

## Signed Edge Domination on Rooted Product Graph

C. Shobha Rani

Department of Mathematics,  
Madanapalle Institute of Technology & Science, Madanapalle-517325, India

\*E-mail: [charapallishobha@gmail.com](mailto:charapallishobha@gmail.com)

S. Jeelani Begum

Department of Mathematics,  
Madanapalle Institute of Technology & Science, Madanapalle-517325, India

E-mail: [sjbmaths@gmail.com](mailto:sjbmaths@gmail.com)

G. S. S. Raju

Department of Mathematics,  
JNTUA College of Engineering,

Pulivendula- 516390, India

E-mail: [rajugss@yahoo.com](mailto:rajugss@yahoo.com)

**Abstract**— Let  $G$  be a rooted product graph of path with a cycle graph with the vertex set  $V$  and the edge set  $E$ . Here  $P_n$  be a Path graph with  $n$  vertices and  $C_m (m \geq 3)$  be a cycle with a sequence of  $n$  rooted graphs  $C_{m1}, C_{m2}, C_{m3}, \dots, C_{mn}$ . Then by  $P_n(C_m)$  we denote the graph obtained by identifying the root of  $C_{mi}$  with the  $i^{\text{th}}$  vertex of  $P_n$ . We call  $P_n(C_m)$  the rooted product of  $P_n$  by  $C_m$  and it is denoted by  $P_n \circ C_m$ . Every  $i^{\text{th}}$  vertex of  $P_n$  is merging with any one vertex in every  $i^{\text{th}}$  copy of  $C_m$ . So in  $G = P_n \circ C_m$ ,  $P_n$  contains  $n$  vertices and  $C_m$  contains  $(m-1)$  vertices in each copy of  $C_m$ . In this paper we discuss some results on rooted product graph of path with a cycle graph.

**Keywords**- Rooted product graph, signed dominating functions, signed domination number.

### I. INTRODUCTION

Graph theory is an important subject in mathematics. Applications in many fields like coding theory, Logical Algebra, Engineering communications and Computer networking. The rooted product graphs are used in internet systems for connecting internet to one system to other systems.

Mostly Product of graphs used in discrete mathematics. In 1978, Godsil and McKay [3] introduced a new product on two graphs  $G_1$  and  $G_2$ , called rooted product denoted by  $G_1 \circ G_2$ . In 1977, Mitchell and Hedetniemi [7] have studied about "Edge domination in trees". In 2001, Xu [2] have studied about "On signed edge domination numbers of graphs". Further we studied about signed edge domination in [1, 4, 5, 6]. Here we can find out signed edge domination related parameters on rooted product graph.

### II. RESULTS ON SIGNED EDGE DOMINATION

**Theorem 2.1:** If  $m$  is divisible by 3 then the signed edge domination number of

$$G = P_n \circ C_m \text{ is } \gamma_s(G) = n \left[ m - \frac{2m}{3} + 1 \right] - 1.$$

**Proof:** Let  $G = P_n \circ C_m$  be a rooted product graph and  $m=3k$ . Where  $k$  is a natural number set.

We define a signed edge dominating function  $f : E \rightarrow [0,1]$  as follows:

$$f(e) = \begin{cases} -1, & \text{for } \frac{m}{3} \text{ edges in each copy of } C_m \text{ in } G, \\ +1, & \text{otherwise.} \end{cases}$$

Then by the definition of the function.

$$f(e_1) = f(e_2) = \dots = f(e_{n-1}) = 1,$$

$$f(h_{ij}) = -1, \text{ if } j \equiv 1 \pmod{3} \text{ in each copy } C_m \text{ of } G,$$

$$f(h_{ij}) = 1, \text{ otherwise.}$$

By the function definition, the values -1 is assigned to  $\frac{m}{3}$  edges in each copy of  $C_m$  and +1 is assigned to remaining vertices in  $G$ .

Case 1: If  $e_i \in P_n$ , where  $i = 1, 2, \dots, (n-1)$ .

