Stochastic modelling of a two phase bulk service queueing system with active Bernoulli feedback, server loss and vacation

Niranjan S.P 1, Chandrasekaran V.M 2*, Indhira K 3
Department of Mathematics, School of Advanced Sciences, VIT University, Vellore-14, Tamil Nadu, INDIA.
1niran.jayan092@gmail.com, 2vmcsn@vit.ac.in, 3kindhira@vit.ac.in

Abstract

In this paper, server provides essential two phase service. In the first essential service completion epoch, if the server got breakdown with probability ($\delta$) then the renewal of service station will be considered. On the other hand if there is no breakdown with probability ($1 - \beta$), then the server provides second essential service successively. After completion of second essential service, a batch of customers may leave from the system with probability ($1 - \beta$), or may join for additional service as feedback with probability ($\beta$). The batch of units which needs feedback joins at the head of the queue and taken for service immediately by the server. In the second essential service completion epoch, if the queue length is less than ‘$a$’ then the server leaves for vacation (secondary job) with probability ($1 - \beta$). If the queue length is still less than ‘$a$’ even after the vacation completion then the server remains idle (dormant) until the queue length reaches the value ‘$a$’. Moreover, if the queue length reaches the value ‘$a$’ after vacation then the server becomes ready to do preparatory work. Though the server completed preparatory work, service starts only if the queue length reaches the...
threshold value ‘$N$’($N > b$). For this proposed model, probability generating function of the queue size at an arbitrary time will be obtained. Various performance measures will also be derived with suitable numerical illustration.

**AMS Subject Classification:** 60K25, 90B22, 68M20

**Key Words:** server loss, Bernoulli feedback, two phase service, renewal time, bulk arrival

### 1 Introduction

Stochastic modelling of bulk arrival and batch service queueing systems are significant, as it deals with effective utilization of resources in real time systems like manufacturing industries, production line systems, communications networks, etc. Queueing system with server vacation allows server to utilize its idle time by doing some supplementary jobs of the server. Bulk arrival and batch service queueing models have been analyzed by many of the researchers. Neuts[5] classified bulk queues with Poisson input. Haridass and Arumuganathan[3] analyzed $M^X/G(a,b)/1$ queueing model with vacation interruption. They derived various performance of queueing system with real time application. Recently Niranjan and Indhira[6] reviewed classical bulk arrival and batch service queueing models. During the service completion, a batch of customer may request for additional service and joins the head of the queue is called queueing system with feedback. In many classical bulk arrival and batch service queueing models customer feedback is not taken into consideration. But $M^b/G(a,b)/1$ queueing model with customer feedback will occur in many real time situations. Choi et al.[2] analysed an M/G/1 queue with multiple types of feedback, FCFS policy and gated vacations. Badamchi Zadeh[1] derived various performance measures of batch arrival multi-phase queueing system with random feedback in service and single vacation policy. Identification of server failure and clear the server breakdown or proper maintenance of the server is called renewal time of the server. Many of the researchers have analyzed queueing system with breakdown. Wu et al.[7] analyzed an M/G/1 queue with threshold, single vacation, unreliable service station and replaceable repair facility. Only few authors have studied queueing
model with renewal or repair period of the server. In many real time situations renewal time of server made effective changes in performance measures of the system. Jeyakumar and Senthilnathan[4] derived steady state condition for $M^X/G(\alpha, \beta)/1$ queueing system with server breakdown without service interruption, multiple vacations and closedown time using supplementary variable technique.

