International Journal of Pure and Applied Mathematics
Volume 116 No. 17 2017, 349-353
ISSN: 1311-8080 (printed version); ISSN: 1314-3395 (on-line version)
url: http://www.ijpam.eu
Special Issue

ijpam.eu

CONVERGENCE OF DIFFERENT TYPES OF RESOLVENT

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Abstract : Convergence of sequence depends on the spectral inclusion and compactness , respectively .The concept of spectral ttheory were first given by Bailey et al.in 1993 for sequences of self-adjoint operators These results can be used to find the approximate values of different operators.

Keywords: strongly convergent, Strong and weak Operator topology

1. Definition: (Strong Operator Topology)

The topology is the weakest topology on $L(X,Y)$ such that the maps $E_x : L(X,Y) \to Y$ given by $E_x(T) = (Tx)$ are all continuous $\forall\, x \in X$, [1-11]
Where $\{x_i\}_{i=1}^{\infty}$ is a finite collection of elements of X and ϵ □□ □□□□□□□□□.

2. Definition (Weak Operator Topology)

The topology on $L(X,Y)$ is weak if
$E_{x,l} : LbecomsC$, Given by (i)
$E_{x.l}(T) = l(Tx)$□□□ □□□ □□□□□□□□□□ $\forall x \in$ (ii)
$X, l \in Y^*$
A basis at the oecomsrigin is given by sets of the form $\{s \,|s\epsilon L(x,y), |l_i(Tx_j)| < \epsilon$,[12-18]

3. Definition

Let T be some subspace of X^2.

(1) A sequence is said to be converge to T if for some $\lambda \in C \setminus R$, $(\lambda I - T_n)^{-1}$ is convergent $to\ (zI - T)^{-1}$
$\| (\lambda I - T)^{-1} f - (\lambda I - T)^{-1} f \| \to 0$ as $n \to \infty$ for any $f \in X$, denoted by $(\lambda I - T_n)^{-1}\ (\lambda I - T)^{-1}$.

(2) $\{T_n\}_{\infty}^{n=1}$ is said to be converge to T in the norm resolvent sense (briefly , NRC) if for some $\lambda \epsilon C \setminus R$, $\| (\lambda I - T_n)^{-1} - (\lambda I - T)^{-1} \| \to 0$ as $n \to \infty$, denoted by
$(\lambda I - T_n)^{-1}\ \ (\lambda I - T)^{-1}$.

Obviously, $\{T_n\}$ is SRC to T if $\{T_n\}$ is NRC to T.

Note :
The condition for convergence.
Let $\{T_n\}_{\infty}^{n=1}$ and T be subspace in X^2. If for any $(x,f)\ inT$, there is $(x_n, y_n) \in T_n$ so that $(x, f) = lim_{n\to\infty}(x_n, y_n)$, then we denote it by $T = lim_{n\to\infty} T_n$.[12-18]

Lemma

Assume that $\{T_n\}_{\infty}^{n=1}$ and T are subspaces of X^2. Then we have the conditions
1. $\{T_n\}$ converges to T;
2. $T = lim_{n\to\infty} T_n$.

Definition

A subspace $\underline{T_0}$ is said to be subspace $T\ if\ \underline{T_0} = T$.

Theorem

Let $\{T_n\}_{n=1}^{\infty}$ and T be a self-adjoint subspace in X^2. Then $\{T_n\}$ convergesto T if :
T has a subspacee T_0 such that $T_0 = lim_{n\to\infty} T_n$;
For $n \geq 1$. $D(T) \subset D(T_0)$, and $T_n - T$ is closed
$\| f_n - f \| \leq a_n (\| x \| + \| f \|), \forall\, (x, f) \in T$,
$\forall\, (x, f_n) \in T_n$
And $lim_{n\to\infty} a_n = 0$.

Proof

Since the sequence and T are subspace of X^2, $i \epsilon \rho(T) \cap \rho(T_n)$, $iI - T$ and $iI - T_n$ are bijective, and $(iI - T)^{-1}$ and $(iI - T_n)^{-1} \in B(X)$ for $n \geq 1$.[19-22]

(i) AS T_0 is a subspace in T, we have $\underline{T_0} = T$. we have $R(iI - T_0) \subset R(iI - T) \subset R(iI - T_0)$, Also $R(iI - T) = X$, denotess that $R(iI - T_0) = X$.
i.e. $R(iI - T)$ is dense in X. In order to show that for any $f \in X$,
$\| (iI - T_n)^{-1} f - (iI - T)^{-1} f \| \to 0$ as $n \to \infty$,
By known result it suffices to show that for any □ $\in R(iI - T_0)$,
$\| (iI - T_n)^{-1}$□ $- (iI - T)^{-1}$□ $\| \to 0$ as $n \to \infty$,

