On Fuzzy Semi-Regular Weakly Continuous Functions in Fuzzy Topological Spaces

R. S. Wali1 and Basayya B. Mathad2

1 Bhandari and Rathi College,
Guledagudd-587 203, Karnataka, India
rsuali@rediffmail.com
2 Rani Channamma University,
Belagavi-591 156, Karnataka, India
bbmath.mathad@gmail.com

Abstract

The aim of this paper is to introduce a new class of fuzzy generalized functions called fuzzy srw-continuous and fuzzy srw-irresolute functions in fuzzy topological spaces and discussed some of their properties and characterizations.

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1 Introduction

2 Preliminaries

Let $X$, $Y$ and $Z$ are non-empty fuzzy sets and $(X, R)$, $(Y, S)$ and $(Z, T)$ are fuzzy topological spaces. Throughout this paper simply use $X$, $Y$ and $Z$ as corresponding fuzzy topological spaces on which no separation axioms are assumed unless explicitly stated.

Now we recall some of definitions which are essential to our discussion.

**Definition 1.** Let a fuzzy subset $A$ of $X$ is said to be;

1. fuzzy regular open set [2], if $\text{int}(\text{cl}(A)) = A$.
2. fuzzy semi-open set [2] if and only if there exists a fuzzy set $G$ in $X$ such that $G \leq A \leq \text{cl}(G)$.
3. fuzzy semi-closed set [2] if and only if there exists a fuzzy set $F$ in $X$ such that $\text{int}(F) \leq A \leq F$.
4. fuzzy semi-preopen set [14] if $A \leq \text{cl}((\text{int}(\text{cl}(A))))$.
5. fuzzy regular weakly (briefly, fuzzy rw) closed set [4], if $\text{cl}(A) \leq G$ whenever $A \leq G$ and $G$ is fuzzy regular semi open set in $X$.
6. fuzzy $\alpha$-regular weakly (briefly, fuzzy $\alpha$rw) closed set [10], if $\alpha\text{cl}(A) \leq G$ whenever $A \leq G$ and $G$ is fuzzy regular weakly open set in $X$.
7. fuzzy generalized semi (briefly, fuzzy gs) closed set [13], if $\text{scl}(A) \leq G$ whenever $A \leq G$ and $G$ is fuzzy open set in $X$.
8. fuzzy generalized semi pre (briefly, fuzzy gsp) closed set [12], if $\text{spcl}(A) \leq G$ whenever $A \leq G$ and $G$ is fuzzy open set in $X$.
9. fuzzy generalized (briefly, fuzzy g) closed set [3], if $\text{cl}(A) \leq G$ whenever $A \leq G$ and $G$ is fuzzy open set in $X$.
10. fuzzy semi-regular weakly (briefly, fuzzy srw) closed set [8], if $\text{scl}(A) \leq G$ whenever $A \leq G$ and $G$ is fuzzy rw-open set in $X$.
11. A fuzzy subset $G$ of a fuzzy topological space $X$ is called a fuzzy semi-regular weakly open (briefly, fuzzy srw-open)[9] set if its complement 1-$G$ is a fuzzy srw-closed set in $X$.

The compliment of the above all fuzzy closed sets becomes corresponding fuzzy open sets in the same fuzzy topological spaces.

**Definition 2.** Let $X$ and $Y$ be two fuzzy topological spaces, then a function $f : X \rightarrow Y$ is called a;

1. fuzzy continuous (briefly, f-continuous) [5], if $f^{-1}(A)$ is a fuzzy open (fuzzy closed) set in $X$, for every fuzzy open (fuzzy closed) set $A$ in $Y$. 

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2. fuzzy semi-continuous [2], if $f^{-1}(A)$ is a fuzzy semi-open (fuzzy semi-closed) set in $X$, for every fuzzy open (fuzzy closed) set $A$ in $Y$.

