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# Topological Credibility Analysis of Digital Images-A Locally Finite Spaces Approach

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## Abstract

This article is a self-evident theory of Topological Space (TS) that are Locally Finite (LF) and the Digital Imagery (DI) through Digital Topology (DT). LF Spaces (LFS) has the advantage of computer representation. New DT axioms are suggested here deriving basic topological ideas investigating properties of LFS and bridging classical topology and computer science. In addition to the foundations, effectual algorithms solving geometrical and topological problems can be achieved using any programming languages facilitating their practical employment. ALFS based new tactic independent of Euclidean Geometry (EG) to Digital Geometry (DG). This is an anthology of the basic research and results in DT, DG and DI leading to new computer based solutions of everyday problems especially in analysis of technical and medical images.

**Key Words:**Topological spaces, euclidean geometry, connectedness, locally finite, ALF spaces, continuous spaces, connected components.

# 1. Introduction

This article grants the most important research with accomplished results in the area of DT,DG and DI. It is ardent to the theory of LFTS and its applied engineering. A LFS is a TS where all elements in it owns a neighborhood (nbd) containing countable elements are explicitly characterized in a computer. Here is an accepted approach to geometry and topology of LFS with applied engineering to DI and to other arenas.

# 2. Aims

(i) It is possible to develop a LFTS independent of the Euclidean Space (ES) well-suited for DI.
(ii) Extant counsels for evolving effectual DI algorithms using LFS and Abstract Cell Complexes (ACC).

The principal topics of the article are:

- DT – An Axiomatic Approach
- ACC – a Special Case
- Continuous LFS Mapping
- Lines and Planes that are Digital
- 3D Space Surface Theory
- Data Edifices
- nD space Border-Tracing Algorithm
- Tagging Linked Segments
- 3D space Surfaces Tracing, Encrypting and Rebuilding
- Discussion on Irrational Numbers, and Derivative's Optimal Estimates
- Resolution of Problems.

# 3. Theory & Methodology

**Locally Finite (LF) Topological Spaces (TS)**

The philosophy of LFTS aids overcoming the inconsistency among theory and applied engineering. Traditionally theory is done using ES where as applied engineering is finite discrete sets. It is not possible to explicitly characterize a small subset of ES, containing infinite number of points, in a computer. LFTS are theoretically consistent and conform to classical topology, and explicitly can be characterized in a computer.

**New Axioms**

The new DT axioms below are counseled to clarify the relation of the classical topology axioms to the DI constraints.

These axioms are associated to the ideas of the subset borders and linkage which are vital for image analysis applications, and deduction of the classical topology.

*Axiom-1*

In space S, $\forall c \in S \exists$ subsets $\supset$ *c* that are *nbd(c)*. Then$\bigcap$*nbd(c)=nbd(c)*and*c* has its Smallest Nbd*SN(c)*.

*Axiom-2*

$\exists$c $\ni$|SN(c)|> 1.

*Axiom-3*

In Fig.1(a, b), the BorderBr(T,S) of any subset T⊂S is thin.

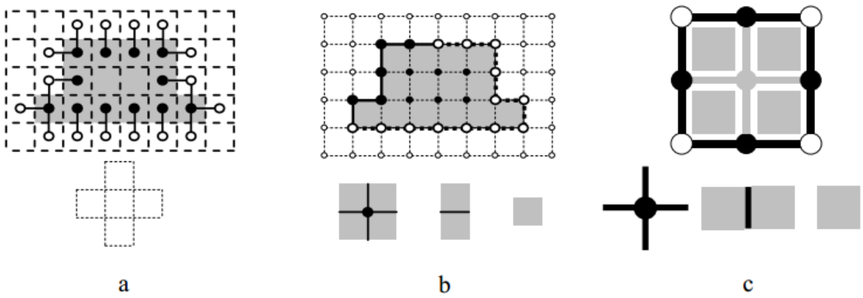


Fig. 1: Samples of Borders: (a) A Thick Border; (b) A Thin Border; (c) A Gapped Border

*Axiom-4*

The border of Br(T,S) =Br(T,S) ⇒Br(Br(T, S), S)=Br(T, S). Importantly, a border is said to be linked if there are no gaps in it. More precisely, the border of a border B is the same as B. From Fig.1c,Axiom-4 is not satisfied due to the gaps represented by white dots. From theFig.1c, a space S has dots, lines and squares. The SN(Line) ⊃incident squares, when the SN(Point) ⊃ incident lines and SN(Point) ⊅ squares. The SN(Square)=SN(Square). Hence nbdis non-transitive. The gaps are:

- The subset T is signified by grey-values and Br(T,S) consists of black dots and black lines ∉ T, and SN ∩ T.
- The white dots ∉B=Br(T,S) because their SN ~∩ T.