2 Model description

In this paper dual control policy for a bulk arrival and two phase batch service queueing model with active Bernoulli feedback, server loss and vacation are considered. Arrival of customers into the system as bulk, follows Poisson distribution with rate $\lambda$. Server provides essential two phase service according to general bulk service rule introduced by Neuts[5]. Server can hold minimum of $'a'$ and maximum of $'b'$ number of customers. The server will be turned on only if the queue length is at least $'a'$. During second essential service completion if the queue length say $\zeta$, is more than $'b'$ then the server will take only $'b'$ number of customers for service, remaining $\zeta - b$ customers has to be wait in queue for next batch of service. In the first essential service completion epoch if the server got breakdown with probability $\delta$ then the renewal of service station will be considered. If the server got breakdown during first essential service then the second essential service will be continued after renewal process. On the other hand if there is no breakdown with probability $(1-\delta)$, then the server provides second essential service successively. In the time of second essential service completion a batch of customers may leave from the system with probability $(1-\beta)$, or may join for additional service as feedback with probability $\beta$. The batch of units which needs feedback joins at the head of the queue and taken for service immediately. During second essential service completion if the queue length is less than $'a'$ then the server leaves for vacation (secondary job) with probability $(1-\beta)$. Upon vacation completion the server needs to do some work before begins the service that is called preparatory work. In the vacation completion, preparatory work will be started only if the queue length reaches the value $'a'$. To provide service for long time without shutdown of the server, service will be initiated only
if the queue length reaches the threshold value ‘$N$’($N > b$) after the preparatory work. After completing a vacation if the queue length is still less than ‘$a$’ then the server remains idle (dormant) until the queue length reaches the value ‘$a$’. For the proposed model probability generating function of the queue size at an arbitrary time will be obtained by using supplementary variable technique. Various performance characteristics are also derived with suitable numerical illustration. Schematic representation of this proposed queueing system is depicted below.

**Figure 1**: Schematic representation of the model: Q-queue length.

### 3 Notations

Let $\lambda$ be the Poisson arrival rate, $X$ be the group size random variable of the arrival, $g_k$ be the Probability that $k$ customers arrive in a batch, $X(z)$ be the probability generating function (PGF) of $X$, $N_q(t)$ be the number of customers waiting for service at time $t$, $N_s(t)$ be the number of customers under the service at time $t$, $\delta$ be the breakdown probability, $\beta$ be the feedback probability. Let $S(x)(s(x))\{\hat{S}(\theta)\}[S^0(x)]$ be the cumulative distribution function (probability density function) \{Laplace-Stieltjes transform \} \{remaining primary service time\} of first essential service. Let $S_2(x)(s_2(x))\{\hat{S}_2(\theta)\}[S_2^0(x)]$ be the cumulative distribution function (probability density function) \{Laplace-Stieltjes transform\} \{remaining secondary service time\} of second essential service. Let $V(x)(v(x))\{\hat{V}(\theta)\}[V^0(x)]$ be the cumulative distribution function (probability density function) \{Laplace-Stieltjes transform\} \{remaining vacation time\} of vacation. Let $R(x)(r(x))\{\hat{R}(\theta)\}[R^0(x)]$ be the cumulative distribution
function (probability density function) \{\text{Laplace-Stieltjes transform}\}\{\text{remaining renewal time}\} of renewal period. Let \( A(x)(a(x)) \{\tilde{A}(\theta)\} [A^0(x)] \) be the cumulative distribution function (probability density function) \{\text{Laplace-Stieltjes transform}\}\{\text{remaining renewal time}\} of preparatory work.

\[
C(t) = \begin{cases} 
0, & \text{when the server is busy with first essential service} \\
1, & \text{when the server is busy with second essential service} \\
2, & \text{when the server is on vacation} \\
3, & \text{when the server is on preparatory work} \\
4, & \text{when the server is on renewal period} \\
5, & \text{when the server is on dormant period}
\end{cases}
\]

The state probabilities are defined as follows

\[
P_{ij}(x,t)dt = Pr \left\{ N_s(t) = i, N_q(t) = j, x \leq S^0(t) \leq x + dt, c(t) = 0 \right\} a \leq i \leq b, n \geq 0
\]

\[
W_{ij}(x,t)dt = Pr \left\{ N_s(t) = i, N_q(t) = j, x \leq S^0(t) \leq x + dt, c(t) = 1 \right\} a \leq i \leq b
\]

\[
Q_n(x,t)dt = Pr \left\{ N_q(t) = n, x \leq V^0(t) \leq x + dt, c(t) = 2 \right\} 0 \leq n \leq a-1
\]

\[
A_n(x,t)dt = Pr \left\{ N_q(t) = n, x \leq A^0(t) \leq x + dt, c(t) = 3 \right\} n \geq 1
\]

\[
R_n(x,t)dt = Pr \left\{ N_q(t) = n, x \leq R^0(t) \leq x + dt, c(t) = 4 \right\} n \geq 1
\]

\[
T_n(t)dt = Pr \left\{ N_q(t) = n, c(t) = 5 \right\} 0 \leq n \leq a-1
\]

