International Journal of Pure and Applied Mathematics
Volume 117 No. 15 2017, 1031-1042
ISSN: 1311-8080 (printed version); ISSN: 1314-3395 (on-line version)
url: http://www.ijpam.eu
Special Issue

ijpam.eu

Bat Algorithm Based Parameter Identification Of Second Order System using Step Response

S.K. Lakshmanaprabu[1,*], D. Najumnissa Jamal[2]

[1]Research Scholar, Electronics and Instrumentation Engineering,
[2]professor, Electronics and Instrumentation Engineering,
[1,2]B S Abdur Rahman University, Chennai, India.
*prabusk.leo@gmail.com

Abstract:

This paper presents the identification of model parameter for Two interacting conical frustum tank process using a bat optimization algorithm. The mathematical model TICFTLP is developed using mass balance equations and then the model parameters are identified using real time experimental data. The ordinary differential equation parameters are identified for a minimum value of Root Mean Square Error (RMSE). The validation results of real time model and identified model are discussed.

Keyword: *Model identification, parameter estimation, RMSE, Bat Algorithm.*

1. Introduction

The mathematical model is essential for engineering and scientific area to understand and analyze about the system. The mathematical model of an industrial process or any system are developed based on the physical laws with assumptions. The developed mathematical model is often complex with some unknown parameters. The identification of unknown model parameter is the central step in the process control engineering. The controller can be designed according to the identified model. But it is very difficult to obtain model parameter of the model accurately.

Schittkowski, K. (2008) used to modify Gauss-Newton method to compute the unknown parameter of the differential equation model. Schenkendorf, R., & Mangold, M. (2014) utilized flatness properties to identify the model parameter of the system. Gennemark, P., & Wedelin, D. (2009) reported method of identifying the model parameter using optimization methods. The model parameters are identified for minimizing the log-likelihood based error function.Garnier, H., Wang, L., & Young, P. C. (2008) discussed the identification of continuous time model using sample data and then issues associated with the identification. Ursem, R. K., & Vadstrup, P. (2003, December) solved a model identification problem using evolutionary algorithms. They focus on identifying the model parameter for an induction motor using an evolutionary algorithm and the authors concluded that the differential evolution algorithm converges faster than other algorithms for identifying model parameter.Brincker, R., Zhang, L., & Andersen, P. (2001) proposed frequency based model identification, where the signals decomposed into spectral density function matrix for identifying the parameter. Zhou, S., Cao, J., & Chen, Y. (2013) proposed a GA based time domain identification procedure for fractional order system. The authors used different excitation signal such as sawtooth wave, square wave, step signal and Pseudo Random Binary Signal (PRBS) for generating experimental data for identification. Alfi, A., & Modares, H. (2011) proposed method to find the optimal model parameter using adaptive particle swarm optimization algorithm. Yu K et.al (2017) presented a method of identifying the model parameter by minimizing the RMSE using an improved JAYA algorithm.

In this paper, the mathematical model for TIFCTLP is developed and then the model parameters are identified from the experimental data using BA. The experimental data are generated using various step input and then BA is applied to find the model parameter for minimum value of RMSE.

2. Two interacting conical frustum tank process Description

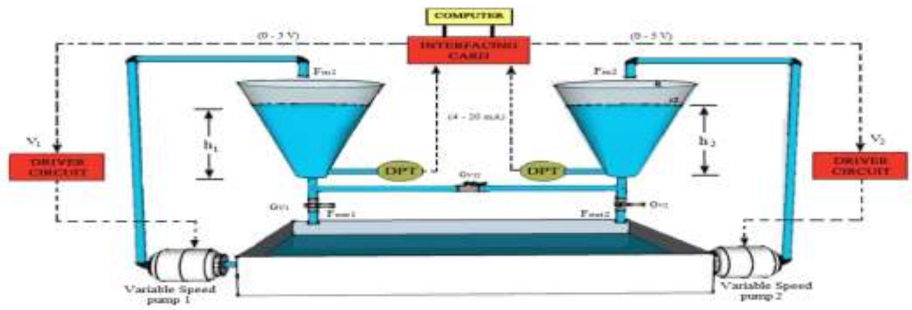

Figure 1. Schematic Diagram of Two Interacting conical frustum Tank Level Process