3. fuzzy $g$-continuous [3], if $f^{-1}(A)$ is a fuzzy $g$-open (fuzzy $g$-closed) set in $X$, for every fuzzy open (fuzzy closed) set $A$ in $Y$.

4. fuzzy almost continuous [2], if $f^{-1}(A)$ is a fuzzy open (fuzzy closed) set in $X$, for every fuzzy regular open (fuzzy closed) set $A$ in $Y$.

5. fuzzy gs-continuous [13], if $f^{-1}(A)$ is a fuzzy gs-open (fuzzy gs-closed) set in $X$, for every fuzzy open (fuzzy closed) set $A$ in $Y$.

6. fuzzy irresolute [15], if $f^{-1}(A)$ is a fuzzy semi-open-open (fuzzy semi-closed) set in $X$, for every fuzzy semi-open (fuzzy semi-closed) set $A$ in $Y$.

The compliment of the above all fuzzy closed sets becomes corresponding fuzzy open sets in the same fuzzy topological spaces.

**Lemma 3.** [9]

1. Every fuzzy open set is fuzzy srw-open set in $X$, but not conversely.
2. Every fuzzy semi-open set is fuzzy srw-open set in $X$, but not conversely.
3. Every fuzzy $\alpha$rw-open set is fuzzy srw-open set in $X$, but not conversely.
4. Every fuzzy srw-open set is fuzzy gs-open set in $X$, but not conversely.
5. Every fuzzy regular open set is fuzzy srw-open set in $X$, but not conversely.

3 Fuzzy Semi-regular Weakly Continuous (briefly, fuzzy srw-continuous) Functions:

In this section, we introduce the concept of fuzzy srw-continuous functions in topological spaces and study their relations with various fuzzy generalized continuous functions. We also discuss some properties of fuzzy srw-continuous functions.

**Definition 4.** Let $X$ and $Y$ are two fuzzy topological spaces. A function $f : X \rightarrow Y$ is called fuzzy srw-continuous, if $f^{-1}(G)$ is fuzzy srw-open (fuzzy srw-closed) set in $X$ for every fuzzy open (fuzzy closed) set in $Y$.

**Example 1.** Let $X=\{a, b, c, d\}$ and $Y=\{p, q\}$ and fuzzy subsets are $0_X = \{(a, 0), (b, 0), (c, 0), (d, 0)\} = 0$, $\alpha_1 = \{(a, 1), (b, 0), (c, 0), (d, 0)\}$, $\alpha_2 = \{(a, 0), (b, 1), (c, 1), (d, 0)\}$, $\alpha_3 = \{(a, 1), (b, 1), (c, 1), (d, 0)\}$, $\alpha_X = \{(a, 1), (b, 1), (c, 1), (d, 1)\}$, $\beta_Y = \{(p, 0), (q, 0)\}$, $\beta_1 = \{(p, 1), (b, 0)\}$, $\beta_2 = \{(p, 0), (q, 1)\}$ and $\beta_X = \{(p, 1), (q, 1)\}$. Then fuzzy topologies of $X$ and $Y$ are $R = \{0_X, \alpha_1, \alpha_2, \alpha_3, \alpha_X\}$ and $S = \{0_Y, \beta_1, \beta_2, \beta_X\}$ respectively. Let the function $f : X \rightarrow Y$ is defined as $f((a, 1)) = f((d, 1)) = (p, 1)$ and $f((b, 1)) = f((c, 1)) = (q, 1)$. Then $f$ is fuzzy srw-continuous function.
Theorem 5. Let a function $f : X \to Y$ is fuzzy srw-continuous if and only if the inverse image of every fuzzy closed set of $Y$ is fuzzy srw-closed set in $X$.

