

SPECIAL CLASSES OF REGULAR SEMIGROUPS

Julie Thomas¹, K.Indhira² and V.M. Chandrasekaran³

Department of Mathematics,

School of Advanced Sciences,

VIT University, Vellore-632 014, INDIA

¹julie.thomas@vit.ac.in, ²kindhira@vit.ac.in, ³vmcsn@vit.ac.in

³Corresponding author

Abstract

Let T be a regular semigroup. An inverse transversal of T is an inverse subsemigroup T^0 that contains exactly a unique inverse for every element $a \in T$. We denote the unique element of $V(a) \cap T^0$ by a^0 , where $V(a)$ denotes the set of all inverses of $a \in T$. In 1982, Blyth and McFadden initiated the concept of inverse transversals. In the last forty years many papers have been devoted to the study of describing regular semigroups with inverse transversals. This notion has come from a paper by McAlister and Blyth (1978) on orthodox semigroups. Many authors have introduced various types of regular semigroups with inverse transversals. In particular an inverse transversal T^0 of T is (1) multiplicative if $a^0abb^0 \in E(T^0)$, (2) weakly multiplicative if $(a^0abb^0)^0 \in E(T^0)$, (3) a quasi ideal if $T^0TT^0 \subseteq T^0$. Rong Hua Zhang and Shou Feng Wang have introduced the concept of regular semigroups with generalized inverse transversals. Chen Jianfei has introduced the concept of orthodox transversals as a generalization of inverse transversals. Chen Jianfei and GuoYuqi discussed the general case of orthodox transversals and obtained some properties of orthodox transversals. In many papers, Xiangjun Kong et.al has discussed regular semigroups with various classes of or-

thodox transversals. In this article we survey what we know about these types of semigroups.

AMS Subject Classification: 20M17.

Key Words and Phrases: Inverse transversal, orthodox transversal, regular $*$ -transversal, split map, regular idempotent.

1 Regular Semigroups with Inverse Transversals

Many papers have been devoted to the theme of describing inverse transversals of regular semigroups from 1978 to till date. Many authors have investigated some important types of regular semigroups with inverse transversals (see the papers and references of McAlister and McFadden (1983), Saito (1984-1989), Blyth (2000), Blyth and Santos (2001, 2008, 2009), Tang (1997, 2010), Zhu (2006)). In particular an inverse transversal T^0 of T is said to be

- (1) multiplicative if $a^0abb^0 \in E(T^0)$ by Blyth and McFadden(1982).
- (2) a quasi ideal if $T^0TT^0 \subseteq T^0$ by McAlister and McFadden(1983).
- (3) weakly multiplicative if $(a^0abb^0)^0 \in E(T^0)$ by Saito(1985).

We can characterize the above concepts in terms of the set product of two sets I and Λ where $I = \{a \in T | a = aa^0\}$ and $\Lambda = \{a \in T | a = a^0a\}$. Note that the above conditions are naturally related.

Theorem 1. *Let T^0 be an inverse transversal of T . Then T^0 is multiplicative if and only if T^0 is weakly multiplicative and a quasi-ideal.*

Saito (1985 and 1989) obtained a general structure theorem for regular semigroups with inverse transversals and structure theorems on regular semigroups with a quasi-ideal inverse transversal. Saito (1986) proved that the quasi-ideal inverse transversals of a regular semigroup T are mutually isomorphic and product of any two quasi-ideal inverse transversals of T is a quasi-ideal inverse transversal of T . Also, if T has a multiplicative inverse transversal, then any quasi-ideal inverse transversal of T is multiplicative inverse transversal. Rajan (1988), Blyth (2000), Blyth and Santos (2001, 2008, 2009), Xilin Tang (1997, 2010), Fenglin Zhu (2006), Junwei Shi and Zhanji (2012), and Pastijn and Al-

bert (2015) have obtained various structure of regular semigroups having other types of inverse transversals. All are based on the coordinatisation $a \equiv (aa^0, a^{00}, a^0a)$ and algebraically we can assert that in general T is isomorphic to the set $W = \{(i, a, l) \in I \times T^0 \times \Lambda; i \mathcal{L} a^{-1} \mathcal{R} l\}$ with usual multiplication $(i, a, l)(j, b, m) = (ialj(alj)^0, a(lj)^{00}b, (ljb)^0ljb m)$. Based on the above definition, we can prove that

