}(\psi(x))} V_A(t)$, $V_A(y_0) = \sup_{t\in\psi^{-1}(\psi(y))} V_A(t)$ respectively, then

$$\begin{aligned} V_h(\psi(x)-\psi(y)) &= \sup_{t\in\psi^{-1}(\psi(x)-\psi(y))} V_A(t) \\ &\geq V_A(x_0-y_0) \\ &\geq min(V_A(x_0),V_A(y_0)) \\ &= min(\sup_{t\in\psi^{-1}(\psi(x))} V_A(t), \sup_{t\in\psi^{-1}(\psi(y))} V_A(t)) \\ &= min(V_h(\psi(x)),V_h(\psi(y))) \end{aligned}$$

and

$$\begin{aligned} V_h(\psi(x)\psi(y)) &= \sup_{t\in\psi^{-1}(\psi(x)\psi(y))} V_A(t) \\ &\geq V_A(x_0y_0) \\ &\geq V_A(y_0)) \\ &= \sup_{t\in\psi^{-1}(\psi(y))} V_A(t) \\ &= V_h(\psi(y) \end{aligned}$$

and

$$\begin{aligned} V_h(\psi(y+x-y)) = V_h(\psi(y)+\psi(x)-\psi(y)) &= \sup_{t\in\psi^{-1}(\psi(y)+\psi(x)-\psi(y))} V_A(t) \\ &\geq V_A(y_0+x_0-y_0) \\ &= V_A(x_0) \\ &= \sup_{t\in\psi^{-1}(\psi(x))} V_A(t) \\ &= V_h(\psi(x)). \end{aligned}$$

this shows that A is a vague left ideal of $\psi(N)$. Next assume that A is a vague right ideal of N. Given $\psi(i) \in \psi(N)$, let $i_0 \in \psi^{-1}(\psi(i)$ such that

$$V_A(i_0) = \sup_{t\in\psi^{-1}(\psi(i))} V_A(t) then$$

$$\begin{aligned} V_h(\psi(x+i)y-xy)) = V_h(\psi(x)+\psi(x))\psi(y)-\psi(x)\psi(y)) &= \sup_{t\in\psi^{-1}(\psi(x)+\psi(x))\psi(y)-\psi(x)\psi(y))} V_A(t) \\ &\geq V_A((x_0+i_0)y_0-x_0y_0) \\ &= V_A(i_0) \\ &= \sup_{t\in\psi^{-1}(\psi(x))} V_A(t) \\ &= V_h(\psi(i)). \end{aligned}$$

this shows that h is a vague right ideal of $\psi(N)$.

□

4 Conclusion

In this paper, we define the new concepts vague sub near-ring defined and discussed and also some interesting concepts vague ideal of near-ring pre-sented. We have observe that a near-ring homomorphic pre-image of a vague left(right) ideal is a vague left(right) ideal and also obverse near-ring homomorphic Image of a vague left(right) ideal having the sup property is a vague left(right) ideal. And developed some propertis related to the vague ideal of a near-ring.

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