Fix any $\square \in R(iI - T_0)$ and set $x_n = (iI - T_n)^{-1}\square$, $x = (iI - T)^{-1}\square$.
Then $(x_n, h) \in iI - T_n$, $(x, h) \in iI - T_0$. Let $= ix - h$ Then $(x, f) \in T_0$.
By the assumption that $T_0 = lim_{n\to\infty} T_n$, there exists $(y_n, g_n) \in T_n$ such that

$$(y_n, g_n) \to (x, f) \quad \text{as } n \to \infty.$$

Set $\square_n = iy_n - g_n$. Then $(y_n, \square_n) \in iI - T_n$ and $\square_n \to h$ as $n \to \infty$, which , together with $(x_n, h) \in iI - T_n$[19-21], yields that $(x_n - y_n, h - \square_n) \in iI - T$
i.e. $x_n - y_n = (iI - T)^{-1}(h - \square_n)$.
Thus it follows from well known Theorem we get

$$\| x_n - y_n \| \le \| (iI - T_n)^{-1} \| \| h - \square_n \| \le \| h - \square_n \| \to 0 \quad \text{as n} \to \infty.$$

Hence $x_n \to x$ as $n \to \infty$ and consequently holds.
Therefore holds. This implies that $\{T_n\}$ is SRC to T.
Hence the assertion (i) is proved.
(ii) Since we know that
$(iI - T_n)^{-1} - (iI - T)^{-1} =$
$(iI - T_n)^{-1}(T_n - T)(iI - T)^{-1}$. $n \ge 1$
Fix any $\in X$, and set $x_n = (iI - T_n)^{-1}f$ and $x = (iI - T)^{-1}f$. then

$$(x_n, ix_n - f) \in T_n \quad , \qquad (x, ix - f) \in T.$$

I $(f, x_n - x) \in (iI - T_n)^{-1}(T_n - T)(iI - T)^{-1}$ .So there exist
$g_n \in X$ Such that [20-21]

$$(x, g_n) \in T_n - T, \quad (g_n, x_n - x) \in (iI - T_n)^{-1}.$$

Also $(x_n - x, i(x_n - x) - g_n o) \in T_n$, and

$$(x, ix - f + g_n) \in T_n. \qquad (1)$$

Hence we have

$$\| g_n \| = \| ix - f + g_n - (ix - f) \| \le a_n(\| x \| + \| ix - f \|).$$

In addition by weiknown theorem

$$\| x_n - x \| \le \| (iI - T_n)^{-1} \| \| g_n \| \le \| g_n \|. \qquad (2)$$

This implies $\| x_n - x \| \to 0$ as $n \to \infty$ by the assumption that

$$lim_{n\to\infty} T_n = 0.$$

Therefore, $\{T_n\}$ is SRC to T. The whole proof is complete.

Theorem
Assume that sequence Tn and T are subspace of X^2
1. Then Tn converges to T if Tns T_s. Converges to T
Proof, we may assume $Tn = T$ for all $n \ge 0$.
By known result " If T is a self-adjoint subspace in X^2, then T_∞ and T_s are self-adjoint subspaces in $T(0)^2$ and $(T(0)^\perp)^2$, respectively ".
By the use above quoted result we have T_s and $T_{n,s}$ $(n \ge 0)$ are self- subspaces in $(T)^2 = (T_n)^2$.
If $(\lambda I - T_n)^{-1}$ $(\lambda I - T)^{-1}$, Fix any $f \epsilon T(0)^\perp$ and let
$x = (\lambda I - T_s)^{-1}f$, $xn = (\lambda I - T_{n,s})^{-1}f$, $n \ge 1$.
Since $T_s \subset T$ and $T_{n,s} \subset T_n$, we get that $(\lambda I - T_s)^{-1} \subset (\lambda I - T)^{-1}$ and $(\lambda I - T_{n,s})^{-1} \subset (\lambda I - T)^{-1}$, and thus $x = (\lambda I - T)^{-1}f$, $x_n = (\lambda I - T_n)^{-1}f$, $n \ge 0$,
It follows from the assumption that Tn converges to T $\to 0$ as $n \to \infty$.
Also $(\square - \square_{\square,\square})^{-1}$ $(\square\,\square - \square_{\square})^{-1}$ for some $\square \in \square \setminus \square$. Fix any $\square \in \square$.
Then f has the decompositions: $\square = \square_1 + \square_2$, where $\square_1 = \square(0)^\perp$, $\square_2 = \square(0)$.
It follows from the equation $(\square - \square)^{-1} = (\square\,\square - \square_{\square})^{-1} \oplus (\square\,\square - \square_{\infty})^{-1}$ that
$(\square - \square)^{-1}\square = (\square\,\square - \square_{\square})^{-1}\square_1, (\square - \square_{\square})^{-1}\square = (\square - \square_{\square,\square})^{-1}\square_1$, $\square \ge 1$,
This implies that