4 Steady state analysis

The following steady state queue size equations are obtained by using supplementary variable technique.

\[
-\frac{d}{dx} P_{io}(x) = -\lambda P_{io}(x) + (1 - \beta) \sum_{m=a}^{b} W_{ma}(0)s(x) + \beta(W_{io}(0)s(x))
\]

\[
-\frac{d}{dx} P_{ij}(x) = -\lambda P_{ij}(x) + \sum_{k=1}^{j} P_{i,j-k}(x)\lambda g_k + \beta(W_{ij}(0)s(x))
\]

\[
-\frac{d}{dx} P_{bj}(x) = -\lambda P_{bj}(x) + (1 - \delta) \sum_{m=a}^{b} W_{m,b+j}(0)s(x) j \geq N - b
\]

\[
+ \sum_{k=1}^{j} W_{b,j-k}(x)\lambda g_k + A_{b+j}(0)s(x) + \beta(W_{bj}(0)s(x))
\]
\(-\frac{d}{dx} P_{bj}(x) = -\lambda P_{bj}(x) + (1 - \delta) \sum_{m=a}^{b} W_{m,b+j}(0) s(x) \) \hspace{1cm} (4)

\(+ \sum_{k=1}^{j} W_{b,j-k}(x) \lambda g_k + \beta(W_{bj}(0) s(x)) \quad 1 \leq j \leq N - b - 1 \)

\(-\frac{d}{dx} W_{io}(x) = -\lambda W_{io}(x) + (1 - \delta) \sum_{m=a}^{b} P_{m0}(0) s_2(x) \) \hspace{1cm} (5)

\(+ R_0(0) s_2(x) \quad a \leq i \leq b \)

\(-\frac{d}{dx} W_{ij}(x) = -\lambda W_{ij}(x) + \sum_{k=1}^{j} W_{i,j-k}(x) \lambda g_k \)

\(+ (1 - \delta) P_{ij}(0) s_2(x) \quad a \leq i \leq b - 1 \quad j \geq 1 \)

\(-\frac{d}{dx} W_{bj}(x) = -\lambda W_{bj}(0) + ((1 - \delta) P_{bj}(0) + R_{b+j}(0)) s_2(x) \quad j \geq 1 \) \hspace{1cm} (7)

\(-\frac{d}{dx} Q_0(x) = -\lambda Q_0(x) + (1 - \beta) \sum_{m=a}^{b} W_{m0}(0) v(x) \) \hspace{1cm} (8)

\(-\frac{d}{dx} Q_n(x) = -\lambda Q_n(x) + \sum_{k=1}^{n} Q_{n-k}(x) \lambda g_k + (1 - \beta) \sum_{m=a}^{b} W_{mn}(0) v(x) \quad 1 \leq n \leq a - 1 \) \hspace{1cm} (9)

\(-\frac{d}{dx} A_n(x) = -\lambda A_n(x) + \sum_{k=1}^{n} A_{n-k}(x) \lambda g_k + \sum_{m=0}^{a} -1D_m \lambda g_{i-m} a(x) \quad n \geq a \) \hspace{1cm} (10)

\(-\frac{d}{dx} R_0(x) = -\lambda R_0(x) + \delta \sum_{m=a}^{b} P_{m0}(0) r(x) \) \hspace{1cm} (11)

\(-\frac{d}{dx} R_n(x) = -\lambda R_n(x) + \delta \sum_{m=a}^{b} P_{mn}(0) r(x) + \sum_{k=1}^{n} R_{n-k}(x) \lambda g_k \quad n \geq 1 \)

\(0 = -\lambda D_0 + Q_0(0) \) \hspace{1cm} (13)

\(0 = -\lambda D_n + Q_n(0) + \sum_{k=1}^{n} D_{n-k} \lambda g_k \quad 1 \leq n \leq a - 1 \) \hspace{1cm} (14)

