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Fuzzy Inventory Models With Decentralized Polices for a Single Vendor Two Buyer System Under Crisp and Fuzzy Decisions Variable

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Abstract

In today business transaction vendor usually offers their buyers a delay period in payment. The paper consider an inventory model with decentralized polices for single vendor two buyer system in fuzzy environment by employing two types of fuzzy numbers which are trapezoidal. we consider the fuzzy total cost under crisp order quantity in order to extend the traditional inventory model to the fuzzy environment. Two fuzzy models are developed .In the first model the inputs parameter are fuzzified while the decision variable are treated as crisp variables. In the second model not only the input parameters but also the decision variables are fuzzified .We use function principle as arithmetic operation of fuzzy total cost and apply the graded mean integration method to defuzzify the fuzzy total cost and then the optimal policy for each model is determined .The optimal policy for the second model is determined by using Kuhn-tucker conditions after the defuzzification of the fuzzy total cost .Numerical examples are also given to demonstrate the developed crisp and fuzzy models solved by using Matlab software.

AMS Subject Classification: 94D.

Key Words and Phrases: One-vendor two buyer inventory system,Permissible delay in payments ,Integrated policy,Decentralized policy,Trapezoidal number,Graded mean integration,Kuhn-tucker condition.

1 Introduction

The fuzzy set concept has been used to treat the classical inventory model recently. In the real world, the parameters and variables in inventory model may be almost uncertain datum. Fuzzy inventory model has been discussed recently. In 1987, Park [**?**] used fuzzy set concept to treat the inventory problem with fuzzy principle. In 1996, Chen et al., [**?**] introduced back ordered fuzzy inventory model under function principle and also discussed fuzzy inventory model for crisp ordered or for fuzzy ordered quantity with generalized trapezoidal fuzzy number. Chang [3] presented the membership function of the fuzzy total cost of inventory model and used the extension principle to obtain the EOQ. In this paper we introduce the fuzzy inventory models in which fuzzy parameters and fuzzy ordered quantity are all trapezoidal fuzzy numbers. In order to simplify the calculations of trapezoidal fuzzy number we use Chen's function principle instead of extension principle to calculate fuzzy total inventory cost of our proposed model. Function principle is proposed as the fuzzy arithmetical operations of fuzzy numbers in 1985. Also the principle is proven that it does not change the type of membership function under fuzzy arithmetical operations of fuzzy numbers. In the fuzzy sense it is reasonable to discuss the grade of each point of support set of fuzzy number for representing fuzzy numbers. Therefore Chen and Hsieh's [2] graded mean integration representation method adopted grade as an important degree of each point of support set of generalized fuzzy number. With this reason we use it to defuzzify the trapezoidal fuzzy total inventory cost. Furthermore, the Kuhn-Tucker method is used to solve inequality constraints in fuzzy single supplier multiple retailer inventory model for fuzzy ordering quantity.

Nomenclature

Subscripts

d_j $\rightarrow$ demand rate at buyer j, $j = 1, 2$
k_j $\rightarrow$ fixed replenishment cost per order at buyer j, $j = 1, 2$
h_j $\rightarrow$ inventory holding cost per unit and time unit at buyer j, $j = 1, 2$
p_0 $\rightarrow$ unit purchase cost of an item, $(p_0 < p_j, \forall\ j)$
p_j $\rightarrow$ unit selling price of an item
M $\rightarrow$ credit period offered by the vendor to the buyer
I_{ej} $\rightarrow$ annual interest earned per monetary unit that buyer j deposits in an account during $[0, M]$, $j = 1, 2$
I_{cj} $\rightarrow$ annual interest charged to be paid per monetary unit that buyer j need to finance to settle the account at instant M, $j = 1, 2$
t_j $\rightarrow$ replenishment interval at buyer j, $j = 1, 2$
C_j $\rightarrow$ total cost per unit time incurred by buyer j, $j = 1, 2$

Mathematical model

If $t_j \leq M$.

