Ranking of pentagonal fuzzy numbers applying incentre of centroids

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Abstract

Fuzzy numbers are based on membership function which have been classified into shape of triangle, trapezoidal, bell etc., even in various different points in real numbers. The human judgment data preference are repeatedly unclear. So that the crisp values are insufficient by using then using uncertain numbers such as triangular, trapezoidal. Even they are not suitable in few case whereas uncertainties arises in more than four points. In such case pentagonal fuzzy number i.e., five points can be used to solve the problems. Our main concept of in this paper we introduced the five different points of pentagonal fuzzy numbers and the new operations addition, subtraction, multiplication, division. It also introduces the ranking of Pentagonal fuzzy numbers and applying incentre of centroid.

Key Words: Fuzzy numbers [FN], Pentagonal fuzzy number [PFN], fuzzy arithmetic operations, alpha cut, Ranking, Centroid.
1 Introduction

L.A.Zadeh\[1\] was introduced the concept of fuzzy numbers and fuzzy arithmetic. Masaharu Mizumoto and Kokichi Tsuka\[6\] have investigated the algebraic properties of fuzzy numbers under addition, subtraction, multiplication, division, joint and meet operations. J.G. Dijkman, H. Van Haeringen and S. J. De Lange\[2\] have investigated nine operations of fuzzy numbers for addition. Triangular fuzzy numbers are frequently used in application. In some cases triangular fuzzy numbers is not suitable whereas uncertainties arises in more than four points. So the important contributes to the theory of fuzzy numbers have five different points by numerous researchers with triangular shape fuzzy numbers. In this paper four different operations like addition, subtraction, multiplication and division have been introduced using alpha cut principle and as new approach for ranking with incentre of centroid using pentagonal fuzzy numbers.

2 Preliminaries and Notations

2.1 Definition(FN): A fuzzy set $A$ is defined on the set of real line, $\mathbb{R}$ is said to be a fuzzy number if its membership function $\mu_A : \mathbb{R} \rightarrow [0, 1]$ satisfies

- Convex and Normal of fuzzy set.
- $A$ is piecewise continuous.

2.1 Definition(TFN): A fuzzy numbers $A$ is called a triangular fuzzy number (TFN) is a particular case of semi symmetric L-R fuzzy number and if its membership function $\mu_A$ is given by

$$
\mu_A(x) = \begin{cases} 
\frac{x-\lambda_1}{\lambda_2-\lambda_1} & \text{for } \lambda_1 \leq x \leq \lambda_2 \\
\frac{\lambda_3-x}{\lambda_3-\lambda_2} & \text{for } \lambda_2 \leq x \leq \lambda_3 \\
0 & \text{otherwise}
\end{cases}
$$

![Figure 1: A triangular fuzzy number $A = (\lambda_1, \lambda_2, \lambda_3)$](image)

3 New PFNs

3.1 Definition(PFN): A fuzzy number $A_{PFN}$ is a PFN denoted by $A_{PFN} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5)$ whereas $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5$ are real numbers and its membership function
\[
\mu_{A_{PFN}}(x) = \begin{cases} 
\frac{1}{2}(x-\lambda_1) & \text{for } \lambda_1 \leq x \leq \lambda_2 \\
\frac{1}{2} + \frac{1}{2}(x-\lambda_2) & \text{for } \lambda_2 \leq x \leq \lambda_3 \\
1 - \frac{1}{2}(x-\lambda_3) & \text{for } \lambda_3 \leq x \leq \lambda_4 \\
\frac{1}{2}(\lambda_4-x) & \text{for } \lambda_4 \leq x \leq \lambda_5 \\
0 & \text{for } x < \lambda_1 \text{ and } x > \lambda_5
\end{cases}
\]

Figure 2: Graphical representation of a normal new PFN for \(x \in [0,1]\)

4 Operations of PFNs:

4.1 Definition

Let \(A_{PFN} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5)\) and \(B_{PFN} = (\beta_1, \beta_2, \beta_3, \beta_4, \beta_5)\) be their corresponding PFN then