Theorem 2. *T is a completely simple semigroup if and only if T^0 is a group.*

Recently in 2017, Blyth and Almeida Santos extended the concept of inverse transversal by taking regular semigroups with a special subgroup. They obtained some important characterisations of weakly prime special subgroups and quasi-ideal special subgroups.

2 Regular Semigroups with Orthodox Transversal

In 1999, Chen Jianfei introduced the concept of orthodox transversals which is a generalization of inverse transversals. Let T be a semigroup and T^0 a subsemigroup of T . We call T^0 an orthodox transversal of T if the following conditions are satisfied:

- (1) $(\forall x \in S) V(x) \cap T^0 \neq \emptyset$.
- (2) If $x, y \in T$ and $\{x, y\} \cap T^0 \neq \emptyset$, then $(V(x) \cap T^0)(V(y) \cap T^0) \subseteq V(yx) \cap T^0$.

Note that T is regular by the condition (1) and T^0 is orthodox by the condition (2). Hence the class of regular semigroups with orthodox transversals includes both the class of orthodox semigroups and the class of inverse transversals of a regular semigroup. Structure theorem for regular semigroups with quasi-ideal orthodox transversals was obtained by Chen Jianfei (1999). In 2001, Chen Jianfei and GuoYuqi described the general case of orthodox transversals and obtained some properties of orthodox transversals. The important results are on the sets $I = \{aa^0 : a \in T, a^0 \in V(a) \cap T^0\}$ and $\Lambda = \{a^0a : a \in T, a^0 \in V(a) \cap T^0\}$, two components of regular semigroups with orthodox transversals. Suppose T^0 is an inverse transversal of T then I and Λ are subbands of T . This result does not hold if T^0 is an orthodox transversal of T . But we have:

Theorem 3. *Let T^0 be an orthodox transversal of T . Then the semibands $\langle I \rangle$ and $\langle \Lambda \rangle$ generated by I and Λ are subbands of the regular semigroup T .*

Later, Chen and Guo have proved that I and Λ are subbands of T if T^0 is a quasi-ideal orthodox transversal of T . From 2003 to 2014, Xiangjun Kong et.al has investigated many classes of orthodox transversals. In 2007, Xiangjun Kong described quasi-ideal orthodox transversals of regular semigroups by a simpler format. In 2009, Xiangjun Kong and Xianzhong Zhao constructed structure theorem for quasi-ideal orthodox transversals of regular semigroups by two orthodox semigroups. Similar to the inverse transversal, the notion of a multiplicative orthodox transversal was introduced by Pei Wang and Xiangjun Kong (2010). Blyth and McFadden have proved that $E(T) = \{a \in T : a^0 \in E(T^0)\}$ in a regular semigroup T with a multiplicative inverse transversal T^0 is the idempotent-generated regular subsemigroup of T . In case of orthodox transversal, we have the following result:

Theorem 4. *Let T^0 be a multiplicative orthodox transversal of a regular semigroup T . Then $E(T) = \{a \in T : V_{T^0}(a) \subseteq E(T^0)\}$ where $V_{T^0}(a)$ is the intersection of $V(a)$ and T^0 .*

In 2011, Xiangjun Kong introduced the concept of generalized orthodox transversals and obtained some characterizations of generalized orthodox transversals. In 2012, Indhira and Chandrasekaran have investigated another generalization for quasi-ideal orthodox transversal of a regular semigroup. Further they studied the class of regular semigroups with T^0 -orthodox transversals. In particular, if T^0 is an T^0 -orthodox transversal of T , then the semibands $\langle I \rangle$ and $\langle \Lambda \rangle$ generated by I and Λ respectively are rectangular subbands of T and they constructed a structure theorem for T^0 -orthodox transversals of a regular semigroup.