$$C_j(t_j) = \frac{h_j}{t_j} + \frac{h_j d_j t_j}{2} - I_{ej} p_j d_j \left(M - \frac{t_j}{2}\right)$$

$$\frac{\partial C_j(t_j)}{\partial t_j} = \frac{-k_j}{t_j^2} + \frac{h_j d_j}{2} + \frac{I_{ej} p_j d_j}{2} = 0$$
$$t_j = \sqrt{\frac{2k_j}{h_j d_j + I_{ej} p_j d_j}}$$
If $t_j > M$
$$C_j(t_j) = \frac{k_j}{t_j} + \frac{h_j d_j t_j}{2} - \frac{I_{ej} p_j d_j}{2t_j} + I_{cj} p_0 d_j M^2 \frac{(t_j - M)^2}{2t_j}$$
$$\frac{\partial C_j(t_j)}{\partial t_j} = \frac{-k_j}{t_j^2} + \frac{h_j d_j}{2} + \frac{I_{ej} p_j d_j M^2}{2t_j^2} + \frac{I_{cj} p_0 d_j}{2} - \frac{I_{cj} p_0 d_j M^2}{2t_j^2} = 0$$
$$t_j = \sqrt{\frac{2k_j + d_j M^2 [I_{cj} p_0 - I_{ej} p_j]}{d_j [h_j + I_{cj} p_p]}}$$

2 The kuhn-tucker condition

Taha (1997) discussed how to solve the optimum solution of nonlinear programming problem subject to inequality constraints by using the Kuhn-Tucker conditions. The development of the Kuhn-Tucker conditions is based on the Lagrangean method.Suppose that the problem is given by
Minimise $y = f(x)$
Subject to $g_i(x) \geq 0$, $i = 1, 2, \ldots m$.

The non negativity constraints $x \geq 0$, if any, are included in the m constraints.The inequality constraints may be converted into equations by using non-negative surplus variables. Let s_i^2 be the surplus quantity added to the ith constraint $g_i(x) \geq 0$. Let $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_m)$, $g(x) = (g_1(x), g_2(x), \ldots, g_m(x))$ and $s_2 = (s_1^2, s_2^2, \ldots, s_m^2)$.

Then the Lagrengean function given by $L(X, S, \lambda) = f(x) - \lambda[g(x) - S^2)]$. Given the constaints $g_i(x) \geq 0$
Taking the partial derivatives of L with respect to x, S and λ, we obtain

$$\frac{\partial L}{\partial X} = \nabla f(x) - \lambda \nabla g(x) = 0$$
$$\frac{\partial L}{\partial S_i} = 2\lambda_i S_i = 0, \ i = 1, 2, \ldots, m$$
$$\frac{\partial L}{\partial \lambda_i} = -g_i(x) + S_i^2 = 0, \ i = 1, 2, \ldots, m$$

From the second and third sets of equations it shows that $\lambda_i g_i(x) = 0$, $i = 1, 2, \ldots, m$

The Kuhn-Tucker conditions need x and λ to be a stastionary point of the minimization problem, which can be summarized as follows:

$$\begin{cases} \lambda \leq 0, & \\ \nabla f(x) - \lambda \Delta g(x) = 0, & \\ \lambda_i g_i(x) = 0, & i = 1, 2, \ldots, m. \\ g_i(x) \geq 0, & i = 1, 2, \ldots, m. \end{cases}$$

Fuzzy Modelling of the EOQ model

In this subsection we consider model in section is fully fuzzified ;i.e., by fuzzifying the input parameters $(k_j, h_j, d_j, I_{ej}, p_j, M)$ and the decision variables (t_j)

If $t_j \leq M$

$$\begin{aligned}
\tilde{C}_j(t_j) &= (C_1, C_2, C_3, C_4) \\
C_1 &= \frac{k_{j1}}{t_j} + \frac{h_{j1}d_{j1}t_j}{2} - I_{ej4}p_{j4}d_{j4}\left(M_4 - \frac{t_j}{2}\right) \\
C_2 &= \frac{k_{j2}}{t_j} + \frac{h_{j2}d_{j2}t_j}{2} - I_{ej3}p_{j3}d_{j3}\left(M_3 - \frac{t_j}{2}\right) \\
C_3 &= \frac{k_{j3}}{t_j} + \frac{h_{j3}d_{j3}t_j}{2} - I_{ej2}p_{j2}d_{j2}\left(M_2 - \frac{t_j}{2}\right) \\
C_4 &= \frac{k_{j4}}{t_j} + \frac{h_{j4}d_{j4}t_j}{2} - I_{ej1}p_{j1}d_{j1}\left(M_1 - \frac{t_j}{2}\right)
\end{aligned}$$

where C_1, C_2, C_3 and C_4 are components of trapezoidal fuzzy number .We defuzzify $\tilde{C}_j(t_j)$ by using graded mean integration representation