If  $adj(e_i) = 5$  then  $\sum_{e \in N[e_i]} f(e) = 1 + 1 + 1 + 1 + (-1) + (-1) = 2$ .

If  $adj(e_i) = 6$  then  $\sum_{e \in N[e_i]} f(e) = 1 + 1 + 1 + 1 + (-1) + (-1) + 1 = 3$ .

Case 2: If  $h_{ij} \in C_m; i = 1, 2, \dots, n; j = 1, 2, 3, \dots, m$ .

Subcase 1: Suppose  $adj(h_{ij}) = 2, N[h_{ij}], j = 1, 2, 3, \dots, m$  there are no edges of  $P_n$  and two edges of  $C_m$  and there are two edges which are drawn from the vertices  $u_{ij}$  and  $u_{i(j+1)}$  of  $C_m$ .

Therefore  $\sum_{e \in N[h_{ij}]} f(e) = 1 + (-1) + 1 = 1$ .

Subcase 2: Suppose  $adj(h_{ij}) = 3, N[h_{ij}], j = 1, 2, 3, \dots, m$  there are two edges of  $C_m$ , one edge of  $P_n$  and there is an edge which are drawn from the vertices

$$u_{ij}, i = 1, 2, \dots, n; j = 1 \text{ or } (m-1) \text{ and } v_i, i = 1 \text{ or } n.$$

Therefore  $\sum_{e \in N[h_{ij}]} f(e) = (-1) + 1 + 1 = 2$ .

Subcase 3: Suppose  $adj(h_{ij}) = 4, N[h_{ij}], j = 1, 2, 3, \dots, m$  there are two edges of  $C_m$ , two edges of  $P_n$  and there is an edge which are drawn from the vertices

$$u_{ij}, i = 1, 2, \dots, n; j = 1 \text{ or } (m-1) \text{ and } v_i, i = 1 \text{ or } n.$$

Therefore  $\sum_{e \in N[h_{ij}]} f(e) = [1 + (-1) + 1] + 1 + 1 = 3$ .

From the above possible cases, we get  $\sum_{e \in E(G)} f(e) \geq 1$ .

This implies  $f$  is a signed edge dominating function.

Now the minimality check for of  $f$ . Define another function  $g : E \rightarrow \{-1, 1\}$  by

$$g(e) = \begin{cases} -1, \text{ for } \frac{m}{3} \text{ edges in each copy of } C_m \text{ in } G, \\ -1, \text{ if } e = e_k \in P_n \text{ for some } k, \\ +1, \text{ otherwise.} \end{cases}$$

Since strict equality not holds at an edge  $e_i \in P_n$ , it follows that  $g < f$ .

Case 1: If  $e_i \in P_n$ , where  $i = 1, 2, \dots, (n-1)$ .

Sub case 1: Let  $e_k \in N[e_i]$ .

If  $adj(e_i) = 5$  then  $\sum_{e \in N[e_i]} g(e) = 1 + (-1) + \underbrace{1 + (-1)}_{2\text{-times}} = 0$ .

If  $adj(e_i) = 6$  then  $\sum_{e \in N[e_i]} g(e) = 1 + (-1) + 1 + \underbrace{(-1) + 1}_{2\text{-times}} = 1$ .

Sub case 2: Let  $e_k \notin N[e_i]$ .

If  $adj(e_i) = 5$  then  $\sum_{e \in N[e_i]} g(e) = 1 + 1 + \underbrace{1 + (-1)}_{2\text{-times}} = 2$ .

If  $adj(e_i) = 6$  then  $\sum_{e \in N[e_i]} g(e) = 1 + 1 + \underbrace{1 + (-1)}_{2\text{-times}} + 1 = 3$ .

Case 2: If  $h_{ij} \in C_m; i = 1, 2, \dots, n; j = 1, 2, 3, \dots, m$ .

Subcase 1: Suppose  $adj(h_{ij}) = 2, N[h_{ij}], j = 1, 2, 3, \dots, m$  there are no edges of  $P_n$  and two edges of  $C_m$  and there are two edges which are drawn from the vertices  $u_{ij}$  and  $u_{i(j+1)}$  of  $C_m$ .