3 Regular Semigroups with Generalized Inverse Transversals

Let T be a regular semigroup and T^0 be an inverse subsemigroup of T . Then we call T^0 a generalized inverse transversal of T if for ev-

ery a in T , $V(a) \cap T^0 \neq \phi$. Clearly, an inverse transversal T^0 of T is a generalized inverse transversal, but the converse is not true, even if T^0 is a quasi-ideal of T . Rong Hua Zhang and Shou Feng Wang (2008) have introduced the concept of regular semigroups with generalized inverse transversals. It is easier to see that a multiplicative generalized inverse transversal of T is an inverse transversal of T . They have derived various characterizations of generalized inverse transversal and constructed a structure theorem for generalized inverse transversals of regular semigroups.

4 Regular Semigroups with Regular $*$ -Transversals

Let T be a regular semigroup. We call T a regular $*$ -semigroup if there is a unary operation $*$ which satisfies the following conditions:
 (1) for every a in T , $aa^*a = a$, (2) $(a^*)^* = a$ for all $a \in T$ and
 (3) $(ab)^* = b^*a^*$ for all $a, b \in T$.

In 1978, Nordahl and Scheiblich introduced and studied the concept of a regular $*$ -semigroup. Again, in 1978 Reilly studied a special regular $*$ -semigroup. Imaoka (1980, 1983) investigated the representation of the fundamental regular $*$ -semigroups and studied the properties of $*$ -congruence on a regular semigroup. In 1981, Yamada characterized the structure of fundamental regular $*$ -semigroup.

Let T be a regular semigroup and T^* a subsemigroup of T . If there exists a unary operation $*$ on T such that

- (1) for all a in T , $a^* \in T^* \cap V(a)$, (2) $(a^*)^* = a$ for all $a \in T^*$ and
 (3) $(a^*b)^* = b^*a^{**}$ and $(ab^*)^* = b^{**}a^*$, for all $a, b \in T$.

where $a^{**} = (a^*)^*$ for all $a \in T$, then we call T^* a regular $*$ -transversal of T . It is easy to see that T^* is a regular $*$ -subsemigroup. Further an inverse transversal of T is a regular $*$ -transversal. If we replace (3) by $(ab)^* = b^*a^*$ for all $a, b \in T$, then T^* is called a strongly regular $*$ -transversal of T . Li (2002, 2003) studied the class of regular semigroups with a strongly regular $*$ -transversal.

Theorem 5. *Let T^* be a strongly regular $*$ -transversal of a regular semigroup T . Then $a \in E(T)$ if and only if $a^* \in E(T^*)$. Therefore, T is an orthodox semigroup if and only if T^* is an orthodox semigroup.*

In 2015, Shoufeng Wang obtained the structure of fundamental regular semigroups with quasi-ideal regular $*$ -transversals.

5 Regular Semigroups with a Split Map

In 1992, Loganathan and Chandrasekaran introduced the concept of a split map and obtained the various properties of regular semigroups with a split map. They proved that if S is a regular subsemigroup of a regular semigroup T , and a split map $\theta : T \rightarrow S$ exists, then T can be written in terms of the regular subsemigroup S . Let T be a regular semigroup and let S be a regular subsemigroup of T . A map $\theta : T \rightarrow S$ is called a split map if it satisfies the following conditions: (1) $a\theta = a$ for all $a \in S$, (2) $V_S(a\theta) \subseteq V(a)$ for all $a \in T$, (3) $(ab)\theta = a\theta \circ (a^*abb^*)\theta \circ (b\theta)$ for all $a, b \in T$, $a^* \in V_S(a\theta)$, $b^* \in V_S(b\theta)$. Here, for any $a \in T$, $V_S(a)$ (resp. $V(a)$) denotes the set of all inverses of a in S (resp. in T). Since for every $a \in T$, let a^0 be the unique inverse of a in S and $a^{00} = (a^0)^0$, we can prove that $\theta^0 : T \rightarrow S$ by $a\theta^0 = a^{00}$ for all $a \in T$ is a split map.