Proof: Let $F$ be any fuzzy closed set in $Y$, then $1-F$ is fuzzy open set in $Y$. Since $f$ is fuzzy srw-continuous, $f^{-1}(1-F)$ is fuzzy srw-open set in $X$. But $f^{-1}(1-F) = 1-f^{-1}(F)$ and hence $f^{-1}(F)$ is fuzzy srw-closed set in $X$. Conversely, by hypothesis the inverse image of every fuzzy closed set in $Y$ is fuzzy srw-closed set in $X$. Let $G$ be any fuzzy open set in $Y$, then $1-G$ is fuzzy closed set in $Y$. By hypothesis $f^{-1}(1-G) = 1-f^{-1}(G)$ is fuzzy srw-closed set in $X$. Thus $f$ is fuzzy srw-continuous function.

Theorem 6. Every fuzzy continuous function is fuzzy srw-continuous function but not conversely.

Proof: Let a function $f$ is fuzzy continuous. Let $G$ any fuzzy open set in $Y$. Since $f$ is fuzzy continuous, $f^{-1}(G)$ fuzzy open set in $X$ and so $f^{-1}(G)$ is fuzzy srw-open set in $X$, Lemma 3(1). Hence $f$ is fuzzy srw-continuous function.

Example 2. From Example 1, Let the function $f : X \to Y$ is defined as $f((a,1)) = f((d,1)) = (p,1)$ and $f((b,1)) = f((c,1)) = (q,1)$. Hence, $f$ is fuzzy srw-continuous function. However, since $\{(b,1),(c,1)\}$ is fuzzy srw-closed set but not fuzzy closed set in $X$.

Theorem 7. Every fuzzy semi-continuous function is fuzzy srw-continuous function but not conversely.

Proof: Let a function $f : X \to Y$ is fuzzy semi-continuous. Let $G$ any fuzzy open set in $Y$. Since $f$ is fuzzy semi-continuous, $f^{-1}(G)$ fuzzy semi-open set in $X$ and so $f^{-1}(G)$ is fuzzy srw-open set in $X$, Lemma 3(2). Hence $f$ is fuzzy srw-continuous function.

Example 3. Let $X=\{a, b, c, d\}$ and $Y=\{p, q\}$ and fuzzy subsets are $0_X = \{(a,0),(b,0),(c,0),(d,0)\} = 0$, $\alpha_1 = \{(a,1),(b,0),(c,0),(d,0)\}$, $\alpha_2 = \{(a,0),(b,1),(c,1),(d,0)\}$, $\alpha_3 = \{(a,1),(b,1),(c,1),(d,0)\}$, $\alpha_X = \{(a,1),(b,1),(c,1),(d,1)\}$, $0_Y = \{(p,0),(q,0)\}$, $\beta_1 = \{(p,1),(b,0)\}$, and $\beta_Y = \{(p,1),(q,1)\}$. Then fuzzy topologies of $X$ and $Y$ are $R = \{0_X, \alpha_1, \alpha_2, \alpha_3, \alpha_X\}$ and $S = \{0_Y, \beta_1, \beta_Y\}$ respectively. Let the function $f : X \to Y$ is defined as $f((a,1)) = f((c,1)) = f((d,1)) = (q,1)$ and $f((b,1)) = (p,1)$. Hence, $f$ is fuzzy srw-continuous function. However, since $\{(a,1),(c,1),(d,1)\}$ is fuzzy srw-closed set but not fuzzy semi-closed set in $X$.

Definition 8. Let $X$ and $Y$ are two fuzzy topological spaces. A function $f : X \to Y$ is called fuzzy $\alpha rw$-continuous, if $f^{-1}(G)$ is fuzzy $\alpha rw$-open (fuzzy $\alpha rw$-closed) set in $X$ for every fuzzy open (fuzzy closed) set in $Y$.

Theorem 9. Every fuzzy $\alpha rw$-continuous function is fuzzy srw-continuous function but not conversely.

Proof: Let a function $f : X \to Y$ is fuzzy $\alpha rw$-continuous. Let $G$ any fuzzy open set in $Y$. Since $f$ is fuzzy $\alpha rw$-continuous, $f^{-1}(G)$ fuzzy $\alpha rw$-open set in $X$ and so $f^{-1}(G)$ is fuzzy srw-open set in $X$, Lemma 3(3). Hence $f$ is fuzzy srw-continuous.
function.