Therefore  $\sum_{e \in N[h_{ij}]} g(e) = 1 + (-1) + 1 = 1$ .

Subcase 2: Suppose  $adj(h_{ij}) = 3, N[h_{ij}], j = 1, 2, 3, \dots, m$  there are two edges of  $C_m$ , one edge of  $P_n$  and there is an edge which are drawn from the vertices  $u_{ij}, i = 1, 2, \dots, n; j = 1$  or  $(m-1)$  and  $v_i, i = 1$  or  $n$ .

Let  $e_k \in N[h_{ij}]$  then  $\sum_{e \in N[h_{ij}]} g(e) = (-1) + 1 + 1 + (-1) = 0$ .

Let  $e_k \notin N[h_{ij}]$  then  $\sum_{e \in N[h_{ij}]} g(e) = (-1) + 1 + 1 + 1 = 2$ .

Subcase 3: Suppose  $adj(h_{ij}) = 4, N[h_{ij}], j = 1, 2, 3, \dots, m$  there are two edges of  $C_m$ , two edges of  $P_n$  and there is an edge which are drawn from the vertices  $u_{ij}, i = 1, 2, \dots, n; j = 1$  or  $(m-1)$  and  $v_i, i = 1$  or  $n$ .

Therefore  $\sum_{e \in N[h_{ij}]} g(e) = \begin{cases} 1 + (-1) + 1 + (-1) + 1 = 1, & \text{if } e_k \in N[h_{ij}] \\ 1 + (-1) + 1 + 1 + 1 = 3, & \text{if } e_k \notin N[h_{ij}] \end{cases}$ .

From the above possible cases, we get

$$\sum_{e \in E(G)} g(e) < 1, \text{ for some } e \in E.$$

This implies  $g$  is not a signed edge dominating function.

Hence  $f$  is a minimal signed edge dominating function.

Now signed edge domination number is

$$\sum_{e \in E(G)} f(e) = \underbrace{\left(\frac{m}{3}\right)(-1) + \left(m - \frac{m}{3}\right)(+1)}_{n\text{-times}} + (n-1) = n \left[ m - \frac{2m}{3} + 1 \right] - 1.$$

**Theorem 2.2:** If  $m$  is not divisible by 3, that is  $m = 3k + 1$  then the signed edge domination number of  $G = P_n \circ C_m$  is

$$\gamma_s(G) = n \left[ m - 2 \left\lfloor \frac{m}{3} \right\rfloor + 1 \right] - 1.$$

*Proof:* Let  $G = P_n \circ C_m$  be a rooted product graph and  $m = 3k + 1$ . Where  $k$  is a natural number set.

We define a signed edge dominating function  $f : E \rightarrow [0, 1]$  as follows:

$$f(e) = \begin{cases} -1, & \text{for } \frac{m}{3} \text{ edges in each copy of } C_m \text{ in } G, \\ +1, & \text{otherwise.} \end{cases}$$

Then by the definition of the function.

$$f(e_1) = f(e_2) = \dots = f(e_{n-1}) = 1,$$

$$f(h_{ij}) = -1, \text{ if } j \equiv 0 \pmod{3} \text{ in each copy } C_m \text{ of } G,$$

$$f(h_{ij}) = 1, \text{ otherwise.}$$

By the function definition, the values  $-1$  is assigned to  $\frac{m}{3}$  edges in each copy of  $C_m$  and  $+1$  is assigned to remaining

vertices in  $G$ .

Case 1: If  $e_i \in P_n$ , where  $i = 1, 2, \dots, (n-1)$ .

If  $adj(e_i) = 5$  then  $\sum_{e \in N[e_i]} f(e) = 1 + 1 + 1 + 1 + 1 + 1 = 6$ .

If  $adj(e_i) = 6$  then  $\sum_{e \in N[e_i]} f(e) = 1 + 1 + 1 + 1 + 1 + 1 + 1 = 7$ .

Case 2: If  $h_{ij} \in C_m; i = 1, 2, \dots, n; j = 1, 2, 3, \dots, m$ .