Let \( \tilde{P}_{in}(\theta), \tilde{W}_{in}(\theta), \tilde{Q}_{n}(\theta), \tilde{A}_{n}(\theta) \) and \( \tilde{R}_{n}(\theta) \) be the Laplace-Stieltjes transform (LST) of \( P_{in}(x), W_{in}(x), Q_{n}(x), A_{n}(x) \) and \( R_{n}(x) \) respectively. Multiplying the equations from (1) – (14) by \( e^{-\theta x} \) and integrating with respect to \( x \) with limits 0 to \( \infty \) then the following equations are obtained:
\[ \theta \tilde{P}_i(\theta) - P_i(0) = \lambda \tilde{P}_i(\theta) - (\beta W_i(0) + (1 - \beta) \sum_{m=a}^{b} W_{mi}(0)) \tilde{S}(\theta) \quad (15) \]

\[ \theta \tilde{P}_{ij}(\theta) - P_{ij}(0) = \lambda \tilde{P}_{ij}(\theta) - \beta W_{ij}(0) \tilde{S}(\theta) - \sum_{k=1}^{j} \tilde{P}_{i,j-k}(\theta) \lambda g_k \quad (16) \]

\[ \theta \tilde{P}_{b_j}(\theta) - P_{b_j}(0) = \lambda \tilde{P}_{b_j}(\theta) - \beta W_{b_j}(0) \tilde{S}(\theta) - (1 - \beta) \sum_{m=a}^{b} W_{m,b+j}(0) \tilde{S}(\theta) \]

\[- \sum_{k=1}^{j} \tilde{P}_{b,j-k}(\theta) \lambda g_k \quad 1 \leq j \leq N - b - 1 \quad (17) \]

\[ \theta \tilde{P}_{bj}(\theta) - P_{bj}(0) = \lambda \tilde{P}_{bj}(\theta) - \beta W_{bj}(0) \tilde{S}(\theta) - (1 - \beta) \sum_{m=a}^{b} W_{m,b+j}(0) \tilde{S}(\theta) \]

\[- \sum_{k=1}^{j} \tilde{P}_{b,j-k}(\theta) \lambda g_k - A_{b+j}(0) \tilde{S}(\theta) \quad j \geq N - b \quad (18) \]

\[ \theta \tilde{W}_{i0}(\theta) - W_{i0}(0) = \lambda \tilde{W}_{i0}(\theta) - (R_i(0) + (1 - \delta) \sum_{m=a}^{b} P_{mi}(0)) \tilde{S}_2(\theta) \quad (19) \]

\[ \theta \tilde{W}_{ij}(\theta) - W_{ij}(0) = \lambda \tilde{W}_{ij}(\theta) - (1 - \delta) P_{ij}(0) \tilde{S}_2(\theta) \quad a \leq i \leq b - 1 \quad (20) \]

\[ \theta \tilde{W}_{b_j}(\theta) - W_{b_j}(0) = \lambda \tilde{W}_{b_j}(\theta) - ((1 - \delta) P_{b_j}(0) - R_{b+j}(0)) \tilde{S}_2(\theta) \quad (21) \]

\[- \sum_{k=1}^{j} \tilde{W}_{b,j-k}(\theta) \lambda g_k \]

\[ \theta \tilde{Q}_0(\theta) - Q_0(0) = \lambda \tilde{Q}_0(\theta) - (1 - \beta) \sum_{m=a}^{b} W_{m0}(0) \tilde{V}(\theta) \quad (22) \]

\[ \theta \tilde{Q}_n(\theta) - Q_n(0) = \lambda \tilde{Q}_n(\theta) - (1 - \beta) \sum_{m=a}^{b} W_{mn}(0) \tilde{V}(\theta) \quad (23) \]

\[- \sum_{k=1}^{j} \tilde{Q}_{n-k}(\theta) \lambda g_k \quad 1 \leq n \leq a - 1 \]

\[ \theta \tilde{A}_n(\theta) - A_n(0) = \lambda \tilde{A}_n(\theta) - \sum_{k=1}^{j} \tilde{A}_{n-k}(\theta) \lambda g_k - (Q_n(0) + \sum_{m=0}^{a-1} D_m \lambda g_{i-m}) \tilde{A}(\theta) \quad (24) \]