However, Br (B,S) ⊃the white dots since SN ∩ B and SN ∩ B\S. Thus, the borderB=Br(T,S) ≠Br(B,S).

## Definition

A LFS fulfilling all the 4 axioms is called Abstract LFS (ALFS).

## Properties of ALFS

The ALFS is a specific instance of *T0* space alone. An ACC[2]whose elements are called cells, is a specific case of ALFS if *dim(c)* assigns an integer > *0* ∀*c* ∋ if *d*∈ *SN(c)*, then *dim(c)*≤ *dim(d)*.

*Definition*

An ACC*C=(S, R, dim)* is a set *S* of abstract elements with an anti-symmetric, ir-reflexive, and transitive relation $R \subset S \times S$ is termed the *Bounding-Face-Relation (BFR)* with a dimension function *dim:* $S \rightarrow N$ from *S* into *N*, where *N*is the set of the integers that are non-negative $\exists dim(c') < dim(c'') \forall$ pairs $(c', c'') \in R$.A cell $\not\subset$ another cell, $c'<c'' \forall (c', c'') \in R$.

The BFR*B* is a poset[3] in *S* indicating the well-ordered duos of elements$(c', c'') \ni c'$ is supposed to bound-face*c''*, and is symbolized as $c' < c''$. Consider the arrangement of cells $a < b < ... < k$ of a cplx*C*, where a Bounding Path (BP),cells bounding the subsequent one, from *a* to $k \in$ C with length len(BP) = |cells in the arrangement|- 1.

*Definition*

The Dimension of a Cell (DC)$c \epsilon C$ is $dim(c,C)$=len(Max(BP($x \rightarrow c$)) where $x \epsilon C$.

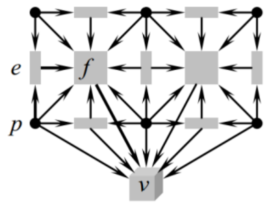


Fig. 2: A Complex with BFR Signified by Arrows

According to Definition, DC is defined relative to a SCplx$\supset c$ since the len(MaxBP) varies for SCplxs. Fig. 2 shows as ample of calibrating the DCs.

The $dim(v)= 3$ since $length(p \rightarrow e \rightarrow f \rightarrow v)= 3$. The path is signified by arrows pointing from *p* to *e* if *p* binds *e*. Using dim prevents errors when an LFS without dimensions is used.

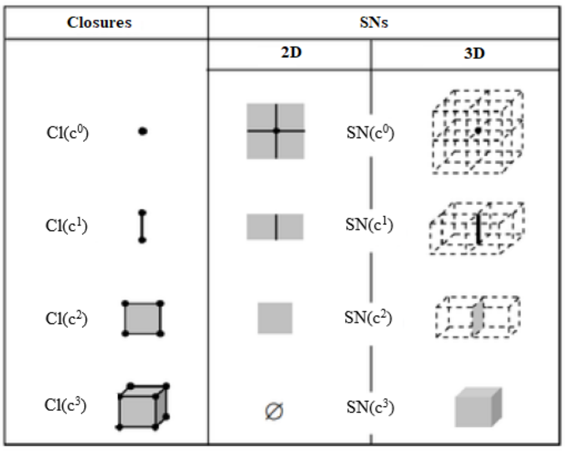


Fig. 3: Cls and SNs of Cells of Cartesian ACC

The introduction of the idea of Cartesian Product (CP) of n 1D Cplxs expressing an nD Cartesian Complex (CC) [4] gives the option to state cell coordinates as combinatorial coordinates. The SNs and the Closures (Cls) of cells ϵCCs of nD are shown in Fig.3.

# 4. Analysis & Elaboration

### Definition

Two sets are said to be homeomorphic when there is a continuous 1-1 and onto mappingamong them. This homeomorphismis applicable to Cplxs,LFSs and ACCsis called the Combinatorial Homeomorphism (CH) and is based on Cells' Elementary Subdivisions (CES) [5].