$$\| (\square - \square_{\square})^{-1}\square - (\square - \square)^{-1}\square \| = \| (\square - \square_{\square,\square})^{-1}\square_1 - (\square\,\square - \square_{\square})^{-1}\square_1 \| \to 0 \quad \text{as } \square \to \infty.$$

Hence, $\{\square_{\square}\}$ is SRC to $\square$ and the proof is complete[5-7].
Theorem
Assume that sequenceTn and $\square$ are subspace $0\square\square^2$. Then $\square\square\square\square\square\square$ to $\square$ for any $\square$ $(\square\square - \square_{\square})^{-1}$ $(\square\square - \square)^{-1}$.
Proof
The sufficient condition follows from the following definition
Let TN and $\square$ be subspace of $\square^2$.
TN converges to $\square$ such thai

$$\| (\square - \square_{\square})^{-1}\square - (\square - \square)^{-1}\square \| \to 0 \quad \text{as } \square \to \infty$$

for any $\square\,\square\,\square$, denoted by $(\square\,\square - \square_{\square})^{-1}$ $(\square - \square)^{-1}$.
Tn converges to $\square$ and $\| (\square - \square_{\square})^{-1} - (\square - \square)^{-1} \| \to 0$ as $\square \to \infty$,
denoted by $(\square - \square_{\square})^{-1}$ $(\square - \square)^{-1}$.
Obviously, $\{\square_{\square}\}$ is SRC to $\square$ if $\{\square_{\square}\}$ is NRC to $\square$.
So we consider the Necessity.
If there exist some $\square_0\,\square\square\,\square \setminus \square$ such that

$$(\square - \square_{\square})^{-1} \qquad (\square_0\,\square - \square)^{-1}.$$

Since we know $\| (\square - \square)^{-1} \| \le |\square\square\square|^{-1}$. By using welknown Theorem () we get

$$\| (\square_0 - \square)^{-1} \| \le |\square\square\square_0|^{-1}, \qquad \| (\square_0\,\square - \square_{\square})^{-1} \| \le |\square\square\square_0|^{-1}, \quad \square \ge 1.$$

In addition it follows from well known Theorem that $(\square - \square)^{-1}$ and $(\square - \square_{\square})^{-1}$ $(\square \ge 1)$ are continous then

$$(\square - \square)^{-1} = \sum_{\square=0}^{\infty} (\square_0 - \square)^{\square}[(\square_0 - \square)^{-1}]^{\square+1},$$

$$(\square - \square_{\square})^{-1} =$$
$\sum_{\square=0}^{\infty} (\square_0 - \square)^{\square} [(\square_0 - \square_{\square})^{-1}]^{\square+1}$,
Which converge in norm in $\square(\square_0, |\square\square\ \square_0|)$.
By induction) we can easily show that for each $\square \geq 0$,
$$\| [(\square_0 - \square_{\square})^{-1}]^{\square+1} - [(\square_0 - \square)^{-1}]^{\square+1} \| \leq$$
$$\| (\square_0 - \square_{\square})^{-1} - (\square_0 - \square)^{-1} \|$$
$$\times \left(\sum \| (\square_0 - \square_{\square})^{-1} \|^{\square-\square} \| (\square_0 - \square)^{-1} \|^{\square}\right)$$
$$\leq (\square + 1)\ |\square\square\ \square_0|^{-\square} \| (\square_0\ \square - \square_{\square})^{-1} - (\square_0 - \square)^{-1} \|,$$
Combining equations
$(\square_0\ \square - \square_{\square})^{-1} \to (\square_0 - \square)^{-1}$ for each $\square \in \square(\square_0, |\square\square\ \square_0|)$.
Thus
$$\| (\square - \square_{\square})^{-1} - (\square - \square)^{-1} \| = \| \{(\square - \square_{\square})^{-1} - (\square - \square)^{-1}\}^{*} \|$$
$$= \| \{(\square - \square_{\square})^{-1} - (\square - \square)^{-1}\} \| .$$
Hence the theorem is proved

4. Conclusion

In this paper we have seen convergence of different types sub spaces through various theorems.

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