$$\begin{aligned}
P\tilde{C}_j(t_j) = \frac{1}{6}\Bigg\{ &\left[\frac{k_{j1}}{t_j} + \frac{h_{j1}d_{j1}t_j}{2} - I_{ej4}p_{j4}d_{j4}\left(M_4 - \frac{t_j}{2}\right)\right] \\
&+ 2\left[\frac{k_{j2}}{t_j} + \frac{h_{j2}d_{j2}t_j}{2} - I_{ej3}p_{j3}d_{j3}\left(M_3 - \frac{t_j}{2}\right)\right] \\
&+ 2\left[\frac{k_{j3}}{t_j} + \frac{h_{j3}d_{j3}t_j}{2} - I_{ej2}p_{j2}d_{j2}\left(M_2 - \frac{t_j}{2}\right)\right] \\
&+2\left[\frac{k_{j4}}{t_j} + \frac{h_{j4}d_{j4}t_j}{2} - I_{ej1}p_{j1}d_{j1}\left(M_1 - \frac{t_j}{2}\right)\right]\Bigg\}
\end{aligned}$$

Now for minimization, differentiating $P\tilde{C}_j(t_j)$ with respect to t_j and equating to zero

$$\begin{aligned}
\frac{\partial P\tilde{C}_j(t_j)}{\partial t_j} = \frac{1}{6}\Bigg\{ &\left[\frac{-k_{j1}}{t_j^2} + \frac{h_{j1}d_{j1}}{2} - \frac{I_{ej4}p_{j4}d_{j4}}{2}\right] + 2\left[\frac{-k_{j2}}{t_j^2} + \frac{h_{j2}d_{j2}}{2} - \frac{I_{ej3}p_{j3}d_{j3}}{2}\right] \\
&+2\left[\frac{-k_{j3}}{t_j^2} + \frac{h_{j3}d_{j3}}{2} - \frac{I_{ej2}p_{j2}d_{j2}}{2}\right] + 2\left[\frac{-k_{j4}}{t_j^2} + \frac{h_{j4}d_{j4}}{2} - \frac{I_{ej1}p_{j1}d_{j1}}{2}\right]\Bigg\} = 0
\end{aligned}$$

$$t_j^* = \sqrt{\frac{2[k_{j1} + k_{j2} + k_{j3} + k_{j4}]}{\begin{array}{c}(h_{j1}d_{j1}t_j + I_{ej4}p_{j4}d_{j4}) + 2(h_{j2}d_{j2} + I_{ej3}p_{j3}d_{j3}) \\ +2(h_{j3}d_{j3} + I_{ej2}p_{j2}d_{j2}) + (h_{j4}d_{j4} + I_{ej1}p_{j1}d_{j1})\end{array}}}$$

Fuzzy inventory EOQ model with fuzzy decision variable

In this section ,we introduce the fuzzy inventory EOQ models by changing the crisp order quantity t_j

be a trapezoidal fuzzy number $t_j = (t_{j1}, t_{j2}, t_{j3}, t_{j4})$ with $0 \leq t_{j1} \leq t_{j2} \leq t_{j3} \leq t_{j4}$

then we get the fuzzy total cost function

$$\begin{aligned}
\tilde{C}_j(t_j) &= (C_1, C_2, C_3, C_4) \\
C_1 &= \frac{k_{j1}}{t_{j4}} + \frac{h_{j1}d_{j1}t_{j1}}{2} - I_{ej4}p_{j4}d_{j4}\left(M_4 - \frac{t_{j1}}{2}\right) \\
C_2 &= \frac{k_{j2}}{t_{j3}} + \frac{h_{j2}d_{j2}t_{j2}}{2} - I_{ej3}p_{j3}d_{j3}\left(M_3 - \frac{t_{j2}}{2}\right) \\
C_3 &= \frac{k_{j3}}{t_{j2}} + \frac{h_{j3}d_{j3}t_{j3}}{2} - I_{ej2}p_{j2}d_{j2}\left(M_2 - \frac{t_{j3}}{2}\right) \\
C_4 &= \frac{k_{j4}}{t_{j1}} + \frac{h_{j4}d_{j4}t_{j4}}{2} - I_{ej1}p_{j1}d_{j1}\left(M_1 - \frac{t_{j4}}{2}\right)
\end{aligned}$$