1. Addition: \(A_{PFN} (+) B_{PFN} = (\lambda_1 + \beta_1, \lambda_2 + \beta_2, \lambda_3 + \beta_3, \lambda_4 + \beta_4, \lambda_5 + \beta_5)\)
2. Subtraction: \(A_{PFN} (-) B_{PFN} = (\lambda_1 - \beta_5, \lambda_2 - \beta_4, \lambda_3 - \beta_3, \lambda_4 - \beta_2, \lambda_5 - \beta_1)\)
3. Multiplication: \(A_{PFN} (*) B_{PFN} = (\lambda_1*\beta_1 \land \lambda_1*\beta_5 \land \lambda_5*\beta_1 \land \lambda_5*\beta_5, \lambda_2*\beta_2 \land \lambda_2*\beta_4 \land \lambda_4*\beta_2 \land \lambda_4*\beta_4, \lambda_3*\beta_3, \lambda_2*\beta_2 \lor \lambda_2*\beta_4 \lor \lambda_4*\beta_2 \lor \lambda_4*\beta_4, \lambda_1*\beta_1 \lor \lambda_1*\beta_5 \lor \lambda_5*\beta_1 \lor \lambda_5*\beta_5)\)
4. Division: \(\frac{A_{PFN}}{B_{PFN}} = \left(\frac{\lambda_1}{\beta_1} \land \frac{\lambda_1}{\beta_5} \land \frac{\lambda_5}{\beta_1} \land \frac{\lambda_5}{\beta_5} \land \frac{\lambda_4}{\beta_1} \land \frac{\lambda_4}{\beta_5} \land \frac{\lambda_2}{\beta_1} \land \frac{\lambda_2}{\beta_5} \lor \frac{\lambda_4}{\beta_1} \lor \frac{\lambda_4}{\beta_5} \lor \frac{\lambda_2}{\beta_1} \lor \frac{\lambda_2}{\beta_5}\right)\)

excluding the case \(\beta_1 = 0\) (or) \(\beta_2 = 0\) (or) \(\beta_3 = 0\) (or) \(\beta_4 = 0\) (or) \(\beta_5 = 0\)

Suppose, \(A_{PFN}\) and \(B_{PFN}\) are the positive real number \(\Re^+\), then

(3) \(A_{PFN} (*) B_{PFN} = (\lambda_1 * \beta_1, \lambda_2 * \beta_2, \lambda_3 * \beta_3, \lambda_4 * \beta_4, \lambda_5 * \beta_5)\) and

(4) \(\frac{A_{PFN}}{B_{PFN}} = \left(\frac{\lambda_1}{\beta_1}, \frac{\lambda_1}{\beta_5}, \frac{\lambda_5}{\beta_1}, \frac{\lambda_5}{\beta_5}, \frac{\lambda_4}{\beta_1}, \frac{\lambda_4}{\beta_5}, \frac{\lambda_2}{\beta_1}, \frac{\lambda_2}{\beta_5}\right)\)

4.2 Example Let \(A_{PFN} = (2, 4, 6, 8, 10)\) and \(B_{PFN} = (3, 6, 9, 12, 15)\) be two PFN then

1. Addition: \(A_{PFN} (+) B_{PFN} = (5, 10, 15, 20, 25)\)
2. Subtraction: \(A_{PFN} (-) B_{PFN} = (-13, -8, -3, 2, 7)\)
3. Multiplication: \(A_{PFN} (*) B_{PFN} = (6, 24, 54, 96, 150)\)
4. Division: $\frac{A_{\text{PFN}}}{B_{\text{PFN}}} = (0.13, 0.33, 0.67, 1.33, 3.33)$

4.3 Definition A $\text{PFN}$ can be defined as $\text{PFN} = P_l(t), Q_l(u), P_u(t), Q_u(u), t \in [0, 0.5], u \in [0.5, 1.0]$ whereas

$$P_l(t) = \frac{1}{2}(\frac{x - \lambda_1}{\lambda_2 - \lambda_1})$$
$$Q_l(t) = \frac{1}{2} + \frac{1}{2}(\frac{x - \lambda_1}{\lambda_2 - \lambda_1})$$
$$P_u(t) = \frac{1}{2}(\frac{\lambda_5 - x}{\lambda_5 - \lambda_4})$$
$$Q_u(t) = 1 - \frac{1}{2}(\frac{\lambda_5 - x}{\lambda_5 - \lambda_4})$$

$P_l(t), Q_l(u)$ is monotonic ascending with bounded under $[0,0.5]$ and $[0.5,1.0]$. $P_u(t), Q_u(u)$ is monotonic descending with bounded under $[0,0.5]$ and $[0.5,1.0]$.