The proposed system consists of two interacting conical frustum tanks connected by interacting pipe. The heights of tanks are 50cm and top and bottom radius of conical tanks are 40cm and 14cm. The gate values Gv_1, Gv_2 and interaction valve $Gv_{12}$are partially opened and kept fixed. The interaction effect of process can be changed by the hand value Gv_{12}. The two tanks getting inflow of water from variable speed pumps. The manipulated inputs of the system are the voltage applied to the pumps. The range of input voltage is 0 to 5V, which is directly proportional to the rate of change of inflow. The differential pressure transmitter used for measuring the level in terms of milliamps. The non-linear equation describing the open loop dynamics of the two interacting conical tank system shown in figure 1 is derived using the conservation of mass and Bernoulli's principle as follows,

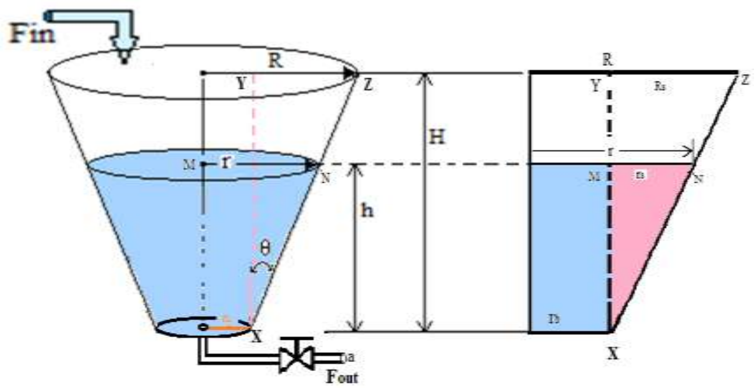

Figure 2. Single conical frustum tank

Rate of accumulation = Rate of inflow – Rate of Outflow

$$\frac{dVol}{dt} = Fin - Fout \tag{1}$$

The volume of water is product of area of the water in the tank and level of water. The Area of tank is nonlinear with respect to height of the tank.

The Volume of cone frustum tank Vol is,

$$vol = \frac{\pi}{3}\left(r_{in}^2 + r^2 + r_{in} r\right) h \tag{2}$$

By considering XYZ, XMN triangle,

$$\frac{NM}{XN} = \frac{YZ}{XY}$$

Incremental radius of liquid level due to slope surface.

$$r_s = \frac{R_s}{H} h = \frac{(R - r_{in})}{H} h \tag{3}$$

The top radius of liquid level, $r = r_{in} + r_s$;

$$r = r_{in} + \frac{(R - r_{in})}{H} h$$

$$Vol = \frac{\pi}{3}\left[3r_{in}^2 h + 3r_{in}\left(\frac{R - r_{in}}{H}\right)h^2 + \left(\frac{R - r_{in}}{H}\right)^2 h^3\right] \tag{4}$$

The equation (1) become,

$$\frac{dh}{dt} = \frac{F_{in} - \beta a\sqrt{2gh}}{\frac{\pi}{3}\left[3r_{in}^2 + 6r_{in}\left(\frac{R - r_{in}}{H}\right)h + 3\left(\frac{R - r_{in}}{H}\right)^2 h^2\right]} \tag{5}$$

Mass balance equation for Tank 1, 2

$$\frac{dh_1}{dt} = \frac{k_{pp1}V_1 - \beta_1 a_1\sqrt{2gh_1} - sign(h_1 - h_2)\beta_{12} a_{12}\sqrt{2g\,|h_1 - h_2|}}{\frac{\pi}{3}\left[3r_{in1}^2 + 6r_{in1}\left(\frac{R_1 - r_{in1}}{H_1}\right)h_1 + 3\left(\frac{R_1 - r_{in1}}{H_1}\right)^2 h_1^{\,2}\right]} \tag{6}$$

$$\frac{dh_2}{dt} = \frac{k_{pp2}V_2 + sign(h_1 - h_2)\beta_{12} a_{12}\sqrt{2g\,|h_1 - h_2|} - \beta_2 a_2\sqrt{2gh_2}}{\frac{\pi}{3}\left[3r_{in2}^2 + 6r_{in2}\left(\frac{R_2 - r_{in2}}{H_2}\right)h_2 + 3\left(\frac{R_2 - r_{in2}}{H_2}\right)^2 h_2^{\,2}\right]} \tag{7}$$