Subcase 1: Suppose  $adj(h_{ij}) = 2, N[h_{ij}], j = 1, 2, 3, \dots, m$  there are no edges of  $P_n$  and two edges of  $C_m$  and there are two edges which are drawn from the vertices  $u_{ij}$  and  $u_{i(j+1)}$  of  $C_m$ .

Therefore  $\sum_{e \in N[h_{ij}]} f(e) = 1 + (-1) + 1 = 1$ .

Subcase 2: Suppose  $adj(h_{ij}) = 3, N[h_{ij}], j = 1, 2, 3, \dots, m$  there are two edges of  $C_m$ , one edge of  $P_n$  and there is an edge which are drawn from the vertices

$u_{ij}, i = 1, 2, \dots, n; j = 1$  or  $(m-1)$  and  $v_i, i = 1$  or  $n$ .

Therefore  $\sum_{e \in N[h_{ij}]} f(e) = \begin{cases} (-1) + 1 + 1 + 1 = 2, & \text{if } -1 \in N[h_{ij}] \\ 1 + 1 + 1 + 1 = 4, & \text{if } -1 \notin N[h_{ij}] \end{cases}$ .

Subcase 3: Suppose  $adj(h_{ij}) = 4, N[h_{ij}], j = 1, 2, 3, \dots, m$  there are two edges of  $C_m$ , two edges of  $P_n$  and there is an edge which are drawn from the vertices

$u_{ij}, i = 1, 2, \dots, n; j = 1$  or  $(m-1)$  and  $v_i, i = 1$  or  $n$ .

Therefore  $\sum_{e \in N[h_{ij}]} f(e) = \begin{cases} (-1) + 1 + 1 + 1 + 1 = 3, & \text{if } -1 \in N[h_{ij}] \\ 1 + 1 + 1 + 1 + 1 = 5, & \text{if } -1 \notin N[h_{ij}] \end{cases}$ .

From the above possible cases, we get  $\sum_{e \in E(G)} f(e) \geq 1$ .

This implies  $f$  is a signed edge dominating function.

Now the minimality check for of  $f$ . Define another function  $g : E \rightarrow \{-1, 1\}$  by

$$g(e) = \begin{cases} -1, & \text{for } \frac{m}{3} \text{ edges in each copy of } C_m \text{ in } G, \\ -1, & \text{if } e = e_k \in P_n \text{ for some } k, \\ +1, & \text{otherwise.} \end{cases}$$

Since strict equality not holds at an edge  $e_i \in P_n$ , it follows that  $g < f$ .

Case 1: If  $e_i \in P_n$ , where  $i = 1, 2, \dots, (n-1)$ .

Sub case 1: Let  $e_k \in N[e_i]$ .

If  $adj(e_i) = 5$  then  $\sum_{e \in N[e_i]} g(e) = 1 + (-1) + \underbrace{1+1}_{2\text{-times}} = 4$ .

If  $adj(e_i) = 6$  then  $\sum_{e \in N[e_i]} g(e) = 1 + (-1) + 1 + \underbrace{1+1}_{2\text{-times}} = 5$ .

Sub case 2: Let  $e_k \notin N[e_i]$ .

If  $adj(e_i) = 5$  then  $\sum_{e \in N[e_i]} g(e) = 1 + 1 + \underbrace{1+1}_{2\text{-times}} = 6$ .

If  $adj(e_i) = 6$  then  $\sum_{e \in N[e_i]} g(e) = 1 + 1 + 1 + \frac{1+1}{2\text{-times}} = 7$ .

Case 2: If  $h_{ij} \in C_m; i = 1, 2, \dots, n; j = 1, 2, 3, \dots, m$ .

Subcase 1: Suppose  $adj(h_{ij}) = 2, N[h_{ij}], j = 1, 2, 3, \dots, m$  there are no edges of  $P_n$  and two edges of  $C_m$  and there are two edges which are drawn from the vertices  $u_{ij}$  and  $u_{i(j+1)}$  of  $C_m$ .