4.4 Definition The alpha cut of the PFN in the set of elements in X is defined as

$$\text{PFN}_\alpha = \{x \in X/\mu_{\text{PFN}}(x) \geq \alpha\} = \begin{cases} [P_l(\alpha), P_u(\alpha)] & \text{for } \alpha \in [0,0.5) \\ [Q_l(\alpha), Q_u(\alpha)] & \text{for } \alpha \in [0.5,1] \end{cases}$$

where $\alpha \in [0,1]$.

4.5 Definition If $P_l(x) = \alpha$ and $P_u(x) = \alpha$, then $\alpha$- cut operations interval $\text{PFN}_\alpha$ is obtained as

- $[P_l(\alpha), P_u(\alpha)] = [2\alpha(\lambda_2 - \lambda_1) + \lambda_1, -2\alpha(\lambda_5 - \lambda_4) + \lambda_5]$ similarly

- $[Q_l(\alpha), Q_u(\alpha)] = [2(\alpha - \frac{1}{2})(\lambda_3 - \lambda_2) + \lambda_2, -2(\alpha - 1)(\lambda_4 - \lambda_3) + \lambda_3]$  

Hence $\alpha$ cut of PFN

$$\text{PFN}_\alpha = \begin{cases} [2\alpha(\lambda_2 - \lambda_1) + \lambda_1, -2\alpha(\lambda_5 - \lambda_4) + \lambda_5] & \text{for } \alpha \in [0,0.5) \\ [2(\alpha - \frac{1}{2})(\lambda_3 - \lambda_2) + \lambda_2, -2(\alpha - 1)(\lambda_4 - \lambda_3) + \lambda_3] & \text{for } \alpha \in [0.5,1] \end{cases}$$

Note: The triangular fuzzy numbers and PFNs are same for the points have the equal intervals and distinct for the points have the unequal intervals.

Figure 3: Graphical representation of a normal PFN $A(2,4,6,8,10)$ and $B=(3,6,9,12,15)$
5 A new operation for Addition, Subtraction, Multiplication, Division on PFN

5.1 Definition Let $A_{PFN}=(\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5)$ and $B_{PFN}=(\beta_1, \beta_2, \beta_3, \beta_4, \beta_5)$, for all $x, \lambda_1, \lambda_2, \ldots, \lambda_5, \beta_1, \beta_2, \ldots, \beta_5 \in \mathbb{R}, \lambda_1 \leq \lambda_2 \leq \ldots \leq \lambda_5, \beta_1 \leq \beta_2 \leq \ldots \leq \beta_5$ be their corresponding PFN then for all $\alpha \in [0,1]$, let us take the membership function on the basis of $\alpha$-cut $A$ and $B$ of $A_{PFN}$ and $B_{PFN}$ by using interval arithmetic.

5.2 Addition of two PFNs:

\[
A_{PFN}(+) B_{PFN} = \begin{cases} 
2\alpha((\lambda_2 + \beta_2) - (\lambda_1 + \beta_1)) + (\lambda_1 + \beta_1), \\
-2\alpha((\lambda_5 + \beta_5) - (\lambda_4 + \beta_4)) + (\lambda_4 + \beta_4) & \text{for } \alpha \in [0,0.5) \\
2\alpha - 1)((\lambda_3 + \beta_3) - (\lambda_2 + \beta_2)) + (\lambda_2 + \beta_2), \\
-2(\alpha - 1)((\lambda_4 + \beta_4) - ((\lambda_3 + \beta_3)) + (\lambda_3 + \beta_3)) & \text{for } \alpha \in [0.5,1]
\end{cases}
\]

In Numerical example 4.2,

\[\begin{array}{cccc}
\alpha(A + B)_{PFN} & \alpha = 0 & \alpha = 0.5 & \alpha = 1 \\
\end{array}\]

Figure 4: Addition of a normal PFNs $A(2,4,6,8,10)$ and $B=(3,6,9,12,15)$

5.3 Subtraction of two PFNs:

\[
A_{PFN}(-) B_{PFN} = \begin{cases} 
2\alpha((\lambda_2 - \lambda_1) + \lambda_1)+ (\lambda_1 + \beta_1), \\
-2\alpha((\lambda_5 - \lambda_4) + (\lambda_3 + \beta_4)) + (\lambda_4 + \beta_4) & \text{for } \alpha \in [0,0.5) \\
2\alpha - \frac{1}{2})(\lambda_3 - \lambda_2) + (\lambda_2 + \beta_2), \\
-2(\alpha - 1)((\lambda_4 - \lambda_3) - ((\lambda_3 + \beta_3)) + (\lambda_3 + \beta_3)) & \text{for } \alpha \in [0.5,1]
\end{cases}
\]