3. Identification of model parameter from experimental data

Figure 3 TICFTLP experimental setup

The different value of input voltage is applied to the pump and flow rate (LPH) at steady state is noted down. The flow rate LPH unit output is converted into cm^3/sec unit. i.e.(1cm^3/sec= 3.6*LPH). The pump characteristics are plotted using input V (v) and output flow rate (cm^3/sec).

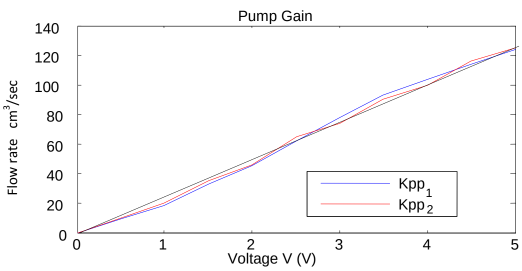

Figure 4 Input output characteristics of pump

The pump gain is determined from the characteristic plot, the slope of Pump gains are Kpp1= 25 cm^3/sec, Kpp2= 25 cm^3/sec. (1 V * pump gain= 25cm^3/sec=90 Liter per hour). The nominal values of the parameters and variables are tabulated in table 1.

Table I

Nominal values of the parameters used

Parameter	Description	Value
R	Top Radius of conical tank	20 cm
H	Maximum height of Tank1 , Tank2	50 cm
Kpp_1 , Kpp_2	Pump gain ($cm^3/V.\sec$)	25 cm^3/sec
F_{in}	Maximum Inflow to Tank1and Tank2	125 $cm^3/\sec$
a_1, a_{12}, a_2	Cross section Area of pipe	1.22 cm^2
r_{in}	Bottom of conical frustum tank radius	12cm

3.1. Experimental Data Generation using repeated step input

In practice, measurement data affected by noise, which leads to uncertainty of parameter estimation. The Measurement data with small variances can be used for identification of model, which may result in a good model. But the data with noises and variation results in non-meaningful model. Hence, the informative measurement data after eliminating noise can be used for identification of model parameter. The unknown parameters of the ratio of valve opening for three gate valves are identified by the minimum value of Root Mean Square Error (RMSE) using BA.

4. Bat Algorithm

The BAT search algorithm is based on the echolocation behavior of micro bats in locating their foods. Some assumptions are made to design BOA based algorithm, the first assumption is each bat has the ability to differentiate the prey and barrier. The second assumption is that all the bat fly randomly with fixed frequency, varying wavelength to adjust the pulse emission rate. The bat movements are updated based on the velocity increment. The β is a random number from a uniform distribution from 0 to 1. The frequency f_i and velocity v_i of i^{th} bat is updated using the following equation,

The frequency of i^{th} bat,

$$\mathbf{f}_i = \mathbf{f}_{min} + (\mathbf{f}_{max} - \mathbf{f}_{min})\,\delta \tag{8}$$

The velocity of i^{th} bat ,

$$\mathbf{v}_i^t = \mathbf{v}_i^{t-1} + (\mathbf{x}_i^t - \mathbf{x}_{best}^t)\,\mathbf{f}_i \tag{9}$$

Where x_{best}^t is the global best location.

The bat new position,

$$\mathbf{x}_i^t = \mathbf{x}_i^{t-1} + \mathbf{v}_i^t \tag{10}$$

The best position is found by random walk,

$$x_{new} = x_{old} + \varepsilon A^t \tag{11}$$

Where 'ε' is a random number from -1 to 1. A^T is an average loudness of all bats. The loudness variation directly proportional to the closeness of prey position, when the bat moves towards to the prey, then it reduces its loudness and increases the pulse emission rate. That is given as,

$$A_i^{t+1} = \mu\, A_i^t \,, \qquad r_i^{t+1} = r_i^0 \left[1 - \exp(-\gamma t)\right] \tag{12}$$

Where μ is a constant in the range of [0, 1] and γ is a positive constant. when time reaches infinite the r_i^t equal to r_i^o.