*CH of Spheres and Balls*

Originally an ACC is very generic and is possible to define a bizarre ACC with <n*0-cells*|*1-cell*>, or *2-cell* with a hole, or *3-cell* torus. Define CES on the basis of the Euclidean Topology and Complexes [5] to avoid any challenges. Since the objective is to establish a philosophy independent of ES, new descriptions based on ACC are suggested. If the concept of CH is notapplicable to any cplx, ∃ a limitation similar to classical limitation that occurs when defining Euclidean Cells as Convex Sets, which excludes the bizarrecplxs.

Try introducingCplxsClass (CxC)thatis similar to Cartesian Class (CrC) with properties:

(i) *1-cell* is bounded by *NOT* more than 2 0-cells
(ii) *2-cell* has *NO* holes
(iii)3-cell asa3D ball

Now define a Topological Ball (TB) and a Topological Sphere (TS) that are essential to define the subdivisions without defining the CxC homeomorphic to CrC is not possible.

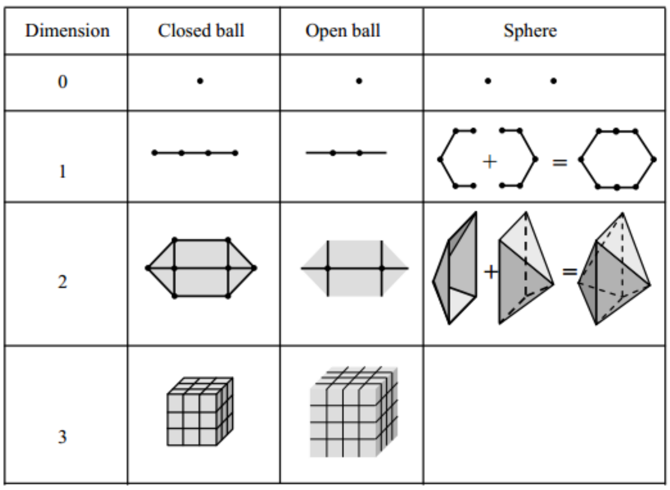


Fig. 4: Samples of CBS of 0-3 Dimensions

When presenting the definitions of Combinatorial Balls and Spheres (CBS), consider the concepts of a TB and TS to Cplxs. Present the concepts of Cplx Cells as replacement for Euclidean convex cells, which avoids the usage of bizarre Cplxs. This idea leads (Fig.4) to the definitions of CBS independent of geometry and metrics.

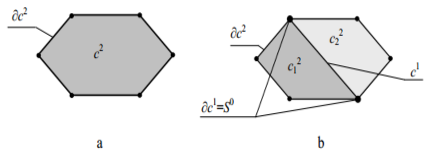

Fig. 5: A Sample of the CES of 2-cell;

(a) Original Cell; (b) Subdivision

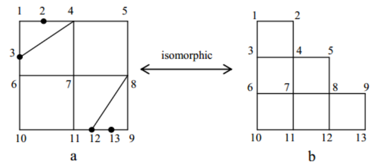


Fig. 6: (a) A CES of Square; (b) Isomorphic to Triangle; "Thick Points" are Newly Created during CES

The definitions of CBS based on CES, Fig.5, are independent of ES justifies the CH for LFS with no reference to ES.

Fig.6 demonstrates the CH of a square and a triangle.The concept of a borderis generalized while introducing the perceptions of an opening border and generalized border.Thus, a punctured 2D sphere is a manifold with an opening border, which according to the classical topology, is not at all a manifold.

## Definition

*Continuous Function (CF)* - Isomorphism is the only homeomorphism possible between 2LFS.

Classically, it is also not possible to continuously map a LFS onto a superior space with more elements. There commended mapping between the spaces X and Y is to assign each cell of X to a subset of Y instead of a single cell [6,7].

Any mapping is termed a *Connectedness Preserving Map* (CPM) if each linked subset of X is mapped to a linked subset of Y.

In covenant with classical *continuity*, a CPM is said to be continuous if the pre-image is open forevery open subset.

Consider the CPM$F$ between the Cplxs$X$ and $Y$,$F: X \rightarrow Y$. The SCplx$F(x)$ which is the image of $x \in X$ and $F\text{-}1(y)$ which is the pre-image of $y \in Y$. Let $V(x,y)$be the linked segment of $F(x) \supset$ y and $H(x,y)$ the linked segmentof $F\text{-}1(y) \supset x$.

A mapping $F$ is called *simple* if $\forall (x,y) \in F$ either $V(x,y)$ or $H(x,y)$contains>1 element. Samples of CPMs are shown in Fig.7.