We defuzzify the fuzzy total cost $\tilde{C}_j(t_j)$ using graded mean integration representation

$$\begin{aligned}
P\tilde{C}_j(t_j) = \frac{1}{6}\Bigg\{ &\left[\frac{k_{j1}}{t_{j4}} + \frac{h_{j1}d_{j1}t_{j1}}{2} - I_{ej4}p_{j4}d_{j4}\left(M_4 - \frac{t_{j1}}{2}\right)\right] \\
&+ 2\left[\frac{k_{j2}}{t_{j3}} + \frac{h_{j2}d_{j2}t_{j3}}{2} - I_{ej3}p_{j3}d_{j3}\left(M_3 - \frac{t_{j3}}{2}\right)\right] \\
&+ 2\left[\frac{k_{j3}}{t_{j2}} + \frac{h_{j3}d_{j3}t_{j3}}{2} - I_{ej2}p_{j2}d_{j2}\left(M_2 - \frac{t_{j3}}{2}\right)\right] \\
&+2\left[\frac{k_{j4}}{t_{j1}} + \frac{h_{j4}d_{j4}t_{j4}}{2} - I_{ej1}p_{j1}d_{j1}\left(M_1 - \frac{t_{j4}}{2}\right)\right]\Bigg\}
\end{aligned}$$

with $0 \le t_{j1} \le t_{j2} \le t_{j3} \le t_{j4}$
Thus ,the optimal solution of $P\tilde{C}_j(t_j)$ given in above equation, subject to the constraints:
$t_{j1} - t_{j2} \le 0$, $t_{j2} - t_{j3} \le 0$, $t_{j3} - t_{j4} \le 0$, $-t_{j4} < 0$
The kuhn Tucker conditions were used to find the optimal solution of $P\tilde{C}_j(t_j)$,subject to inequalities conditions Then
$\lambda \le 0$
$\nabla f(P\tilde{C}_j(t_{ji}) - \lambda \nabla g(t_{ji})) = 0$
$\lambda g(t_{ji}) = 0$
$g(t_{ji}) \le 0$
These conditions simplify to the following:
$\lambda_1, \lambda_2, \lambda_3, \lambda_4 \ge 0$
$\frac{1}{6}\left[\frac{h_{j1}d_{j1}}{2} + \frac{I_{ej4}p_{j4}d_{j4}}{2} + k_{j4}\right] - \lambda_1 + \lambda_4 = 0$
$\frac{2}{6}\left[\frac{h_{j2}d_{j2}}{2} + \frac{I_{ej3}p_{j3}d_{j3}}{2} + k_{j3}\right] - \lambda_2 + \lambda_1 = 0$
$\frac{2}{6}\left[\frac{h_{j3}d_{j3}}{2} + \frac{I_{ej2}p_{j2}d_{j2}}{2} + k_{j2}\right] - \lambda_3 + \lambda_2 = 0$
$\frac{1}{6}\left[\frac{h_{j4}d_{j4}}{2} + \frac{I_{ej1}p_{j1}d_{j1}}{2} + k_{j1}\right] + \lambda_3 = 0$
$\lambda_1(t_{j2} - t_{j1}) = 0$
$\lambda_2(t_{j3} - t_{j2}) = 0$
$\lambda_1(t_{j4} - t_{j3}) = 0$
$\lambda_4 t_{j1} = 0$

$t_{j1} - t_{j2} \leq 0$
$t_{j2} - t_{j3} \leq 0$
$t_{j3} - t_{j4} \leq 0$
$t_{j1} > 0$
Because $t_{j1} > 0$ and $\lambda_4 t_{j1} = 0$ then $\lambda_4 = 0$ If $\lambda_1 = \lambda_2 = \lambda_3 = 0$ then $t_{j4} < t_{j3} < t_{j2} < t_{j1}$,
It does not satisfy the constains $0 \leq t_{j1} \leq t_{j2} \leq t_{j3} \leq t_{j4}$
Therefore, $t_{j2} = t_{j1}$, $t_{j3} = t_{j2}$, $t_{j4} = t_{j3}$, that is $t_{j1} = t_{j2} = t_{j3} = t_{j4} = t_j^*$,

$$t_j^* = \sqrt{\frac{2[k_{j1} + k_{j2} + k_{j3} + k_{j4}]}{\begin{array}{c}(h_{j1}d_{j1} + I_{ej4}p_{j4}d_{j4}) + 2(h_{j2}d_{j2} + I_{ej3}p_{j3}d_{j3}) \\ +2(h_{j3}d_{j3} + I_{ej2}p_{j2}d_{j2}) + (h_{j4}d_{j4} + I_{ej1}p_{j1}d_{j1})\end{array}}}$$