\[ \theta \tilde{R}_0(\theta) - R_0(0) = \lambda \tilde{R}_0(\theta) - \delta \sum_{m=a}^{b} P_{m0}(0) \tilde{R}(\theta) \quad (25) \]

\[ \theta \tilde{R}_n(\theta) - R_n(0) = \lambda \tilde{R}_n(\theta) - \delta \sum_{m=a}^{b} P_{mn}(0) \tilde{R}(\theta) - \sum_{k=1}^{j} \tilde{R}_{n-k}(\theta) \lambda g_k \quad (26) \]
Define the probability generating function as follows:

\[ \tilde{P}_i(z, \theta) = \sum_{n=0}^{\infty} \tilde{P}_{in}(\theta) z^n \]
\[ P_i(z, 0) = \sum_{n=0}^{\infty} P_{in}(0) z^n \]
\[ \tilde{W}_i(z, \theta) = \sum_{n=1}^{\infty} \tilde{W}_{in}(\theta) z^n \]
\[ W_i(z, 0) = \sum_{n=1}^{\infty} W_{in}(0) z^n \]
\[ \tilde{R}(z, \theta) = \sum_{n=0}^{\infty} \tilde{R}_n(\theta) z^n \]
\[ R(z, 0) = \sum_{n=0}^{\infty} R_n(0) z^n \]
\[ A(z, 0) = \sum_{n=0}^{\infty} A_n(0) z^n \]

5 Probability generating function of the queue size at an arbitrary time epoch

\[ P(z) = \sum_{m=a}^{b-1} \tilde{P}_m(z, 0) + \tilde{P}_b(z, 0) + Q(z, 0) + R(z, 0) + \tilde{A}(z, 0) + \sum_{m=a}^{b-1} \tilde{W}_m(z, 0) + \tilde{W}_b(z, 0) \]

From the above equations (16) – (27), using the PGF defined above and doing some algebra, the \( P(z) \) of the proposed model is derived below.

\[
\begin{align*}
P(z) &= \frac{\left( \tilde{S}(\lambda - \lambda X(z)) - 1 \right) K_1 + \left( \tilde{A}(\lambda - \lambda X(z)) - 1 \right) K_2}{M_1(-\lambda + \lambda X(z))} \\
&\quad + \left( \tilde{V}(\lambda - \lambda X(z)) - 1 \right) (1 - \beta) \sum_{n=0}^{a-1} w_n z^n + M_2 \\
&\quad + K_1 \delta \tilde{R}(\lambda - \lambda X(z)) \left( \tilde{S}_2(\lambda - \lambda X(z)) - 1 \right) + D(Z) (-\lambda + \lambda X(z)) \\
&\quad + M_3 \delta + \delta \tilde{R}(\lambda - \lambda X(z)) \tilde{S}(\lambda - \lambda X(z)) \\
&\quad + \left( \tilde{S}(\lambda - \lambda X(z)) - 1 \right) K_3 \\
&\quad + \left( \tilde{S}_2(\lambda - \lambda X(z)) - 1 \right) \tilde{S}(\lambda - \lambda X(z))(1 - \beta)
\end{align*}
\]
where
\[ M_1 = \beta z (1 - \beta) - (1 - \beta) \tilde{S}(\lambda - \lambda X(z)) ((1 - \delta) \tilde{S}(\lambda - \lambda X(z)) + \delta \tilde{R}(\lambda - \lambda X(z))) \]
\[ M_2 = (\tilde{R}(\lambda - \lambda X(z)) - 1) \tilde{S}(\lambda - \lambda X(z)) + \tilde{R}(\lambda - \lambda X(z)) \tilde{S}(\lambda - \lambda X(z)) \tilde{S}(\lambda - \lambda X(z)) \]
\[ K_1 = \beta \tilde{S}(\lambda - \lambda X(z)) \sum_{n=a}^{b-1} d_n z^n - (1 - \beta) \sum_{n=a}^{b-1} w_n d_n = A_n + P_n + R_n \]
\[ K_2 = \tilde{V}(\lambda - \lambda X(z))(1 - \beta) \sum_{n=0}^{a-1} w_n z^n - \left( \sum_{n=0}^{a-1} q_n + \sum_{m=0}^{a-1} D_m \lambda g_{i-m} \right) \]
\[ K_3 = \tilde{\lambda}(\lambda - \lambda X(z)) K_2 + (\delta \tilde{R}(\lambda - \lambda X(z)) + (1 - \beta)) (\tilde{S}(\lambda - \lambda X(z)) \sum_{n=a}^{b-1} d_n - \sum_{n=0}^{b-1} d_n z^n) \]

