

QUOTIENT-3 CORDIAL LABELING FOR PATH RELATED GRAPHS-PART-I

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Abstract

In this paper, a new type of labeling namely quotient-3 cordial labeling f is introduced. Let $G(V, E)$ be a simple graph of order p and size q . Let $f : V(G) \rightarrow Z_4 - \{0\}$ be a function. For each edge $E(G)$ define the labeling $f^* : E(G) \rightarrow Z_3$ by $f^*(uv) = \left\lceil \frac{f(u)}{f(v)} \right\rceil \pmod{3}$ where $f(u) \geq f(v)$. The function f is called quotient-3 cordial labeling of G if the number of vertices labeled with i and the number of vertices labeled with j differ by at most 1, the number of edges labeled with k and the number of edges labeled with l differ by at most 1. $1 \leq i, j \leq 3, i \neq j$ and $0 \leq k, l \leq 2, k \neq$

l. Here we proved that P_n and some path related graphs like Y -tree, $(P_n; S_2)$ and $(P_n; C_3)$ are quotient-3 cordial graphs.

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Key Words and Phrases: Path, Y -tree, quotient-3 cordial graph.

1 Introduction

Here the graphs considered are finite, simple, undirected and non trivial. Graph theory has a good development in the graph labeling and has a broad range of applications. Refer Gallian [1] for more information. The cordial labeling concept was first introduced by Cahit [2]. H - and H_2 -cordial labeling was introduced by Freeda S and Chellathurai R. S [3]. Mean Cordial Labeling was introduced by Albert William, Indra Rajasingh, and Roy S [4]. 3-product cordial labeling was introduced by Jeyanthi P and Maheswari A [5]. Quotient-3 cordial labeling was introduced by Sumathi P, Mahalakshmi A and Rathi A [6]. we follow [7] for notations and terminology. A graph G is said to be quotient-3 cordial graph if it receives quotient-3 cordial labeling. Let $v_f(i)$ denotes the number of vertices labeled with i and $e_f(k)$ denotes the number of edges labeled with k , $1 \leq i \leq 3$, $0 \leq k \leq 2$.

2 Definitions

Definition 1. A Y -tree is a graph obtained from the path P_n by joining an edge to a vertex of the path P_n adjacent to an end vertex.

Definition 2. $(P_n; S_2)$ is a graph obtained by attaching the root of a star S_2 at each vertex of a path P_n through an edge.

Definition 3. A graph which is obtained from a path P_n by joining the vertex of a cycle C_3 to every vertex of a path through an edge is denoted by $(P_n; C_3)$

3 Main Result

Definition 4. Let $G(V, E)$ be a simple graph of order p and size q . Let $f : V(G) \rightarrow Z_4 - \{0\}$ be a function. For each edge set $E(G)$ define the labeling $f^* : E(G) \rightarrow Z_3$ by $f^*(uv) = \left\lceil \frac{f(u)}{f(v)} \right\rceil \pmod{3}$ where $f(u) \geq f(v)$. The function f is called quotient-3 cordial labeling of G if the number of vertices labeled with i and the number of vertices labeled with j differ by at most 1, the number of edges labeled with k and the number of edges labeled with l differ by at most 1. $1 \leq i, j \leq 3$, $i \neq j$ and $0 \leq k, l \leq 2$, $k \neq l$.

Illustration 5. A quotient-3 cordial graph

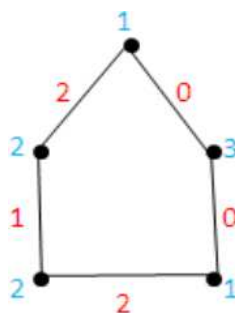


Figure 1:

Theorem 6. All paths P_n are quotient-3 cordial graph.

Proof. Let $V(G) = \{u_i : 1 \leq i \leq n\}$ and $E(G) = \{(u_i u_{i+1}) : 1 \leq i \leq n-1\}$

Here $|V(G)| = n$, $|E(G)| = n-1$.

Define $f : V(G) \rightarrow Z_4 - \{0\}$

Case (i): When $n \equiv 0, 1, 4, 5 \pmod{6}$

For i , $1 \leq i \leq n$

$f(u_i) = 1$, if $i \equiv 1, 2 \pmod{6}$

$f(u_i) = 3$, if $i \equiv 0, 3 \pmod{6}$

$f(u_i) = 2$, if $i \equiv 4, 5 \pmod{6}$

Case (ii): When $n \equiv 2 \pmod{6}$

Labeling of u_i , $1 \leq i \leq n-1$ are similar to case (i).

Assign $f(u_n) = 3$.

Case (iii): When $n \equiv 3 \pmod{6}$

Labeling of u_i , $1 \leq i \leq n-2$ are similar to case (i).