For any case, $0 < \mu < 1$ and $\gamma > 0$

$$A_i^t \to 0, \qquad r_i^t \to r_i^0, as \quad t \to \infty \tag{13}$$

For the easy implementation of BOA, the standard recommended μ, γ values are chosen as 0.9.

4.1 Identification Of Process Parameter Using BA

System identification is the method of developing a model and estimating the model parameters from the system data. In this work, the BA is applied to identify the nonlinear system parameter for minimizing the Root Mean Square Error (RMSE). The Model parameters are identified using the experimental data. The identified model simulation response and experimental output response are compared and validated. The identified model is accurate with minimum values of RMSE, MSE and ISE.

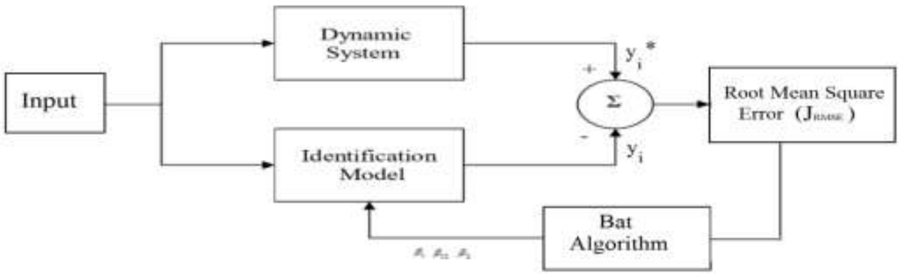

Figure 5 Parameter identification using BA

Root Mean Squared Error

$$J_{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(y_i^* - y_i)^2} \qquad (14)$$

Minimize JRMSE subject to

$$\beta_1^{\min} \le \beta_1 \le \beta_1^{\max};$$
$$\beta_{12}^{\min} \le \beta_{12} \le \beta_{12}^{\max};$$
$$\beta_2^{\min} \le \beta_2 \le \beta_2^{\max}$$

The Identification of model parameter is obtained using BA. The BA parameters such as population and iteration numbers are fixed for identification. The parameters $\beta_1, \beta_{12}, \beta_2$ are obtained for minimum RMSE of 0.043. The identified parameters are tabulated in table 2.

Table II

The parameters identified

Unknown Parameter	Description	The range of guess Value
β_1	Valve coefficient of MV1	0.35
β_{12}	Valve coefficient of Mv12	0.93
β_2	Valve coefficient of Mv2	0.31

5. Time domain Response

The step point is applied to real time model and an identified model for validating the identified model. The comparison results show that the proposed identified model is a perfect replica of TICFLP. The RMSE values also very less for the identified model. The simulation response and experimental output are validated for the input shown in the figure 4.