Therefore  $\sum_{e \in N[h_{ij}]} g(e) = 1 + (-1) + 1 = 1$ .

Subcase 2: Suppose  $adj(h_{ij}) = 3, N[h_{ij}], j = 1, 2, 3, \dots, m$  there are two edges of  $C_m$ , one edge of  $P_n$  and there is an edge which are drawn from the vertices  $u_{ij}, i = 1, 2, \dots, n; j = 1$  or  $(m-1)$  and  $v_i, i = 1$  or  $n$ .

Let  $e_k \in N[h_{ij}] \Rightarrow \sum_{e \in N[h_{ij}]} g(e) = \begin{cases} (-1) + 1 + 1 + (-1) = 0, & \text{if } -1 \in N[h_{ij}] \\ 1 + 1 + 1 + (-1) = 2, & \text{if } -1 \notin N[h_{ij}] \end{cases}$ .

Let  $e_k \notin N[h_{ij}] \Rightarrow \sum_{e \in N[h_{ij}]} g(e) = \begin{cases} (-1) + 1 + 1 + 1 = 2, & \text{if } -1 \in N[h_{ij}] \\ 1 + 1 + 1 + 1 = 4, & \text{if } -1 \notin N[h_{ij}] \end{cases}$ .

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Let  $e_k \in N[h_{ij}] \Rightarrow \sum_{e \in N[h_{ij}]} g(e) = \begin{cases} (-1) + 1 + 1 + 1 + (-1) = 1, & \text{if } -1 \in N[h_{ij}] \\ 1 + 1 + 1 + 1 + (-1) = 3, & \text{if } -1 \notin N[h_{ij}] \end{cases}$ .

Let  $e_k \notin N[h_{ij}] \Rightarrow \sum_{e \in N[h_{ij}]} g(e) = \begin{cases} (-1) + 1 + 1 + 1 + 1 = 3, & \text{if } -1 \in N[h_{ij}] \\ 1 + 1 + 1 + 1 + 1 = 5, & \text{if } -1 \notin N[h_{ij}] \end{cases}$ .

From the above possible cases, we get

$$\sum_{e \in E(G)} g(e) < 1, \text{ for some } e \in E.$$

This implies  $g$  is not a signed edge dominating function.

Hence  $f$  is a minimal signed edge dominating function, if  $m = 3k + 1$ .

Now signed edge domination number is

$$\sum_{e \in E(G)} f(e) = \underbrace{\left( \left\lfloor \frac{m}{3} \right\rfloor (-1) + \left( m - \left\lfloor \frac{m}{3} \right\rfloor \right) (+1) \right)}_{n\text{-times}} + (n-1) = n \left[ m - 2 \left\lfloor \frac{m}{3} \right\rfloor + 1 \right] - 1.$$

**Theorem 2.3:** If  $m$  is not divisible by 3, that is  $m = 3k + 2$  then the function  $f : E \rightarrow [0, 1]$  is defined by

$$f(e) = \begin{cases} -1, & \text{for } \frac{m}{3} \text{ edges in each copy of } C_m \text{ in } G, \\ +1, & \text{otherwise.} \end{cases}$$

It becomes not a minimal signed edge dominating function of  $G = P_n \circ C_m$ .

*Proof:* Let  $G = P_n \circ C_m$  be a rooted product graph and  $m = 3k + 2$ . Where  $k$  is a natural number set.

We define a signed edge dominating function as in the hypothesis.

Then by the definition of the function.

$$f(e_1) = f(e_2) = \dots = f(e_{n-1}) = 1,$$

$$f(h_{ij}) = -1, \text{ if } j \equiv 0 \pmod{3} \text{ in each copy } C_m \text{ of } G,$$

$$f(h_{ij}) = 1, \text{ otherwise.}$$

By the function definition, the values  $-1$  is assigned to  $\frac{m}{3}$  edges in each copy of  $C_m$  and  $+1$  is assigned to remaining vertices in  $G$ .

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If  $adj(e_i) = 5$  then  $\sum_{e \in N[e_i]} f(e) = 1 + 1 + 1 + 1 + 1 + 1 = 6$ .