Assign $f(u_{n-1}) = 3$, $f(u_n) = 2$.

Table 1 gives the quotient-3 cordial for path P_n

Table 1:

Nature of n	$v_f(1)$	$v_f(2)$	$v_f(3)$
$n \equiv 0, 3 \pmod{6}$	$\frac{n}{3}$	$\frac{n}{3}$	$\frac{n}{3}$
$n \equiv 1, 4 \pmod{6}$	$\left(\frac{n-1}{3}\right) + 1$	$\left(\frac{n-1}{3}\right)$	$\left(\frac{n-1}{3}\right)$
$n \equiv 2 \pmod{6}$	$\left(\frac{n+1}{3}\right)$	$\left(\frac{n+1}{3}\right) - 1$	$\left(\frac{n+1}{3}\right)$
$n \equiv 5 \pmod{6}$	$\left(\frac{n-2}{3}\right) + 1$	$\left(\frac{n-2}{3}\right) + 1$	$\left(\frac{n-2}{3}\right)$

Nature of n	$e_f(0)$	$e_f(1)$	$e_f(2)$
$n \equiv 0 \pmod{6}$	$\frac{n}{3} - 1$	$\frac{n}{3}$	$\frac{n}{3}$
$n \equiv 1, 4 \pmod{6}$	$\left(\frac{n-1}{3}\right)$	$\left(\frac{n-1}{3}\right)$	$\left(\frac{n-1}{3}\right)$
$n \equiv 2 \pmod{6}$	$\left(\frac{n+1}{3}\right)$	$\left(\frac{n+1}{3}\right) - 1$	$\left(\frac{n+1}{3}\right) - 1$
$n \equiv 3 \pmod{6}$	$\frac{n}{3}$	$\frac{n}{3} - 1$	$\frac{n}{3}$
$n \equiv 5 \pmod{6}$	$\left(\frac{n-2}{3}\right)$	$\left(\frac{n-2}{3}\right) + 1$	$\left(\frac{n-2}{3}\right)$

□

Theorem 7. *The Y-tree is quotient-3 cordial for $n > 2$.*

Proof. Let $V(G) = \{u, u_i : 1 \leq i \leq n\}$ and $E(G) = \{(u_i u_{i+1}), (u u_{n-1}) : 1 \leq i \leq n-1\}$

Here $|V(G)| = n+1$, $|E(G)| = n$.

Define $f : V(G) \rightarrow Z_4 - \{0\}$

When $n = 3$, define $f(u_1) = f(u_2) = 1$, $f(u_3) = 3$ and $f(u) = 2$.

Clearly this gives the quotient-3 cordial.

When $n > 3$

In this, when $n \equiv 1 \pmod{6}$, $n \equiv 2 \pmod{6}$, $n \equiv 3 \pmod{6}$, $n \equiv 4 \pmod{6}$ and $n \equiv 5 \pmod{6}$

Labeling of u_i , $1 \leq i \leq n$ are similar to the respective cases as in theorem 6

When $n \equiv 0 \pmod{6}$

For i , $1 \leq i \leq n$

$f(u_1) = 1$

$f(u_i) = 3$, if $i \equiv 2, 5 \pmod{6}$

$f(u_i) = 2$, if $i \equiv 3, 4 \pmod{6}$

$f(u_i) = 1$, if $i \equiv 0, 1 \pmod{6}$

Labeling of the vertex u is given below.

When $n \equiv 0, 1, 3, 5 \pmod{6}$, $f(u) = 3$

When $n \equiv 2, 4 \pmod{6}$, $f(u) = 2$.

Table 2 gives the quotient-3 cordial graph for Y -tree for $n > 3$

Table 2:

Nature of n	$v_f(1)$	$v_f(2)$	$v_f(3)$
$n \equiv 0, 3 \pmod{6}$	$\frac{n}{3}$	$\frac{n}{3}$	$\frac{n}{3} + 1$
$n \equiv 1 \pmod{6}$	$\left(\frac{n+2}{3}\right)$	$\left(\frac{n+2}{3}\right) - 1$	$\left(\frac{n+2}{3}\right)$
$n \equiv 2, 5 \pmod{6}$	$\left(\frac{n+1}{3}\right)$	$\left(\frac{n+1}{3}\right)$	$\left(\frac{n+1}{3}\right)$
$n \equiv 4 \pmod{6}$	$\left(\frac{n-1}{3}\right) + 1$	$\left(\frac{n-1}{3}\right) + 1$	$\left(\frac{n-1}{3}\right)$