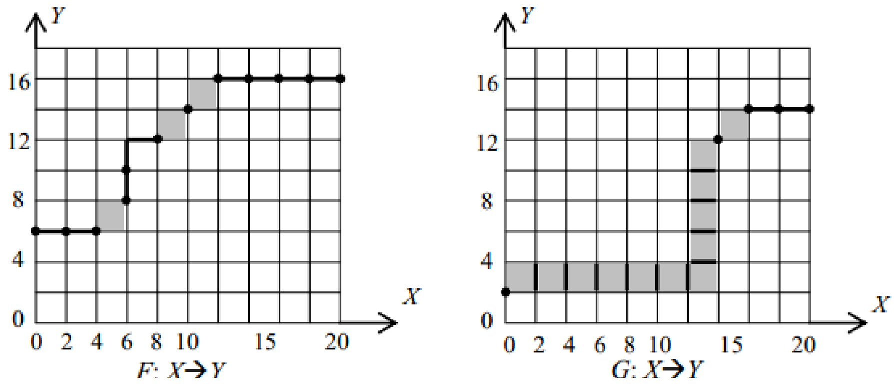


Fig. 7: Samples of CPM: F is Non-Continuously Simple; G is Continuously Non-simple

From the Definition of CH,*X~Y* distinctively postulates a continuous CPM *F: X→Y* with inverse as continuous CPM.

## DG– Digital Planes and Lines

DG introduces theory and definitions of Digital Planes (DP)and Digital Lines (DL) which are independent of ES notions⇒ a DL is not digitalizing a classical line.

*Definition*

AHalf-Plane (HP)is{emts(2D CC)}with its co-ordinates satisfying

(i) linear inequality
(ii) Digital Straight Segment(DSS) as a linked subset of the border

*Definition*

Digital curves in a 2-D space are Visual Curves (VC) and Border Curves (BC). VCs are arrangement of 2D pixels. These are well-suited for representing curves in any image. BCs are arrangement of cells of 0D and 1D. These are well-suited for image analysis. In the shy, BCs are considered than VCs.

Considering the theory of DSS as BCs, denoting this kind of DSS and DSS recognition algorithm analogous to equations and algorithms for lines with some vital variances.

An algorithm for CES of a digital border curve into lengthiest DSS, a method [1]is described which is careful and no-loss encrypting arrangements of DSSs. *No-loss* is meant as a partitioned image can be accurately rebuilt from the details of the partition borders. They are considered as borders of HP, similar to DSSs, and ⊅ voxels (volumetric pixel or 3D pixel) but ⊇ 0D to 2D cells where the theory of surfaces and arcs in3D space are defined.

### Applied Engineering of the DSSs

The following are the applied engineering of DSSs:

- Estimation of the perimeter of a 2D space subset.
- Objects representation for shape analysis of 2D images as polygons.
- Cost-effective and exhaustive image encryption.

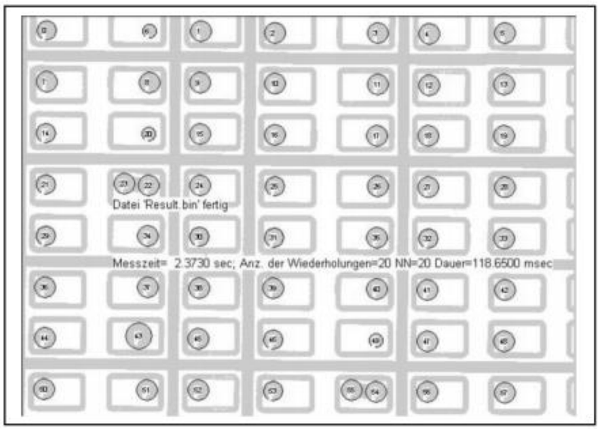


Fig. 8: Sample of Disk-shaped Objects in Wafer Image

There are multiple DSSs passing through any 2 given points, and additionally L, M,N are the integer parameters stated to discriminate the points. M/N = Slope(Base(DSS)), Base Equation (BE) is H(x, y) = 0, L =LHS(BE) at DSS starting point. A DSS can be exactly rebuilt from the Co-Ordinates(End-Points(DSS), Additional Parameters). An arrangement of DSSs can be thriftily encrypted using 2.3 byte/DSS on average (Fig.8).