In Numerical example 4.2,

\[\begin{array}{cccc}
\alpha(A - B)_{PFN} & \alpha = 0 & \alpha = 0.5 & \alpha = 1 \\
(10\alpha - 13, -10\alpha + 7) & [-13, 7] & [-8, 2] & [-3, 3] & (-13, -8, -3, 2, 7)
\end{array}\]

5.4 Multiplication of Two PFN:

\[
A_{PFN}(\ast) B_{PFN} = \begin{cases} 
2\alpha(\lambda_2 - \lambda_1) + \lambda_1 * 2\alpha(\beta_2 - \beta_1) + \beta_1, \\
-2\alpha((\lambda_5 - \lambda_4) + \lambda_3 * -2\alpha(\beta_5 - \beta_4) + \beta_5) & \text{for } \alpha \in [0,0.5) \\
2\alpha - \frac{1}{2})(\lambda_3 - \lambda_2) + \lambda_2 * 2(\alpha - \frac{1}{2})(\beta_3 - \beta_2) + \beta_1 + \beta_2, \\
-2(\alpha - 1)(\lambda_4 - \lambda_3) + \lambda_3 * -2(\alpha - 1)(\beta_4 - \beta_3) + \beta_3) & \alpha \in [0.5,1]
\end{cases}
\]

In Numerical example 4.2,
Figure 5: Subtraction of a normal PFNs $A(2,4,6,8,10)$ and $B=(3,6,9,12,15)$

$$\alpha (A \ast B)_{PFN} = 0 \quad \alpha = 0 \quad 0.0.5 \quad \alpha = 1 \quad PFN$$

<table>
<thead>
<tr>
<th>$\alpha (A \ast B)_{PFN}$</th>
<th>$\alpha = 0$</th>
<th>$\alpha = 0.5$</th>
<th>$\alpha = 1$</th>
<th>$PFN$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(24\alpha^2 + 24\alpha + 6, 24\alpha^2 - 120\alpha + 150)$</td>
<td>$[6,150]$</td>
<td>$[24,96]$</td>
<td>$[54,54]$</td>
<td>$(-6,24,54,96,150)$</td>
</tr>
</tbody>
</table>

Figure 6: Multiplications of a normal PFN $A(2,4,6,8,10)$ and $B=(3,6,9,12,15)$

5.5 Division of Two PFN:

$$\left[ \begin{array}{c} \frac{A}{PFN} \\ \frac{B}{PFN} \end{array} \right] = \left\{ \begin{array}{c} \left[ \frac{-2\alpha(\lambda_3 - \lambda_4) + \lambda_3}{\frac{1}{2}(\lambda_3 - 2\lambda_4 + \lambda_2) + \lambda_2}, \frac{-2\alpha(\lambda_5 - \lambda_6) + \lambda_5}{\frac{1}{2}(\lambda_5 - 2\lambda_6 + \lambda_4) + \lambda_4} \right] \\ \left[ \frac{2(\alpha - 1)(\lambda_3 - \lambda_4) + \lambda_3}{2(\alpha - 1)(\lambda_5 - \lambda_6) + \lambda_5} \right] \end{array} \right\} \quad \text{for} \quad \alpha \in [0,0.5)$$

In numerical example 4.2,

$$\left[ \begin{array}{c} A_{PFN} \\ B_{PFN} \end{array} \right] = \left\{ \begin{array}{c} 0.13, 3.33 \\ 0.13, 3.33 \end{array} \right\} \quad \text{for} \quad \alpha \in [0.5,1]$$

Figure 7: Division of a normal PFN $A(2,4,6,8,10)$ and $B=(3,6,9,12,15)$
6 Proposed ranking method of PFN :

In the PFN having the five different points they are A, M, O, N and E. The M and N points meets at points B and D. Also, join the points BO and DO. Now the normal pentagonal has been divided into two triangles and one quadrilateral ABO, ODE and OBCD respectively. Let the three plain figures are $G_1$, $G_2$ and $G_3$ with the centroid.