Nature of n	$e_f(0)$	$e_f(1)$	$e_f(2)$
$n \equiv 0, 3 \pmod{6}$	$\frac{n}{3}$	$\frac{n}{3}$	$\frac{n}{3}$
$n \equiv 1 \pmod{6}$	$\left(\frac{n+2}{3}\right) - 1$	$\left(\frac{n+2}{3}\right)$	$\left(\frac{n+2}{3}\right) - 1$
$n \equiv 2 \pmod{6}$	$\left(\frac{n+1}{3}\right)$	$\left(\frac{n+1}{3}\right) - 1$	$\left(\frac{n+1}{3}\right)$
$n \equiv 4 \pmod{6}$	$\left(\frac{n-1}{3}\right)$	$\left(\frac{n-1}{3}\right)$	$\left(\frac{n-1}{3}\right) + 1$
$n \equiv 5 \pmod{6}$	$\left(\frac{n+1}{3}\right) - 1$	$\left(\frac{n+1}{3}\right)$	$\left(\frac{n+1}{3}\right)$

□

Theorem 8. *The graph $(P_n; S_2)$ is quotient-3 cordial.*

Proof. Let $V(G) = \{u_i, v_i, w_{ij} : 1 \leq i \leq n, 1 \leq j \leq 2\}$

$E(G) = \{[(u_i u_{i+1}) : 1 \leq i \leq n-1] \cup [(u_i v_i) : 1 \leq i \leq n] \cup [(v_i w_{ij}) : 1 \leq i \leq n, 1 \leq j \leq 2]\}$

Here $|V(G)| = 4n$, $|E(G)| = 4n - 1$.

Define $f : V(G) \rightarrow Z_4 - \{0\}$ by

$f(u_i) = 2$, $1 \leq i \leq n$

$f(v_i) = 1$, $1 \leq i \leq n$

Labeling of w_{ij} , $1 \leq i \leq n$, $1 \leq j \leq 2$ is given below.

Case (i): When $n \equiv 0 \pmod{3}$

Without loss of generality, first set of $\left(\frac{n}{3}\right)$ vertices by the label 1, next set of $\left(\frac{n}{3}\right)$ vertices by the label 2 and the remaining set of $\left(\frac{4n}{3}\right)$ vertices by the label 3.

Case (ii): When $n \equiv 1 \pmod{3}$

Without loss of generality, first set of $\left(\frac{n+2}{3}\right)$ vertices by the label 1, next set of $\left(\frac{n-1}{3}\right)$ vertices by the label 2 and the remaining set of $\left(\frac{4n-1}{3}\right)$ vertices by the label 3.

Case (iii): When $n \equiv 2 \pmod{3}$

Without loss of generality, first set of $\left(\frac{n+1}{3}\right)$ vertices by the label 1, next set of $\left(\frac{n-2}{3}\right)$ vertices by the label 2 and the remaining set of $\left(\frac{4n+1}{3}\right)$ vertices by the label 3.

The following table 3 gives that the graph $(P_n; S_2)$ is quotient-3 cordial.

Table 3:

Nature of n	$v_f(1)$	$v_f(2)$	$v_f(3)$
$n \equiv 0 \pmod{3}$	$\frac{4n}{3}$	$\frac{4n}{3}$	$\frac{4n}{3}$
$n \equiv 1 \pmod{3}$	$\frac{4n-1}{3} + 1$	$\frac{4n-1}{3}$	$\frac{4n-1}{3}$
$n \equiv 2 \pmod{3}$	$\frac{4n+1}{3}$	$\frac{4n+1}{3} - 1$	$\frac{4n+1}{3}$

Nature of n	$e_f(0)$	$e_f(1)$	$e_f(2)$
$n \equiv 0 \pmod{3}$	$\frac{4n}{3}$	$\frac{4n}{3} - 1$	$\frac{4n}{3}$
$n \equiv 1 \pmod{3}$	$\frac{4n-1}{3}$	$\frac{4n-1}{3}$	$\frac{4n-1}{3}$
$n \equiv 2 \pmod{3}$	$\frac{4n+1}{3}$	$\frac{4n+1}{3} - 1$	$\frac{4n+1}{3} - 1$

□

Theorem 9. The graph $(P_n; C_3)$ is quotient-3 cordial.

Proof. Let $V(G) = \{u_i, v_i, w_{ij} : 1 \leq i \leq n, 1 \leq j \leq 2\}$
 $E(G) = \{[(u_i u_{i+1}) : 1 \leq i \leq n-1] \cup [(u_i v_i) : 1 \leq i \leq n] \cup [(v_i w_{ij}) : 1 \leq i \leq n, 1 \leq j \leq 2] \cup [(w_{ij} w_{ij+1}) : 1 \leq i \leq n, j = 1]\}$

Here $|V(G)| = 4n$, $|E(G)| = 5n - 1$.