If  $adj(e_i) = 6$  then  $\sum_{e \in N[e_i]} f(e) = 1 + 1 + 1 + 1 + 1 + 1 + 1 = 7$ .

Case 2: If  $h_{ij} \in C_m; i = 1, 2, \dots, n; j = 1, 2, 3, \dots, m$ .

Subcase 1: Suppose  $adj(h_{ij}) = 2, N[h_{ij}], j = 1, 2, 3, \dots, m$  there are no edges of  $P_n$  and two edges of  $C_m$  and there are two edges which are drawn from the vertices  $u_{ij}$  and  $u_{i(j+1)}$  of  $C_m$ .

Therefore  $\sum_{e \in N[h_{ij}]} f(e) = 1 + (-1) + 1 = 1$ .

Subcase 2: Suppose  $adj(h_{ij}) = 3, N[h_{ij}], j = 1, 2, 3, \dots, m$  there are two edges of  $C_m$ , one edge of  $P_n$  and there is an edge which are drawn from the vertices

$u_{ij}, i = 1, 2, \dots, n; j = 1$  or  $(m-1)$  and  $v_i, i = 1$  or  $n$ .

Therefore  $\sum_{e \in N[h_{ij}]} f(e) = 1 + 1 + 1 + 1 = 4$ .

Subcase 3: Suppose  $adj(h_{ij}) = 4, N[h_{ij}], j = 1, 2, 3, \dots, m$  there are two edges of  $C_m$ , two edges of  $P_n$  and there is an edge which are drawn from the vertices

$u_{ij}, i = 1, 2, \dots, n; j = 1$  or  $(m-1)$  and  $v_i, i = 1$  or  $n$ .

Therefore  $\sum_{e \in N[h_{ij}]} f(e) = 1 + 1 + 1 + 1 + 1 = 5$ .

From the above possible cases, we get  $\sum_{e \in E(G)} f(e) \geq 1$ .

This implies  $f$  is a signed edge dominating function.

Now minimality check for of  $f$ . Define another function  $g : E \rightarrow \{-1, 1\}$  by

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Sub case 1: Let  $e_k \in N[e_i]$ .

If  $adj(e_i) = 5$  then  $\sum_{e \in N[e_i]} g(e) = 1 + (-1) + \underbrace{1+1}_{2\text{-times}} = 4$ .

If  $adj(e_i) = 6$  then  $\sum_{e \in N[e_i]} g(e) = 1 + (-1) + 1 + \underbrace{1+1}_{2\text{-times}} = 5$ .

Sub case 2: Let  $e_k \notin N[e_i]$ .

If  $adj(e_i) = 5$  then  $\sum_{e \in N[e_i]} g(e) = 1 + 1 + \underbrace{1+1}_{2\text{-times}} = 6$ .

If  $adj(e_i) = 6$  then  $\sum_{e \in N[e_i]} g(e) = 1 + 1 + 1 + \underbrace{1+1}_{2\text{-times}} = 7$ .

Case 2: If  $h_{ij} \in C_m; i = 1, 2, \dots, n; j = 1, 2, 3, \dots, m$ .

Subcase 1: Suppose  $adj(h_{ij}) = 2, N[h_{ij}], j=1, 2, 3, \dots, m$  there are no edges of  $P_n$  and two edges of  $C_m$  and there are two edges which are drawn from the vertices  $u_{ij}$  and  $u_{i(j+1)}$  of  $C_m$ .

Therefore  $\sum_{e \in N[h_{ij}]} g(e) = 1 + (-1) + 1 = 1$ .

Subcase 2: Suppose  $adj(h_{ij}) = 3, N[h_{ij}], j=1, 2, 3, \dots, m$  there are two edges of  $C_m$ , one edge of  $P_n$  and there is an edge which are drawn from the vertices  $u_{ij}, i=1, 2, \dots, n; j=1$  or  $(m-1)$  and  $v_i, i = 1$  or  $n$ .