Theorem 6. *The split map $\theta^0 : T \rightarrow S$ from T to S preserves Green's relations.*

Theorem 7. *Suppose S is a quasi ideal inverse transversal of a regular semigroup T , then T can be embedded as an ideal in a locally inverse semigroup V which contains an idempotent h with $fh \mathcal{R} f \mathcal{L} hf$ for every $f \in S$.*

6 Regular Semigroups with a Regular Idempotent

McAlister and McFadden (1983) have proved that a regular semigroup T contains a quasi ideal inverse transversal if and only if T can be embedded as an ideal in a locally inverse semigroup S which contains an idempotent u with $fu \mathcal{R} f \mathcal{L} uf$ for each $f = f^2$ in T . This u is a maximum idempotent of T and it can be characterized as an idempotent with the property that $f = fuf$ for which $f \in \langle E(T) \rangle$, the idempotent generated part of T . Any idempotent satisfying this property is called medial normal idempotent.

Blyth and McFadden (1981), proved a structure theorem for all regular semigroups having a maximum idempotent with some natural order. An idempotent u of T is called medial idempotent if $x = xux$ for every x in $\langle E(T) \rangle$. Loganathan (1987) characterized the structure of regular semigroup with a medial idempotent.

Indhira and Chandrasekaran (2011) introduced the concept of regular idempotent. An idempotent u of T is called a regular idempotent if $fu \mathcal{R} f \mathcal{L} uf$ for each $f \in E(T)$. If u is a regular idempotent of T then T can be described in terms of $T^0 = uTu$ which is a regular subsemigroup of T . The class of regular semigroups with a regular idempotent includes the following classes of regular semigroups: regular semigroups with (1) a medial idempotent, (2) a midunit and (3) a weak middle unit. Let u be a regular idempotent of a regular semigroup T . Set $I = \{i \in E(T) : uiu \in E(uTu) \text{ and } i \mathcal{L} uiu\}$ and $\Lambda = \{\lambda \in E(T) : u\lambda u \in E(uTu) \text{ and } \lambda \mathcal{R} u\lambda u\}$. It is easy to prove that $I = E(Tu)$, $\Lambda = E(uT)$ and $E(uTu) \subset I \cap \Lambda$. These two sets I and Λ play an important role in the structure of regular semigroups with a regular idempotent. The main theorem characterizes the structure of regular semigroups with regular idempotent. This theorem generalizes the structure theorem of regular semigroups with a medial idempotent.

Theorem 8. *Let T be a regular semigroup. Then the following are equivalent:*

- (i) u is a regular idempotent of T .
- (ii) $V(f) \cap uTu \neq \emptyset$ for any $f \in E(T)$.
- (iii) $V(a) \cap uTu \neq \emptyset$ for any $a \in T$.

Now let us see the relation between an inverse transversal and a regular idempotent of a regular semigroup.

Theorem 9. *Suppose T is a locally inverse semigroup, then an idempotent u of T is regular idempotent if and only if uTu is an inverse transversal of T .*

Since uTu is also a quasi-ideal of T , as a corollary we have,

Corollary 10. *Let T be a regular semigroup. Then T contains a quasi ideal inverse transversal if and only if T can be embedded as an ideal in a locally inverse semigroup which contains a regular idempotent.*

7 Other Classes of Regular Semigroups

An inverse transversal T^0 of a regular semigroup T is said to be
 (i) E -inverse transversal if $a^0abb^0 \in E(T)$ for all $a, b \in T$, by Fenglin Zhu and Weijiang Liu in 2001. (ii) strongly E -inverse transversal if $a^0abb^0 \in E(T)$, $aa^0b^0b \in E(T)$ for all $a, b \in T$, by Yuxiao Gao and Yingchun Wang in 2012. Gao and Wang have proved the following theorems.