5.1 Steady state condition

The probability generating function \( P(z) \) has to satisfy \( P(1) = 1 \). In order to satisfy this condition, applying L’Hôpital’s rule and evaluating the expression to 1, it is derived that, \( \rho < 1 \) is the condition to be satisfied for the existence of steady state for the model under consideration, where
\[ \rho = \frac{\lambda^2 (E(X))^2 (1 - \beta) (\lambda E(S) E(X) + (1 - \beta) E(S_2) + \delta E(R))}{b(1 - \beta)} \]

6 Performance Measures

6.1 Expected queue length

\[ E(Q) = \lim_{z \to 1} P'(z) \]
\[ E(Q) = \frac{2U_3''(U_1'' + U_2'' + U_3'') - 2U_3''(U_1'' + U_2'' + U_3'') - S_1}{24 \left( \lambda E(X) \left( (1 - \beta) b - \lambda E(S) E(X) - (1 - \delta) E(S_2) \lambda E(X) - \delta E(R) E(X) \right) \right)^2} \]

where
\[ U_1 = M_1 (N_1 + N_2 + N_3) \quad U_2 = K_3 (N_4 + N_5) \]
\[ U_3 = (\lambda + \lambda X(z)) M_1 \quad S_1 = 3U_3''(U_1' + U_2' + U_3') \]
\[ N_1 = (\tilde{S}(\lambda - \lambda X(z)) - 1) K_1 + \tilde{\lambda}(\lambda - \lambda X(z)) - 1) K_2 \]
\[ N_2 = \tilde{V}(\lambda - \lambda X(z)) - 1) (1 - \beta) \sum_{n=0}^{a-1} w_n z^n + M_2 \]
\[ N_3 = (\tilde{S}_2(\lambda - \lambda X(z)) - 1)\delta K_1 \tilde{R}(\lambda - \lambda X(z)) + T(z)big(-\lambda + \lambda X(z)) \]
\[ N_4 = \tilde{R}(\lambda - \lambda X(z))\tilde{S}(\lambda - \lambda X(z))\delta + M_2\delta + S(\lambda - \lambda X(z)) - 1) \]
\[ N_5 = (\tilde{S}_2(\lambda - \lambda X(z)) - 1)\tilde{S}(\lambda - \lambda X(z))(1 - \beta) \]

### 6.2 Expected length of idle period

Let \( \alpha_j, j = 0, 1, 2, ..., a - 1 \), is the probability that the system state visits \( j \) during an idle period. \( E(I) = \frac{1}{\lambda} \sum_{j=0}^{a-1} \alpha_j \) where \( \frac{1}{\lambda} \) is the expected staying time in the state \( j \) during an idle period.

### 6.3 Expected length of busy period

\[ E(B) = \frac{E(T)}{(1 - \beta) \sum_{i=0}^{a-1} d_i} \]

where \( E(T) = E(S) + E(W) + \delta E(R) \)

### 7 Numerical Illustration

The results obtained for the proposed model are justified numerically with the following assumptions. First essential service time distribution is 4-Erlang with parameter \( \mu \), second essential service time distribution is 2-Erlang with parameter \( \mu_1 \), Batch size distribution of the arrival is geometric with mean 2, Vacation time is exponential with parameter \( \xi \), Preparatory time is exponential with parameter \( \gamma \), Renewal time is exponential with parameter \( \eta \), minimum capacity \( a \) is 2, maximum capacity \( b \) is 4, Threshold \( N \) is 7, feedback probability is \( \beta = 0.2 \).