If $t_j > M$
$\tilde{C}_j(t_j) = (C_1, C_2, C_3, C_4)$

$$C_1 = \frac{k_{j1}}{t_j} + \frac{h_{j1}d_{j1}t_j}{2} - \frac{I_{ej4}p_{j4}d_{j4}M_4^2}{2t_j} + \frac{I_{cj1}p_{01}d_{j1}t_j}{2} + \frac{I_{cj1}p_{01}d_{j1}M_1^2}{2t_j} - I_{cj4}p_{04}d_{j4}$$
$$C_2 = \frac{k_{j2}}{t_j} + \frac{h_{j2}d_{j2}t_j}{2} - \frac{I_{ej3}p_{j3}d_{j3}M_3^2}{2t_j} + \frac{I_{cj2}p_{02}d_{j2}t_j}{2} + \frac{I_{cj2}p_{02}d_{j2}M_2^2}{2t_j} - I_{cj3}p_{03}d_{j3}$$
$$C_3 = \frac{k_{j3}}{t_j} + \frac{h_{j3}d_{j3}t_j}{2} - \frac{I_{ej2}p_{j2}d_{j2}M_2^2}{2t_j} + \frac{I_{cj3}p_{03}d_{j3}t_j}{2} + \frac{I_{cj3}p_{03}d_{j3}M_3^2}{2t_j} - I_{cj4}p_{04}d_{j4}$$
$$C_4 = \frac{k_{j4}}{t_j} + \frac{h_{j4}d_{j4}t_j}{2} - \frac{I_{ej1}p_{j1}d_{j1}M_1^2}{2t_j} + \frac{I_{cj4}p_{04}d_{j4}t_j}{2} + \frac{I_{cj4}p_{04}d_{j4}M_4^2}{2t_j} - I_{cj1}p_{01}d_{j1}$$

We defuzzify $\tilde{C}_j(t_j)$ by using graded mean integration representation

$$\begin{aligned} P\tilde{C}_j(t_j) = \frac{1}{6}\Bigg\{ & \left[\frac{k_{j1}}{t_j} + \frac{h_{j1}d_{j1}t_j}{2} - \frac{I_{ej4}p_{j4}d_{j4}M_4^2}{2t_j} + \frac{I_{cj1}p_{01}d_{j1}t_j}{2} + \frac{I_{cj1}p_{01}d_{j1}M_1^2}{2t_j} - I_{cj4}p_{04}d_{j4}\right] \\ & + 2\left[\frac{k_{j2}}{t_j} + \frac{h_{j2}d_{j2}t_j}{2} - \frac{I_{ej3}p_{j3}d_{j3}M_3^2}{2t_j} + \frac{I_{cj2}p_{02}d_{j2}t_j}{2} + \frac{I_{cj2}p_{02}d_{j2}M_2^2}{2t_j} - I_{cj3}p_{03}d_{j3}\right] \\ & + 2\left[\frac{k_{j3}}{t_j} + \frac{h_{j3}d_{j3}t_j}{2} - \frac{I_{ej2}p_{j2}d_{j2}M_2^2}{2t_j} + \frac{I_{cj3}p_{03}d_{j3}t_j}{2} + \frac{I_{cj3}p_{03}d_{j3}M_3^2}{2t_j} - I_{cj4}p_{04}d_{j4}\right] \\ & + \left[\frac{k_{j4}}{t_j} + \frac{h_{j4}d_{j4}t_j}{2} - \frac{I_{ej1}p_{j1}d_{j1}M_1^2}{2t_j} + \frac{I_{cj4}p_{04}d_{j4}t_j}{2} + \frac{I_{cj4}p_{04}d_{j4}M_4^2}{2t_j} - I_{cj1}p_{01}d_{j1}\right]\Bigg\} \end{aligned}$$

Now for minimization ,differentiating $P\tilde{C}_j(t_j)$ with respect to t_j and equating to zero