Theorem 11. *Let T be a regular semigroup with an E -inverse transversal T^0 . Then*

- (i) $e^0 \in E(T^0)$ for any $e \in E(T)$;
- (ii) T^0 is a weakly multiplicative inverse transversal of T .

Theorem 12. *Let T^0 be an inverse transversal of a regular semigroup T . Then the following statements are equivalent:*

- (i) T is an orthodox semigroup;
- (ii) T^0 is a strongly E -inverse transversal of T .

As a generalization, one can define E -orthodox transversal, strongly E -orthodox transversal and discuss the natures of the regular semigroup with E -orthodox transversals and strongly E -orthodox transversals.

References

- [1] M.H. Almeida Santos and T.S. Blyth, Naturally ordered regular semigroups with inverse monoid transversal, *Semigroup Forum*, **76** (2008), 71-86.
- [2] T.S. Blyth, Inverse transversals- a guided tour, Semigroups (Braga, 1999), *World Sci. Publ., River Edge, N.* (2000), 26-43.
- [3] T.S. Blyth and M.H. Almeida Santos, Special subgroups of regular semigroups, *Comm. Algebra*, **45**, No.10(2017), 4246-4256.
- [4] T.S. Blyth and M.H. Almeida Santos, On inverse transversals of ordered regular semigroups, *Comm. Algebra*, **37**, No.7(2009), 2200-2212.

- [5] T.S. Blyth and M.H. Almeida Santos, Naturally ordered regular semigroups with inverse monoid transversal, *Semigroup Forum*, **76**(2008), 71-86.
- [6] T.S. Blyth and M.H. Almeida Santos, A classification of inverse transversals, *Comm. Algebra*, **29**, No.2(2001), 611-624. .
- [7] T.S. Blyth and M.H. Almeida Santos, On weakly multiplicative inverse transversals, *Proc. Edin. Math. Soc.*, **37**(1993), 91-99.
- [8] T.S. Blyth and M.H. Almeida Santos, Congruences associated with inverse transversals, *Collect. Math*, **46**(1-2) (1995), 35-48.
- [9] T.S. Blyth and J.F. Chen, Inverse transversals are mutually isomorphic, *Comm. Algebra*, **29**(2) (2001), 799-804.
- [10] T.S. Blyth and R.B. McFadden, On the construction of a class of regular semigroups, *Journal of Algebra*, **81**(1983), 1-22.
- [11] T.S. Blyth and R.B. McFadden, Regular semigroups with a multiplicative inverse transversal, *Proc. Roy. Soc. Edinburgh*, **92A**(1982), 253-270.
- [12] T.S. Blyth and R.B. McFadden, Naturally ordered regular semigroups with a greatest idempotent, *Proc. Roy. Soc. Edinburgh*, **91A**(1981), 107-122.
- [13] K. Byleen, Regular four-spiral semigroups, idempotent generated semigroups and the Rees construction, *Semigroup Forum*, **22**(1981), 97-100.
- [14] K. Byleen, J. Meakin and F. Pastjin, The double fourspiral semigroup, *Simon Stevin*, **54**(1980), 75-105.
- [15] K. Byleen, J. Meakin and F. Pastjin, The fundamental four-spiral semigroup, *Journal of Algebra*, **54**(1978), 6-26.
- [16] V.M. Chandrasekaran and K. Indhira, A note on split morphism, *Global Journal of Pure and Applied Mathematics*, **12**(3) (2016), 1105-1108.
- [17] V.M. Chandrasekaran and K. Indhira, On double four-spiral semigroup, *Indian Journal of Mathematics and Mathematical sciences*, **5**(2) (2009), 61-64.