**Example 4.** From Example 2, Let the function \( f \) is fuzzy srw-continuous function. However, since \( \{(b,1), (c,1)\} \) is fuzzy srw-closed set but not fuzzy orw-closed set in \( X \).

**Theorem 10.** Every fuzzy srw-continuous function is fuzzy gs-continuous function but not conversely.

Proof: Let a function \( f : X \rightarrow Y \) is fuzzy srw-continuous. Let \( G \) any fuzzy open set in \( Y \). Since \( f \) is fuzzy srw-continuous, \( f^{-1}(G) \) fuzzy srw-open set in \( X \) and so \( f^{-1}(G) \) is fuzzy gs-open set in \( X \), Lemma 3(4).

**Example 5.** From Example 3, Let the function \( f : X \rightarrow Y \) is defined as \( f((a,1)) = f((b,1)) = (p,1) \), \( f(c,1) = (q,1) \) and \( f(d,1) = (r,1) \). Hence, \( f \) is fuzzy gs-continuous function. However, since \( \{(c,1), (d,1)\} \) is fuzzy gs-closed set but not fuzzy srw-closed set in \( X \).

**Theorem 11.** Let a function \( f : X \rightarrow Y \) is fuzzy srw-continuous, then

1. \( f(fsrw-\text{cl}(A) \leq cl(f(A)) \) where \( A \) is any fuzzy subset of \( X \).
2. \( f(fsrw-\text{cl}(f^{-1}(B)) \leq (f^{-1}(cl(B))) \) where \( B \) is any fuzzy subset in \( X \).

Proof: Let a function \( f \) is fuzzy srw-continuous, then

1. Let \( A \) be any fuzzy subset of \( X \). Now \( cl(f(A)) \) is fuzzy closed set in \( Y \). Since \( f \) is fuzzy srw-continuous, \( f^{-1}(cl(f(A)) \) is fuzzy srw-closed set in \( X \). Hence \( f^{-1}(cl(f(A)) \leq (f^{-1}(cl(f(A)))). Hence \( f(fsrw-\text{cl}(A) \leq cl(f(A)).

2. Similar as (1) by replacing \( A \) by \( f^{-1}(B) \).

**Theorem 12.** If \( f : X \rightarrow Y \) is fuzzy almost continuous function, then \( f \) is fuzzy srw-continuous but not conversely.

Proof: Let a function \( f : X \rightarrow Y \) is fuzzy almost continuous function and \( G \) be any fuzzy open set of \( Y \). Then \( f^{-1}(G) \) is a fuzzy regular open set in \( X \). Hence \( f^{-1}(G) \) is a fuzzy srw-open set in \( X \), Lemma 3(5). Therefore \( f \) is fuzzy srw-continuous.

**Example 6.** From Example 2, \( f \) is fuzzy srw-continuous but not fuzzy almost continuous. However, since \( \{(b,1), (c,1)\} \) is fuzzy srw-closed set but not closed set in \( X \) for \( \{(q,1) \) fuzzy regular closed set in \( Y \). Hence \( f \) is not fuzzy almost continuous function.

**Remark 13.** From the above results we get the following diagram: The following diagram shows the relationships of fuzzy srw-continuous function with some other fuzzy continuous functions.
In the below figure, \( A \rightarrow B \) represents \( A \) implies \( B \) but not conversely and \( A \leftrightarrow B \) represents \( A \) and \( B \) are independent each other.

\[ \text{fuzzy continuous} \rightarrow \text{fuzzy g-continuous} \]

\[ \text{fuzzy semi-continuous} \rightarrow \text{fuzzy srw-continuous} \]

\[ \text{fuzzy srw-continuous} \rightarrow \text{fuzzy gs-continuous} \]

\[ \text{fuzzy srw-continuous} \rightarrow \text{fuzzy rw-continuous} \]

**Theorem 14.** If \( f : X \rightarrow Y \) is fuzzy srw-continuous and \( g : Y \rightarrow Z \) is \( f \)-continuous functions, then \( g \circ f : X \rightarrow Z \) is a fuzzy srw-continuous function.