$$\begin{aligned} \frac{\partial P\tilde{C}_j(t_j)}{\partial t_j} = \frac{1}{6}\Bigg\{ & \left[\frac{-k_{j1}}{t_j^2} + \frac{h_{j1}d_{j1}}{2} - \frac{I_{ej4}p_{j4}d_{j4}M_4^2}{2t_j^2} + \frac{I_{cj1}p_{01}d_{j1}}{2} + \frac{I_{cj1}p_{01}d_{j1}M_1^2}{2t_j^2}\right] \\ & + 2\left[\frac{k_{j2}}{t_j^2} + \frac{h_{j2}d_{j2}}{2} - \frac{I_{ej3}p_{j3}d_{j3}M_3^2}{2t_j^2} + \frac{I_{cj2}p_{02}d_{j2}}{2} + \frac{I_{cj2}p_{02}d_{j2}M_2^2}{2t_j^2}\right] \\ & + 2\left[\frac{k_{j3}}{t_j^2} + \frac{h_{j3}d_{j3}}{2} - \frac{I_{ej2}p_{j2}d_{j2}M_2^2}{2t_j^2} + \frac{I_{cj3}p_{03}d_{j3}}{2} + \frac{I_{cj3}p_{03}d_{j3}M_3^2}{2t_j^2}\right] \\ & + 2\left[\frac{k_{j4}}{t_j} + \frac{h_{j4}d_{j4}}{2} - \frac{I_{ej1}p_{j1}d_{j1}M_1^2}{2t_j^2} + \frac{I_{cj4}p_{04}d_{j4}}{2} + \frac{I_{cj4}p_{04}d_{j4}M_4^2}{2t_j^2}\right]\Bigg\} \end{aligned}$$

$$t_j^* = \sqrt{\frac{\begin{array}{l}2[k_{j1}+k_{j2}+k_{j3}+k_{j4}][d_{j4}M_4^2(I_{cj4}p_{04}-I_{ej4}p_{j4})+2d_{j3}M_3^2(I_{cj3}p_{03}-I_{ej3}p_{j3})\\ +2d_{j2}M_2^2(I_{cj2}p_{02}-I_{ej2}p_{j2})+d_{j1}M_1^2(I_{cj1}p_{01}-I_{ej1}p_{j1})\end{array}}{d_{j1}[h_{j1}+I_{cj1}p_{01}]+2d_{j2}[h_{j2}+I_{cj2}p_{02}]+d_{j3}[h_{j3}+I_{cj3}p_{03}]+d_{j4}[h_{j4}+I_{cj4}p_{04}]}}$$

Fuzzy inventory EOQ model with fuzzy decision variable

In this section ,we introduce the fuzzy inventory EOQ models by changing the crisp order quantity t_j

be a trapezoidal fuzzy number $t_j = (t_{j1}, t_{j2}, t_{j3}, t_{j4})$ with $0 \le t_{j1} \le t_{j2} \le t_{j3} \le t_{j4}$

then we get the fuzzy total cost function

$$\begin{aligned}
\tilde{C}_j(t_j) &= (C_1, C_2, C_3, C_4)\\
C_1 &= \frac{k_{j1}}{t_{j4}} + \frac{h_{j1}d_{j1}t_{j1}}{2} - \frac{I_{ej4}p_{j4}d_{j4}M_4^2}{2t_{j1}} + \frac{I_{cj1}p_{01}d_{j1}t_{j1}}{2} + \frac{I_{cj1}p_{01}d_{j1}M_1^2}{2t_{j4}} - I_{cj4}p_{04}d_{j4}\\
C_2 &= \frac{k_{j2}}{t_{j3}} + \frac{h_{j2}d_{j2}t_{j2}}{2} - \frac{I_{ej3}p_{j3}d_{j3}M_3^2}{2t_{j2}} + \frac{I_{cj2}p_{02}d_{j2}t_{j2}}{2} + \frac{I_{cj2}p_{02}d_{j2}M_2^2}{2t_{j3}} - I_{cj3}p_{03}d_{j3}\\
C_3 &= \frac{k_{j3}}{t_{j2}} + \frac{h_{j3}d_{j3}t_{j3}}{2} - \frac{I_{ej2}p_{j2}d_{j2}M_2^2}{2t_{j3}} + \frac{I_{cj3}p_{03}d_{j3}t_{j3}}{2} + \frac{I_{cj3}p_{03}d_{j3}M_3^2}{2t_{j2}} - I_{cj4}p_{04}d_{j4}\\
C_4 &= \frac{k_{j4}}{t_{j1}} + \frac{h_{j4}d_{j4}t_{j4}}{2} - \frac{I_{ej1}p_{j1}d_{j1}M_1^2}{2t_{j4}} + \frac{I_{cj4}p_{04}d_{j4}t_{j4}}{2} + \frac{I_{cj4}p_{04}d_{j4}M_4^2}{2t_{j1}} - I_{cj1}p_{01}d_{j1}
\end{aligned}$$