# 5. Other Applied Engineering and Algorithms

It is recommended, in addition to adjacency relations, to use geometrical and topological perceptions in conjunction from the ACC or ALFS point of view. For 2D, use 4 and 8adjacencies in the 2D case, and for 3D, use 6, 18 and 26 adjacencies.

The presence of the *dim(cells)* makes the toil easier avoiding inconsistencies with a TS, and hence making ACCs better than ALFS. The *dim(cells)* are descriptive enough for developing algorithm in DI using ACC. For 2D images and 2D subspaces of nDspaces, it leads to several tracing and border encrypting algorithms. Global algorithm is one of those, for tracing borders of 2D portions in nD images, where $2 < n < 4$. Benefits of the algorithms:

(i) No changes required in nDspaces [8],
(ii) Cost-effective and no-loss image encryption
(iii)Provides full geometrical/topological information
(iv)Well-suited for the analyzing image
(v) Extracts incidence, inclusion, adjacency etc. amongst image subsets.

# 6. Discussions and Problems

This article is ardent to debate on

(i) the possibility and necessity avoiding irrational number usage

(ii) the optimal method of calibrating derivatives of functions defined with a restricted precision.

(iii) avoiding real numbers usage (e.g., $\sqrt{3}+\sqrt{5}$ is a problem statement than a result name; only countable subset of real numbers has names like e or $\pi$)

(iv) usage of rational numbers (e.g., ⅜ has result name as numerator/ denominator or decimal number; avoid zero division)

Without names, objects arithmetic cannot exist even tolerating an error, $< 10^{-10101}$, which is as small as desired. These difficulties can be overcome using a LFS as the number axis and the CPM as continuous function.

# 7. Conclusion

This article presents the 1st attempt developing an axiomatic theory of LFS, a notion of DG discrete to EG and Hausdorff Topology, and shows means to apply the theoretical results to DI and other arenas.

# References


[1]. Kovalevsky V., Geometry of Locally Finite Spaces, Publishing House Dr. Baerbel Kovalevski, Berlin (2008).

[2]. Steinitz E., Beiträgezur Analysis, Sitzungsbericht Berliner Mathematischer Gesellschaft 7 (1908), 29-49.

[3]. Birkhoff G., Lattice Theory, American Mathematical Society (1961).

[4]. Kovalevsky V., On the topology of digital spaces, Proceedings of the Seminar Digital Image Processing, Technical University of Dresden (1986), 1-16.

[5]. Stillwell J., Classical Topology and Combinatorial Group Theory, Springer (1995).

[6]. Kovalevsky V., Digital Geometry based on the Topology of Abstract Cell Complexes, Proceedings of the Third International Colloquium "Discrete Geometry for Computer Imagery". University of Strasbourg (1993), 259-284.

[7]. Kovalevsky V., A New Concept for Digital Geometry, Shape in Picture, Springer (1994), 37-51.

[8]. Kovalevsky V., Finite Topology as Applied to Image Analysis. Computer Vision, Graphics and Image Processing 45(2) (1989), 141-161.

[9]. Rajesh, M. & Gnanasekar, J.M. Wireless Pers Commun (2017). https://doi.org/10.1007/s11277-017-4565-9

[10]. Rajesh, M., and J. M. Gnanasekar. "GCCover Heterogeneous Wireless Ad hoc Networks." Journal of Chemical and Pharmaceutical Sciences (2015): 195-200.

[11]. Rajesh, M., and J. M. Gnanasekar. "Consistently neighbor detection for MANET." Communication and Electronics Systems (ICCES), International Conference on. IEEE, 2016.

[12]. Rajesh, M., and J. M. Gnanasekar. "Hop-by-hop Channel-Alert Routing to Congestion Control in Wireless Sensor Networks." Control Theory and Informatics 5.4 (2015): 1-11

[13]. Rajesh, M., and J. M. Gnanasekar. "Annoyed Realm Outlook Taxonomy Using Twin Transfer Learning" International Journal of Pure and Applied Mathematics, 116.21 (2017) 547-558.

[14]. Rajesh, M., and J. M. Gnanasekar. "Get-Up-And-Go Efficientmemetic Algorithm Based Amalgam Routing Protocol." International Journal of Pure and Applied Mathematics, 116.21 (2017)537-547.

[15]. Rajesh, M., and J. M. Gnanasekar. "Congestion Control Scheme for Heterogeneous Wireless Ad Hoc Networks Using Self-Adjust Hybrid Model." International Journal of Pure and Applied Mathematics, 116.21 (2017)537-547.