Proof: Let \( G \) be fuzzy open set in \( Z \), then \( g^{-1}(G) \) is a fuzzy open set in \( Y \) as \( g \) is \( f \)-continuous. Since \( f \) is fuzzy srw-continuous, \( (f^{-1}(g^{-1}(G))) \) is a fuzzy srw-open set in \( X \). Now \( (g \circ f)^{-1}(G) = (f^{-1}(g^{-1}(G))) \) is a fuzzy srw-open set in \( X \).

**Corollary 15.** If \( f : X \rightarrow Y \) is fuzzy \( f \)-irresolute and \( g : Y \rightarrow Z \) is fuzzy srw-continuous functions, then \( g \circ f : X \rightarrow Z \) is a fuzzy srw-continuous function.

Proof: Follows from Definition 2(6), Theorem 7 and Lemma 3(2).

**Theorem 16.** If \( f : X \rightarrow Y \) and \( g : Y \rightarrow Z \) are two fuzzy srw-continuous functions and \( Y \) is a fuzzy topological space where every fuzzy srw-open subset is fuzzy open. Then \( g \circ f : X \rightarrow Z \) is a fuzzy srw-continuous function.

Proof: Let \( G \) be any fuzzy closed set in \( Z \). Since \( g \) is fuzzy srw-continuous function, \( g^{-1}(G) \) is fuzzy srw-open set in \( Y \). By hypothesis, every fuzzy srw-open set in \( Y \) is fuzzy open i.e. \( g^{-1}(G) \) is fuzzy open set in \( Y \). Since \( f \) is fuzzy srw-continuous, \( (f^{-1}(g^{-1}(G))) \) is fuzzy srw-open set in \( X \). But \( (f^{-1}(g^{-1}(G))) = ((g \circ f)^{-1}(G)) \) and hence \( (g \circ f) \) is fuzzy srw-continuous.

Now we defined a new type of fuzzy generalized functions called fuzzy srw-irresolute function and its properties have been studied.

**Definition 17.** Let \( X \) and \( Y \) are two fuzzy topological spaces. A function \( f : X \rightarrow Y \) is called fuzzy srw-irresolute, if \( f^{-1}(G) \) is fuzzy srw-open set in \( Y \) for every fuzzy srw-open (fuzzy srw-closed) set in \( X \) for every fuzzy srw-open (fuzzy srw-closed) set in \( Y \).

**Theorem 18.** Let a function \( f : X \rightarrow Y \) is fuzzy srw-irresolute, if and only if the inverse image of every fuzzy srw-open set in \( Y \) is a fuzzy srw-open set in \( X \).

Proof: Suppose a function \( f : X \rightarrow Y \) is fuzzy srw-irresolute. Let \( G \) be a fuzzy srw-open set in \( Y \), then \( 1 - G \) is fuzzy srw-closed set in \( Y \). Since \( f \) is fuzzy srw-irresolute, \( f^{-1}(1-G) \) is a fuzzy srw-closed set in \( X \). But \( f^{-1}(1-G) = 1 - f^{-1}(G) \) and hence \( f^{-1}(G) \) is a fuzzy srw-open set in \( X \). Conversely, assume that the inverse
image of every fuzzy srw-open set in Y is a fuzzy srw-open set in X. Let F be any fuzzy srw-closed set in Y, then 1-F is a fuzzy srw-open set in Y. By hypothesis, \(f^{-1}(1-F)\) is a fuzzy srw-open set in X. Hence f is a fuzzy srw-irresolute function.