We defuzzify the fuzzy total cost $\tilde{C}_j(t_j)$ using graded mean integration representation

$$\begin{aligned}
P\tilde{C}_j(t_j) = \frac{1}{6}\Bigg\{&\left[\frac{k_{j1}}{t_{j4}} + \frac{h_{j1}d_{j1}t_{j1}}{2} - \frac{I_{ej4}p_{j4}d_{j4}M_4^2}{2t_{j1}} + \frac{I_{cj1}p_{01}d_{j1}t_{j1}}{2} + \frac{I_{cj1}p_{01}d_{j1}M_1^2}{2t_{j4}} - I_{cj4}p_{04}d_{j4}\right]\\
&+2\left[\frac{k_{j2}}{t_{j3}} + \frac{h_{j2}d_{j2}t_{j2}}{2} - \frac{I_{ej3}p_{j3}d_{j3}M_3^2}{2t_{j2}} + \frac{I_{cj2}p_{02}d_{j2}t_{j2}}{2} + \frac{I_{cj2}p_{02}d_{j2}M_2^2}{2t_{j3}} - I_{cj3}p_{03}d_{j3}\right]\\
&+2\left[\frac{k_{j3}}{t_{j2}} + \frac{h_{j3}d_{j3}t_{j3}}{2} - \frac{I_{ej2}p_{j2}d_{j2}M_2^2}{2t_{j3}} + \frac{I_{cj3}p_{03}d_{j3}t_{j3}}{2} + \frac{I_{cj3}p_{03}d_{j3}M_3^2}{2t_{j2}} - I_{cj4}p_{04}d_{j4}\right]\\
&+\left[\frac{k_{j4}}{t_{j1}} + \frac{h_{j4}d_{j4}t_{j4}}{2} - \frac{I_{ej1}p_{j1}d_{j1}M_1^2}{2t_{j4}} + \frac{I_{cj4}p_{04}d_{j4}t_{j4}}{2} + \frac{I_{cj4}p_{04}d_{j4}M_4^2}{2t_{j1}} - I_{cj1}p_{01}d_{j1}\right]\Bigg\}
\end{aligned}$$

with $0 \le t_{j1} \le t_{j2} \le t_{j3} \le t_{j4}$

Thus ,the optimal solution of $P\tilde{C}_j(t_j)$ given in above equation ,subject to the constraints:

$t_{j1} - t_{j2} \le 0,\ t_{j2} - t_{j3} \le 0,\ t_{j3} - t_{j4} \le 0,\ -t_{j4} < 0$

The kuhn - Tucker conditions were used to find the optimal solution of $P\tilde{C}_j(t_j)$,subject to inequalities conditions Then

$\lambda \le 0$

$\nabla f(P\tilde{C}_j(t_{ji}) - \lambda \nabla g(t_{ji})) = 0$

$\lambda g(t_{ji}) = 0$

$g(t_{ji}) \le 0$

These conditions simplify to the following:

$\lambda_1, \lambda_2, \lambda_3, \lambda_4 \ge 0$

$\frac{1}{6}\left[\frac{h_{j1}d_{j1}}{2} + \frac{I_{ej4}p_{j4}d_{j4}M_4^2}{2t_{j1}^2} + \frac{I_{cj1}p_{01}d_{j1}}{2} - \frac{k_{j4}}{t_{j1}^2} - \frac{I_{cj4}p_{04}d_{j4}M_4^2}{2t_{j1}^2}\right] - \lambda_1 + \lambda_4 = 0$

$\frac{2}{6}\left[\frac{h_{j2}d_{j2}}{2} + \frac{I_{ej3}p_{j3}d_{j3}M_3^2}{2t_{j2}^2} + \frac{I_{cj2}p_{02}d_{j2}}{2} - \frac{k_{j3}}{t_{j2}^2} - \frac{I_{cj3}p_{03}d_{j3}M_3^2}{2t_{j2}^2}\right] - \lambda_2 + \lambda_1 = 0$

$$\frac{2}{6}\left[\frac{h_{j3}d_{j3}}{2}+\frac{I_{ej2}p_{j2}d_{j2}M_2^2}{2t_{j3}^2}+\frac{I_{cj3}p_{03}d_{j3}}{2}-\frac{k_{j2}}{t_{j3}^2}-\frac{I_{cj2}p_{02}d_{j2}M_2^2}{2t_{j3}^2}\right]-\lambda_3+\lambda_2=0$$

$$\frac{1}{6}\left[\frac{h_{j4}d_{j4}}{2}+\frac{I_{ej1}p_{j1}d_{j1}M_1^2}{2t_{j1}^2}+\frac{I_{cj4}p_{04}d_{j4}}{2}-\frac{k_{j1}}{t_{j4}^2}-\frac{I_{cj1}p_{01}d_{j1}M_1^2}{2t_{j4}^2}\right]+\lambda_3=0$$