Let a normalized PFN $A_{PFN} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5)$. These three plane figures centroid are

$$G_1 = \{\frac{\lambda_1 + \lambda_2 + \lambda_3}{3}, \frac{1}{6}\}, G_2 = \{\frac{\lambda_2 + 2\lambda_3 + \lambda_4}{4}, \frac{1}{3}\} \text{ and } G_3 = \{\frac{\lambda_3 + \lambda_4 + \lambda_5}{3}, \frac{1}{6}\}$$

respectively. We define the incentre

$$I_{AP}(\tilde{x}_0, \tilde{y}_0) = \left[\frac{\alpha_{AP}\left[\frac{\lambda_1 + \lambda_2 + \lambda_3}{3}\right] + \beta_{AP}\left[\frac{\lambda_2 + 2\lambda_3 + \lambda_4}{4}\right] + \gamma_{AP}\left[\frac{\lambda_3 + \lambda_4 + \lambda_5}{3}\right]}{\alpha_{AP} + \beta_{AP} + \gamma_{AP}}, \frac{\alpha_{AP}\left[\frac{1}{6}\right] + \beta_{AP}\left[\frac{1}{3}\right] + \gamma_{AP}\left[\frac{1}{6}\right]}{\alpha_{AP} + \beta_{AP} + \gamma_{AP}}\right]$$

Whereas $\alpha_{AP} = \sqrt{\lambda_1 + \lambda_2 - \lambda_3 - 3\lambda_4}^2 + 16/12$, $\beta_{AP} = \sqrt{\lambda_1 + \lambda_2 - \lambda_3 - \lambda_5}^2$ and $\gamma_{AP} = \sqrt{\lambda_2 + 4\lambda_3 - 2\lambda_4 - 3\lambda_5}^2 + 16/12$

The ranking function of the normalized PFN $A_{PFN} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5)$. A set of real numbers which map the set of all fuzzy numbers is defined as: $R_{\tilde{F}_p} = \sqrt{\tilde{x}_0^2 + \tilde{y}_0^2}$. This is the incentre of the centroid with Euclidean distance. In the following steps expresses the sum and the incentre of the centroids are the using the rank of two fuzzy numbers $A_{PFN}$ and $B_{PFN}$. Let $A_{PFN} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5)$ and $B_{PFN} = (\beta_1, \beta_2, \beta_3, \beta_4, \beta_5)$ be two normalized PFN then,
step 1 : Find $\alpha_{AP}, \beta_{AP}, \gamma_{AP}$ and $\alpha_{BP}, \beta_{BP}, \gamma_{BP}$

step 2 : Find $I_{AP}(\tilde{x}_0, \tilde{y}_0)$ and $I_{BP}(\tilde{x}_0, \tilde{y}_0)$

step 3 : Find $R_{AP} = \sqrt{\tilde{x}_0^2 + \tilde{y}_0^2}$ and $R_{BP} = \sqrt{\tilde{x}_0^2 + \tilde{y}_0^2}$.

and by using the ranking of fuzzy numbers i.e.,

i) If $R_{AP} > R_{BP}$ then $A_P > B_P$

ii) If $R_{AP} < R_{BP}$ then $A_P < B_P$

iii) If $R_{AP} \approx R_{BP}$ then $A_P \approx B_P$

6.1 Numerical example : Let $A_P = (2, 4, 6, 8, 10)$ and $B_P = (3, 6, 9, 12, 15)$ be two normalized fuzzy numbers

step 1 : Find $\alpha_{AP} = 2.027, \beta_{AP} = 4.000, \gamma_{AP} = 2.0276$ and $\alpha_{BP} = 3.0185, \beta_{BP} = 6.0000, \gamma_{BP} = 3.0185$

step 2 : Find $I_{AP} = [6.0000, 0.3322]$ and $I_{BP} = [9.0000, 0.3328]$

step 3 : Find $R_{AP} = 6.0092$ and $R_{BP} = 9.0062$ so $R_{AP} < R_{BP}$ then $A_P \approx B_P$

7 Conclusion

In this research paper we introduced a new membership function of PFN has been introduced with operations of addition, subtraction, multiplication and division and also illustrate with numerical examples. Hence this paper provides a very simple method to find the ranking of PFNs using the incentre of centroids.

References