- [18] V.M. Chandrasekaran and M. Loganathan, Structure of the double four- spiral semigroup, *Kyungpook Math. Journal*, **43**(4) (2003), 503-512.
- [19] Chen Jianfei, On regular semigroups with orthodox transversals, *Comm. Algebra*, **27** (9) (1999), 4275-4288.
- [20] Chen Jianfei and GuoYuqi, Orthodox transversals of regular semigroups, *Internat. J. Algebra Comput.*, **11**(2) (2001), 269-279.
- [21] Fenglin Zhu, Regular semigroups with inverse transversals, *Semigroup Forum*, **73**(2006), 207-208.
- [22] Fenglin Zhu, Orthodox semigroups with inverse transversals, *J. Univ. Sci. Tech. of China*, **33**(3) (2003), 262-267.
- [23] Fenglin Zhu, Weijiang Liu, Regular semigroup with an E-inverse transversal, *Journal of Mathematical Study*, **34**(2001), 105-108.
- [24] S. Guangtian and Z. Fenglin, Left semiregular orthodox semigroups and regular orthodox semigroups with inverse transversals, *South east Asian Bull. Math.*, **28**(2004), 1149-1160.
- [25] S. Guangtian and Z. Fenglin, Fundamental regular semigroups with inverse transversals, *Semigroup Forum*, **67**(1) (2003), 37-49.
- [26] K. Indhira, Kurian C. Soman and V.M. Chandrasekaran, Structure of bands, *Int. Journal of Algebra*, **5**(15) (2011), 735-738.
- [27] K. Indhira and V.M. Chandrasekaran, H-coextension of the double four spiral semigroup $D\text{Sp}_4$, *Adv. Stud. Contemp. Math., Kyungshang*, **24**(3) (2014), 377-387.
- [28] K. Indhira and V.M. Chandrasekaran, Idempotent separating congruence on a regular semigroup with a regular idempotent, *ASTP*, **7** (2013), 107-114.

- [29] K. Indhira and V.M. Chandrasekaran, Regular Semigroup with a S^0 orthodox Transversal, *International Mathematical Forum*, **7**(34) (2012), 1655-1667.
- [30] K. Indhira and V.M. Chandrasekaran, Construction of generalized inverse semigroups, *Int. Journal of Algebra*, **5**(15) (2011), 739-746.
- [31] K. Indhira and V.M. Chandrasekaran, Structure of regular semigroups with a regular idempotent, *Int. Journal of Contemporary Math sciences*, **6**(12) (2011), 557-570.
- [32] Junwei Shi and Zhenji Tian, Regular semigroups with inverse transversals, *International Mathematical Forum*, **7**, No.31(2012), 1547-1552.
- [33] Lili Wang and Aifa Wang, Congruences on abundant semigroups with quasi-ideal S-adequate transversals, *Commun. Korean Math. Soc.*, **29**, No.1(2014), 1-8.
- [34] Ling Yongfa and Chen Jianfei, Some notes on regular semigroups with orthodox transversals, *Far East J. Math. Sci.*, **11**, No.2(2003), 137-145.
- [35] M. Loganathan, Regular semigroups with a medial idempotent, *Semigroup Forum*, **36**(1987), 69-74.
- [36] M. Loganathan and V.M. Chandrasekaran, Regular semigroups with a splitmap, *Semigroup Forum*, **44**(1992), 199-212.
- [37] D.B. McAlister and R.B. McFadden, Semigroups with inverse transversals as matrix semigroups, *Quart.J.Math., Oxford*, **35**(1984), 455-474.
- [38] D.B. McAlister and R.B. McFadden, Regular semigroups with inverse transversals, *Quart.J.Math., Oxford(2)*, **34**(1983), 459-479.
- [39] D.B. McAlister and T.S. Blyth, Split orthodox semigroups, *Journal of Algebra*, **51**(1978), 491-525.
- [40] F. Pastijn and J. Albert, Free split bands, *Semigroup Forum*, **90**(2015), 753-752.