$$\lambda_1(t_{j2}-t_{j1})=0$$

$$\lambda_2(t_{j3}-t_{j2})=0$$

$$\lambda_1(t_{j4}-t_{j3})=0$$

$$\lambda_4 t_{j1}=0$$

$$t_{j1}-t_{j2}\leq 0$$

$$t_{j2}-t_{j3}\leq 0$$

$$t_{j3}-t_{j4}\leq 0$$

$$t_{j1}>0$$

Because $t_{j1}>0$ and $\lambda_4 t_{j1}=0$ then $\lambda_4=0$ If $\lambda_1=\lambda_2=\lambda_3=0$ then $t_{j4}<t_{j3}<t_{j2}<t_{j1}$,

It does not satisfy the constrains $0<t_{j1}\leq t_{j2}\leq t_{j3}\leq t_{j4}$

Therefore, $t_{j2}=t_{j1}$, $t_{j3}=t_{j2}$, $t_{j4}=t_{j3}$, that is $t_{j1}=t_{j2}=t_{j3}=t_{j4}=t_j^*$,

$$t_j^*=\sqrt{\frac{\begin{array}{l}2[k_{j1}+k_{j2}+k_{j3}+k_{j4}][d_{j4}M_4^2(I_{cj4}p_{04}-I_{ej4}p_{j4})+2d_{j3}M_3^2(I_{cj3}p_{03}-I_{ej3}p_{j3})\\+2d_{j2}M_2^2(I_{cj2}p_{02}-I_{ej2}p_{j2})+d_{j1}M_1^2(I_{cj1}p_{01}-I_{ej1}p_{j1})\end{array}}{d_{j1}[h_{j1}+I_{cj1}p_{01}]+2d_{j2}[h_{j2}+I_{cj2}p_{02}]+d_{j3}[h_{j3}+I_{cj3}p_{03}]+d_{j4}[h_{j4}+I_{cj4}p_{04}]}}$$

Numerical example

The data for buyer stored in separate sheet in excel and it acts as a backend. Using Matlab the datasheet is accessed for all the buyer to find the Fuzzy total cost.

If $t_j\leq M$

we consider an instance of the two buyer system with input data,

for $j=1$

	A	B	C	D	E	F
1	28	29	31	32	K	30
2	48	49	51	52	D	50
3	38	39	41	42	H	40
4	0	0.01	0.03	0.04	Iej	0.02
5	18	19	21	22	Pj	20

for $j=2$

	A	B	C	D	E	F
1	48	49	51	52	K	50
2	78	79	81	82	D	80
3	58	59	61	62	H	60
4	0	0.01	0.03	0.04	Iej	0.02
5	23	24	26	27	Pj	25

```
file='sample.xlsx'
[status,sheets] = xlsfinfo(file)


for s = 1:numel(sheets)
    numbers=xlsread(file,s)
 disp (s)

 m=(numbers(3,1)*numbers(2,1)+(numbers(4,4)*numbers(5,4)*numbers(2,4)));
 disp(m);
 n=2*((numbers(3,2)*numbers(2,2))+(numbers(4,3)*numbers(5,3)*numbers(2,3)));
 disp(n);
 o=2*(numbers(3,3)*numbers(2,3)+(numbers(4, 2)*numbers(5,2)*numbers(2,2)));
 disp(o);
 z=numbers(3,4)*numbers(2,4) + (numbers(4,1)*numbers(5,1)*numbers(2,1));
 disp(z);
 nume=2*(numbers(1,1)+2*numbers(1,2)+2*numbers(1,3)+numbers(1,4));
 disp(nume);
 t(s)= sqrt(nume/(m+n+o+z));
 disp('t value is ');
 disp(t(s))
 end
```

Similarly we can prove for $t_j > M$.

Buyer j	Crisp total cost	Fuzzy total cost
1	0.1724	0.1725
2	0.1437	0.1438

3 Conclusion

We focus on the integrated one vendor two buyer inventory system with permissible delay in payments.In the fuzzy environment it may be possible and reasonable to discuss the fuzzy production full backordering with trapezoidal fuzzy number for crisp production quantity. In addition we find that the fuzzy production quantity with special type of trapezoidal fuzzy number, and the optimal solution of our proposed models are all real numbers. It means that the optimal solution of our proposed model can be specified to meet the classical production inventory models are encountable & useful in the real world. We have implemented a Matlab source code it is possible to find the fuzzy total cost for infinite number of buyers.

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