Let  $e_k \in N[h_{ij}]$  then  $\sum_{e \in N[h_{ij}]} g(e) = 1 + 1 + 1 + (-1) = 2$ .

Let  $e_k \notin N[h_{ij}]$  then  $\sum_{e \in N[h_{ij}]} g(e) = 1 + 1 + 1 + 1 = 4$ .

Subcase 3: Suppose  $adj(h_{ij}) = 4, N[h_{ij}], j=1, 2, 3, \dots, m$  there are two edges of  $C_m$ , two edges of  $P_n$  and there is an edge which are drawn from the vertices  $u_{ij}, i=1, 2, \dots, n; j=1$  or  $(m-1)$  and  $v_i, i = 1$  or  $n$ .

Let  $e_k \in N[h_{ij}]$  then  $\sum_{e \in N[h_{ij}]} g(e) = 1 + 1 + 1 + 1 + (-1) = 3$ .

Let  $e_k \notin N[h_{ij}]$  then  $\sum_{e \in N[h_{ij}]} g(e) = 1 + 1 + 1 + 1 + 1 = 5$ .

From the above possible cases, we get  $\sum_{e \in E(G)} g(e) \geq 1$ .

This implies  $g$  is also a signed edge dominating function. Hence  $f$  is not a minimal signed edge dominating function, if  $m=3k+2$ .

REFERENCES

[1] B. Zelinka, "On signed edge domination numbers of trees", *Mathematica Bohemica*, Vol. 127(1), pp. 49-55, 2002.

[2] B. Xu, "On signed edge domination numbers of graphs", *Discrete Mathematics*, Vol. 239(1-3), pp. 179-189, 2001.

[3] C. D. Godsil & B. D. McKay, "A new graph product and its spectrum", *Bulletin of the Australian Mathematical Society*, Vol. 18(1), pp. 21-28, 1978.

[4] H. Karami, A. Khodkar and S. M. Sheikholeslami, "An improved upper bound for signed edge domination numbers in graphs", *Utilitas Math.*, Vol. 78, pp. 121-128, 2009.

[5] H. Karami, A. Khodkar and S. M. Sheikholeslami, "Signed edge domination numbers in trees", *Ars Combinatoria*, Vol. 93, pp.451-457, 2009.

[6] H. Xia, F. Wei, & J. Xu Chunlei, "Signed edge total domination numbers of two classes of graphs", *International Journal of Pure and Applied Mathematics*, Vol. 81(4), pp. 581-590, 2012.

[7] S. Mitchell and S. T. Hedetniemi, "Edge domination in trees", *Congr. Numer*, Vol. 19, pp. 489-509, 1977.

[8] RAJESH, M. "A SYSTEMATIC REVIEW OF CLOUD SECURITY CHALLENGES IN HIGHER EDUCATION." *The Online Journal of Distance Education and e - Learning* 5.4 (2017): 1.

[9] Rajesh, M., and J. M. Gnanasekar. "Protected Routing in Wireless Sensor Networks: A study on Aimed at Circulation." *Computer Engineering and Intelligent Systems* 6.8: 24-26.

[10] Rajesh, M., and J. M. Gnanasekar. "Congestion control in heterogeneous WANET using FRCC." *Journal of Chemical and Pharmaceutical Sciences* ISSN 974 (2015): 2115.

[11] Rajesh, M., and J. M. Gnanasekar. "Hop-by-hop Channel-Alert Routing to Congestion Control in Wireless Sensor Networks." *Control Theory and Informatics* 5.4 (2015): 1-11.

[12] Rajesh, M., and J. M. Gnanasekar. "Multiple-Client Information Administration via Forceful Database Prototype Design (FDPD)." *IJRESTS* 1.1 (2015): 1-6.

[13] Rajesh, M. "Control Plan transmit to Congestion Control for AdHoc Networks." *Universal Journal of Management & Information Technology (UJMIT)* 1 (2016): 8-11.

[14] Rajesh, M., and J. M. Gnanasekar. "Consistently neighbor detection for MANET." *Communication and Electronics Systems (ICCES), International Conference on.* IEEE, 2016.