#### 7.1 Effects of various parameters on performance measures

Impacts of various performance measures for a fixed threshold values are presented. Table.1 shows the way in which performance measures are varying for different arrival rates with the parameters \( \xi = 10, \gamma = 5, \eta = 10, a = 2, b = 4, N = 7, \beta = 0.2 \) and \( \delta = 0.3 \), it
is clear that, if the arrival rate increases then the expected queue length, the expected busy period and the expected waiting time increases whereas the expected idle period decreases.

In table 2 for $\xi = 9$, $\gamma = 6$, $\eta = 10$, $a = 2$, $b = 4$, $N = 7$ and $\beta = 0.2$ performance measures are compared with breakdown probability. It is observed that if the probability of breakdown increases then the busy period of the server and $E(Q)$ also increases.

In table 3 for $\xi = 9$, $\gamma = 6$, $a = 2$, $b = 4$, $N = 7$, $\beta = 0.2$ and $\delta = 0.2$ performance measures are compared with renovation rate. It is observed that when the renovation rate increases then the busy period of the server and $E(Q)$ decreases.

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$E(Q)$</th>
<th>$E(B)$</th>
<th>$E(I)$</th>
<th>$E(W)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0</td>
<td>2.7531</td>
<td>4.7562</td>
<td>1.6531</td>
<td>0.3561</td>
</tr>
<tr>
<td>2.5</td>
<td>3.6214</td>
<td>5.6381</td>
<td>0.9821</td>
<td>0.5651</td>
</tr>
<tr>
<td>3.0</td>
<td>5.2976</td>
<td>6.2865</td>
<td>0.6295</td>
<td>0.7523</td>
</tr>
<tr>
<td>3.5</td>
<td>8.3262</td>
<td>8.6325</td>
<td>0.4192</td>
<td>0.6218</td>
</tr>
<tr>
<td>4.0</td>
<td>9.5763</td>
<td>9.5731</td>
<td>0.2963</td>
<td>0.4592</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\delta$</th>
<th>$E(Q)$</th>
<th>$E(B)$</th>
<th>$E(I)$</th>
<th>$E(W)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>4.5371</td>
<td>4.5236</td>
<td>1.9243</td>
<td>0.4469</td>
</tr>
<tr>
<td>0.2</td>
<td>5.2964</td>
<td>4.9134</td>
<td>1.1049</td>
<td>0.6397</td>
</tr>
<tr>
<td>0.3</td>
<td>5.9739</td>
<td>5.7792</td>
<td>0.5260</td>
<td>0.9246</td>
</tr>
<tr>
<td>0.4</td>
<td>7.3641</td>
<td>6.2384</td>
<td>0.3347</td>
<td>1.1379</td>
</tr>
<tr>
<td>0.5</td>
<td>8.4092</td>
<td>8.1347</td>
<td>0.1752</td>
<td>1.6324</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Renewal rate</th>
<th>$E(Q)$</th>
<th>$E(B)$</th>
<th>$E(I)$</th>
<th>$E(W)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>7.9326</td>
<td>4.9125</td>
<td>0.97533</td>
<td>2.5384</td>
</tr>
<tr>
<td>4</td>
<td>7.1124</td>
<td>4.5386</td>
<td>1.2342</td>
<td>2.2719</td>
</tr>
<tr>
<td>5</td>
<td>6.5476</td>
<td>4.0954</td>
<td>1.4375</td>
<td>1.9853</td>
</tr>
<tr>
<td>6</td>
<td>5.4981</td>
<td>3.2361</td>
<td>1.5783</td>
<td>1.8742</td>
</tr>
<tr>
<td>7</td>
<td>4.9731</td>
<td>2.5769</td>
<td>1.7752</td>
<td>1.7594</td>
</tr>
</tbody>
</table>
8 Conclusion

In this paper bulk arrival and two phase batch service queueing system with active Bernoulli feedback, server loss and vacation is analysed. For this proposed queueing model probability generating function of the queue size at an arbitrary time epoch is obtained by using supplementary variable technique. Various performance characteristics are also computed with suitable numerical illustration.

Acknowledgement: This work was supported by the <NBHM DAE, Government of India ><Ref.No.2/48(6)/2015/NBHM(R.P)/R&D11/14129 >
References