- [41] Pei Wang and Xiangjun Kong, Regular semigroups with multiplicative orthodox transversals, *Journal of Mathematical Sciences: Advances and Applications*, **4**, No.2(2010), 381-393.
- [42] A.R. Rajan, Combinatorial regular semigroups with inverse transversals, *Bull. Cal. Math. Society*, **80**(4)(1988), 241-249.
- [43] Rong Hua Zhang and Shou Feng Wang, On generalized inverse transversals, *Acta Math. Sin. (Engl. Ser.)*, **24**, No.7(2008), 1193-1204.
- [44] T. Saito, Construction of regular semigroups with inverse transversals, *Proc. Edinburgh Math Soc.*, **32**(1989), 41-51.
- [45] T. Saito, Naturally ordered regular semigroups with maximum inverses, *Proc. Edinburgh Math.Soc.*, **32** (1989), 33-39.
- [46] T. Saito, Quasi-orthodox semigroups with inverse transversals, *Semigroup Forum*, **36**(1987), 47-54.
- [47] T. Saito, A note on regular semigroups with inverse transversals, *Semigroup Forum*, **33**(1986), 149-152.
- [48] T. Saito, Relationship between the inverse transversals of a regular semigroup, *Semigroup Forum*, **33**(1986), 245-250.
- [49] T. Saito, Construction of regular semigroups with inverse transversal, *Proc. 9th symposium on semigroups, Naruto Univ.Teacher Ed.*, (1985), 16-23.
- [50] T. Saito, Regular semigroups with a weakly multiplicative inverse transversal, *Proc. 8th symposium on semigroups, Shimane Univ.*,(1985), 22-25.
- [51] T. Saito, Structure of regular semigroups with a quasi-ideal inverse transversal, *Semigroup Forum*, **31**(1985), 291-296.
- [52] T. Saito, Construction of a class of regular semigroups with an inverse transversal, *Math.Ges. D.D.R.Conference on the theory and Applications of Semigroups*, Griefswald, (1984), 108-112.
- [53] Wang Pei and Xiangjun Kong, Regular semigroups with multiplicative orthodox transversals, *J. Math. Sci. Adv. Appl.*, **4**, No.2(2010), 381-393.

- [54] Xiangjun Kong, On generalized orthodox transversals, *Communications in Algebra*, **42**(4)(2014), 1431-1447.
- [55] Xiangjun Kong, Regular semigroups with quasi-ideal orthodox transversals, *Semigroup Forum*, **74**(2007), 247-258.
- [56] Xiangjun Kong and FanweiMeng, The generalization of two basic results for orthodox semigroups, *Semigroup Forum*, **89**(2014), 394-402.
- [57] Xiangjun Kong and Wang Pei, Abundant semigroups with quasi-ideal S -adequate transversals, *Commun. Korean Math. Soc.*, **26**(1) (2011), 1-12.
- [58] Xiangjun Kong and Wang Pei, Relationship between the orthodox transversals of a regular semigroup, *Xibei Shifan Daxue Xuebao Ziran Kexue*, **42**, No.2(2006), 16-18.
- [59] Xiangjun Kong and Wang Pei, Weakly multiplicative orthodox transversals, *Qufu Shifan Daxue Xuebao Ziran Kexue*, **32**, No.4(2006), 43-45.
- [60] Xiangjun Kong and Xianzhong Zhao, A new construction for regular semigroups with quasi-ideal orthodox transversals, *J. Aust. Math. Soc.*, **86**(2009), 177-187.
- [61] Xiangjun Kong and Yanfeng Luo, Regular semigroups with simplistic orthodox transversals, *SCIENTIA SINICA Mathematica*, **41**(2011), 745-757.
- [62] Xilin Tang, Free orthodox semigroups and free bands with inverse transversals, *Sci. China Math.*, **53**(11) (2010), 3015-3026.
- [63] Xilin Tang, Regular semigroups with inverse transversals, *Semigroup Forum*, **55**(1997), 24-32.
- [64] Wang Pei and Xiangjun Kong, Regular semigroups with multiplicative orthodox transversals, *J. Math. Sci. Adv. Appl.*, **4**(2) (2010), 381-393.
- [65] Yuxiao Gao and Yingchun Wang, Regular Semigroups with strongly E -inverse transversals, *International Mathematical Forum*, **7**(15) (2012), 737-743.