**Theorem 19.** Every fuzzy srw-irresolute function is a fuzzy srw-continuous function but not conversely.

Proof: Let a function \(f : X \to Y\) is fuzzy srw-irresolute. Let G be any fuzzy srw-open set in Y, then G is a fuzzy srw-open set in Y, Lemma 3(1). Since f is fuzzy srw-irresolute, \(f^{-1}(G)\) is a fuzzy srw-open set in X. Hence f is a fuzzy srw-irresolute function.

**Example 7.** Let \(X=\{a, b, c, d\}\) and \(Y=\{p, q\}\) and fuzzy subsets are \(0_X = \{(a,0), (b,0), (c,0), (d,0)\}\), \(\alpha_1 = \{(a,1), (b,0), (c,0), (d,0)\}\), \(\alpha_2 = \{(a,0), (b,1), (c,1), (d,0)\}\), \(\alpha_3 = \{(a,1), (b,1), (c,1), (d,0)\}\), \(\alpha_X = \{(a,1), (b,1), (c,1), (d,1)\}\), \(\beta_X = \{(a,0), (b,0), (c,0), (d,0)\}\), \(\beta_1 = \{(a,1), (b,0), (c,0), (d,0)\}\), \(\beta_2 = \{(a,0), (b,1), (c,0), (d,0)\}\), \(\beta_3 = \{(a,1), (b,0), (c,1), (d,0)\}\), \(\beta_Y = \{(a,1), (b,1), (c,1), (d,1)\}\). Then fuzzy topologies of X and Y are \(\mathcal{S}_X = \{0_X, \alpha_1, \alpha_2, \alpha_3, \alpha_X\}\) and \(\mathcal{S}_Y = \{0_Y, \beta_X, \beta_1, \beta_2, \beta_3, \beta_Y\}\) respectively. Let the function \(f : X \to Y\) is defined as \(f((a,1)) = (b,1), f((b,1)) = (c,1), f(c,1) = (d,1)\) and \(f(d,1) = (a,1)\). Hence f is fuzzy srw-continuous but not fuzzy srw-irresolute. However, since \(\{(c,1), (d,1)\}\) is fuzzy srw-open set in Y but \(f^{-1}\{(a,1), (d,1)\}\) = \(\{(c,1), (d,1)\}\) which is not fuzzy srw-open set in X.

**Theorem 20.** If \(f : X \to Y\) and \(g : Y \to Z\) are two fuzzy srw-irresolute functions, then \(g \circ f : X \to Z\) is a fuzzy srw-irresolute function.

Proof: Let \(f : X \to Y\) and \(g : Y \to Z\) are two fuzzy srw-irresolute functions. Let G be any fuzzy srw-open set in Z, then \(g^{-1}(G)\) is a fuzzy srw-open set in Y. Since f is fuzzy srw-irresolute, \(f^{-1}(g^{-1}(G))\) is a fuzzy srw-open set in X, i.e. \((g \circ f)^{-1}(1-G) = f^{-1}(g^{-1}(1-G))\) is a fuzzy srw-open set in X. Hence \((g \circ f)^{-1}(1-G)\) is a fuzzy srw-irresolute function.

**Theorem 21.** If \(f : X \to Y\) and \(g : Y \to Z\) are two fuzzy functions. If f is fuzzy srw-irresolute function and g is a fuzzy srw-continuous function, then \(g \circ f : X \to Z\) is a fuzzy srw-continuous function.

Proof: Let \(f : X \to Y\) be a fuzzy srw-irresolute function and \(g : Y \to Z\) be a fuzzy srw-continuous function. Let G be any fuzzy open set in Z, then \(g^{-1}(1-G)\) is a fuzzy srw-open set in Y. Since f is a fuzzy srw-irresolute, \(f^{-1}(g^{-1}(1-G))\) is a fuzzy srw-open set in X, i.e. \(((g \circ f)^{-1}(1-G)) = f^{-1}(g^{-1}(1-G))\) is a fuzzy srw-open set in X. Hence \(g \circ f\) is a fuzzy srw-continuous function.

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