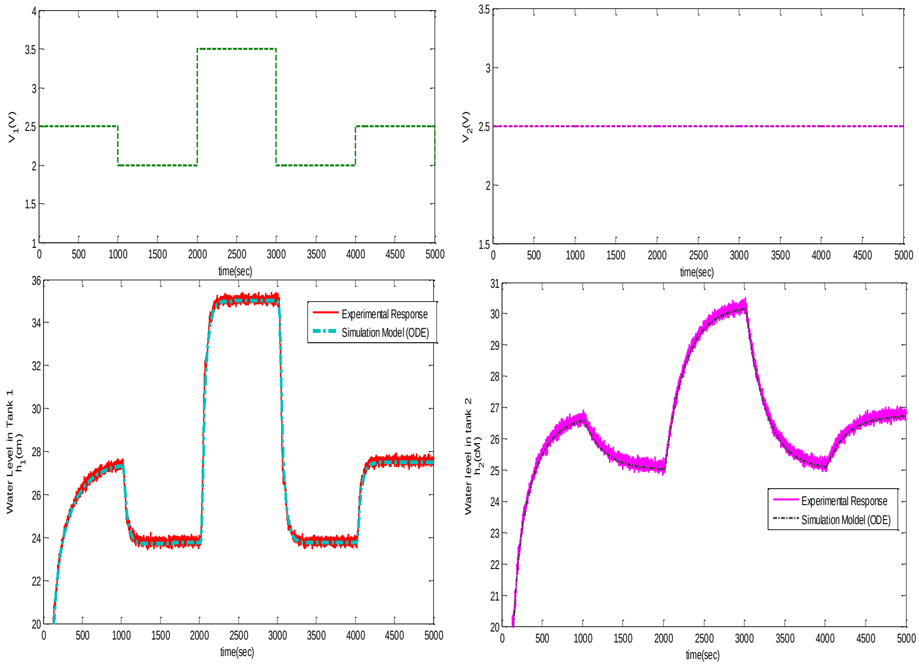

Figure 6 The comparison of simulation model and experimental response.

It is clearly seen from the figure 6 is that, the identified model has accurately matched to the experimental TIFCTL process. The transient response and steady state response of the experimental setup and identified ODE are perfectly matching with small distraction. Hence, the simulation model can be used for various analysis and controller design.

6. Conclusion

This paper presents an optimization method for identifying second order TICFLP using PRBS response. The BA is applied to identify the model parameters by matching the open loop response of the model. The ODE parameter identification problem is converted into optimization problem and the parameters are identified for a minimum value of RMSE. The grey box model is developed by combining the mathematical model with experimental data. The experimental model and simulation models are validated for step variation in the input signal. On analysis of the RMSE and MSE between the estimated model and experimental model, it is evident that the values are minimum specifying that the identified model closely resembles the original model.

Reference

[1]. Schittkowski, K. (2008). Parameter identification and model verification in systems of partial differential equations applied to transdermal drug delivery. *Mathematics and Computers in Simulation*, *79*(3), 521-538.

[2]. Schenkendorf, R., & Mangold, M. (2014). Parameter identification for ordinary and delay differential equations by using flat inputs. *Theoretical Foundations of Chemical Engineering*, *48*(5), 594-607.

[3]. Ursem, R. K., & Vadstrup, P. (2003, December). Parameter identification of induction motors using differential evolution. In *Evolutionary Computation, 2003. CEC'03. The 2003 Congress on* (Vol. 2, pp. 790-796). IEEE.

[4]. Gennemark, P., & Wedelin, D. (2009). Benchmarks for identification of ordinary differential equations from time series data. *Bioinformatics*, *25*(6), 780-786.

[5]. Garnier, H., Wang, L., & Young, P. C. (2008). Direct identification of continuous-time models from sampled data: Issues, basic solutions and relevance. In *Identification of continuous-time models from sampled data* (pp. 1-29). Springer London.

[6]. Ljung, L. (1998). System identification. In *Signal analysis and prediction* (pp. 163-173). Birkhäuser Boston.

[7]. Brincker, R., Zhang, L., & Andersen, P. (2001). Modal identification of output-only systems using frequency domain decomposition. *Smart materials and structures*, *10*(3), 441.

[8]. Zhou, S., Cao, J., & Chen, Y. (2013). Genetic algorithm-based identification of fractional-order systems. *Entropy*, *15*(5), 1624-1642.

[9]. Alfi, A., & Modares, H. (2011). System identification and control using adaptive particle swarm optimization. *Applied Mathematical Modelling*, *35*(3), 1210-1221.

[10]. Wibowo, T. C. S., Saad, N., & Karsiti, M. N. (2009, June). System identification of an interacting series process for real-time model predictive control. In *American Control Conference, 2009. ACC'09.* (pp. 4384-4389). IEEE.

[11]. Yu, K., Liang, J. J., Qu, B. Y., Chen, X., & Wang, H. (2017). Parameters identification of photovoltaic models using an improved JAYA optimization algorithm. *Energy Conversion and Management*, *150*, 742-753.

[12]. Yang, X. S. (2011). Bat algorithm for multi-objective optimisation. *International Journal of Bio-Inspired Computation*, *3*(5), 267-274.

[13]. Chen, L., Li, J., & Ding, R. (2011). Identification for the second-order systems based on the step response. *Mathematical and computer modelling*, *53*(5), 1074-1083.