Define $f : V(G) \rightarrow Z_4 - \{0\}$

Case (i): When $n \equiv 0 \pmod{3}$

$f(u_i) = 2, 1 \leq i \leq n$

$f(v_i) = 1, 1 \leq i \leq n$

$f(w_{ij}) = 1, 1 \leq i \leq \frac{n}{3}, j = 1$

$f(w_{ij}) = 3, 1 \leq i \leq \frac{n}{3}, j = 2$

$f(w_{ij}) = 3, \frac{n}{3} + 1 \leq i \leq \frac{2n}{3}, 1 \leq j \leq 2$

$f(w_{ij}) = 3, \frac{2n}{3} + 1 \leq i \leq n, j = 1$

$f(w_{ij}) = 2, \frac{2n}{3} + 1 \leq i \leq n, j = 2$

Case (ii): When $n \equiv 1 \pmod{3}$

$$f(u_i) = 2, 1 \leq i \leq n$$

$$f(v_i) = 1, 1 \leq i \leq n$$

$$f(w_{ij}) = 1, 1 \leq i \leq \frac{n-1}{3}, j = 1$$

$$f(w_{ij}) = 3, 1 \leq i \leq \frac{n-1}{3}, j = 2$$

$$f(w_{ij}) = 3, \frac{n+2}{3} \leq i \leq \frac{2n+1}{3}, 1 \leq j \leq 2$$

$$f(w_{ij}) = 3, \frac{2n+4}{3} \leq i \leq n, j = 1$$

$$f(w_{ij}) = 2, \frac{2n+4}{3} \leq i \leq n, j = 2$$

Case (iii): When $n \equiv 2 \pmod{3}$

$$f(u_i) = 2, 1 \leq i \leq n$$

$$f(v_i) = 1, 1 \leq i \leq n-1$$

$$f(v_i) = 2, i = n$$

$$\text{Assume } f(w_{11}) = f(w_{n1}) = 1$$

$$f(w_{12}) = f(w_{n2}) = 3$$

$$f(w_{ij}) = 1, 2 \leq i \leq \frac{n+1}{3}, j = 1$$

$$f(w_{ij}) = 3, 2 \leq i \leq \frac{n+1}{3}, j = 2$$

$$f(w_{ij}) = 3, \frac{n+4}{3} \leq i \leq \frac{2n-1}{3}, 1 \leq j \leq 2$$

$$f(w_{ij}) = 3, \frac{2n+2}{3} \leq i \leq n-1, j = 1$$

$$f(w_{ij}) = 2, \frac{2n+2}{3} \leq i \leq n-1, j = 2$$

The following table 4 gives the quotient-3 cordial for $(P_n; C_3)$

Table 4:

Nature of n	$v_f(1)$	$v_f(2)$	$v_f(3)$
$n \equiv 0 \pmod{3}$	$\frac{4n}{3}$	$\frac{4n}{3}$	$\frac{4n}{3}$
$n \equiv 1 \pmod{3}$	$\frac{4n-1}{3}$	$\frac{4n-1}{3}$	$\frac{4n-1}{3} + 1$
$n \equiv 2 \pmod{3}$	$\frac{4n+1}{3}$	$\frac{4n+1}{3}$	$\frac{4n+1}{3} - 1$

Nature of n	$e_f(0)$	$e_f(1)$	$e_f(2)$
$n \equiv 0 \pmod{3}$	$\frac{5n}{3}$	$\frac{5n}{3} - 1$	$\frac{5n}{3}$
$n \equiv 1 \pmod{3}$	$\frac{5n+1}{3}$	$\frac{5n+1}{3} - 1$	$\frac{5n+1}{3} - 1$
$n \equiv 2 \pmod{3}$	$\frac{5n-1}{3}$	$\frac{5n-1}{3}$	$\frac{5n-1}{3}$

□

Illustration 10. A quotient-3 cordial labeling of $(P_7; C_3)$ is given in figure 2

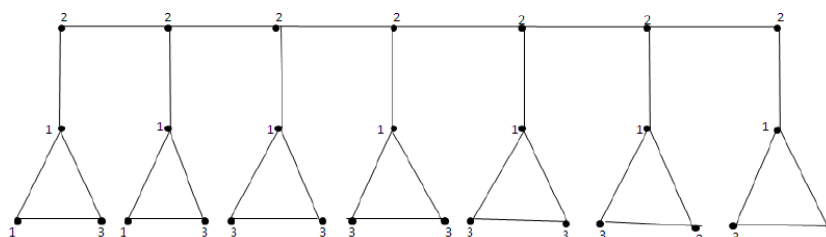


Figure 2:

4 Conclusion

It is very interesting to find out graphs or graph families which are quotient-3 cordial. In this paper, a quotient-3 cordial for some path related graphs has been found. The quotient-3 cordial labeling of some more graphs and graph families shall be explored further in future.

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