

On intuitionistic soft fuzzy compactness

T.Yogalakshmi.

Assistant Professor of Mathematics,

School of Advanced Sciences,

VIT University, Vellore,

TamilNadu, India.

email:mathyoga14@gmail.com

Abstract: In this paper, intuitionistic soft fuzzy topological space and intuitionistic soft fuzzy compact space are introduced. In this connection, several characterizations of it are studied. Moreover, the concepts of various compactness such as almost compact, nearly compact, etc are discussed. Some interesting covering properties of the intuitionistic soft fuzzy filters are established.

2000 Mathematics Subject Classification. 54A40, 03E72

Keywords: Intuitionistic soft fuzzy sets, intuitionistic soft fuzzy continuous functions, intuitionistic soft fuzzy compact spaces, intuitionistic soft fuzzy almost compact spaces, intuitionistic soft fuzzy nearly compact spaces, intuitionistic soft fuzzy filters.

1 Introduction:

To solve the complicated problems in social sciences, economics, engineering and environment, etc., classical methods are not useful. The solution of such problems involve the use of mathematical principles based on uncertainty and imprecision. To handle such problems of uncertainty, many theories such as fuzzy set theory, interval fuzzy theory, intuitionistic fuzzy sets, rough sets, soft sets, etc are used.

The fuzzy concept has penetrated almost all branches of Mathematics since the introduction of the concept of fuzzy set by Zadeh [10]. Fuzzy sets have applications in many fields such as engineering, information and control. Fuzzy topological space was introduced by C.L.Chang [3]. Ismail U.Tiryaki [7] introduced and studied the notion of soft fuzzy set. V.Visalakshi, M.K.Uma, E.Roja [8] introduced the concept of soft fuzzy topological spaces. T.Yogalakshmi, E.Roja, M.K.Uma studied the properties of soft fuzzy compact spaces in [9]. D.Coker [4] studied the notion of intuitionistic sets and several properties are also discussed. Besides, intuitionistic fuzzy sets were introduced by K.Atanassov [1]. Many authors studied about the topological properties such as compactness, connectedness, e.t.c.

In this paper, the concepts such as intuitionistic soft fuzzy topological spaces, intuitionistic soft fuzzy continuous functions, intuitionistic soft fuzzy compactness are introduced and studied. Properties related to those concepts are established. The notion of various compactness such as intuitionistic soft fuzzy almost compact spaces, intuitionistic soft fuzzy nearly compact spaces, etc are introduced and several properties are discussed. Some interesting covering properties of the intuitionistic soft fuzzy filters are investigated.

2 Preliminaries :

Definition: 2.1. [8] Let X be a nonempty set. Let μ be a fuzzy subset of X such that $\mu : X \rightarrow [0, 1]$ and M be any crisp subset of X . Then, the pair (μ, M) is called as a **soft fuzzy set** in X . The family of all soft fuzzy subsets of X , will be denoted by $\mathbf{SF}(X)$. The **complement** of a soft fuzzy set (μ, M) is defined as $(\mu, M)' = (1 - \mu, X \setminus M)$.

Definition: 2.2. [9] Let $x \in X$ and $\lambda : X \rightarrow [0, 1]$. Define,

$$x_\lambda(y) = \begin{cases} \lambda & (0 < \lambda \leq 1), \\ 0, & \text{if } x = y \\ & \text{otherwise} \end{cases}$$

Then, the soft fuzzy set $(x_\lambda, \{x\})$ is called as the **soft fuzzy point** (inshort. SFP) in $SF(X)$, with **support**, x and **value**, λ .

Definition: 2.3. [?, 1] An intuitionistic fuzzy set A in a non-empty set X is an object having the form $A = \{\langle x, \lambda_A(x), \mu_A(x) \rangle : x \in X\}$ where the functions $\lambda_A : X \rightarrow [0, 1]$ and $\mu_A : X \rightarrow [0, 1]$ denote the degree of membership of each element $x \in X$ to the set A respectively and $0 \leq \mu_A(x) + \lambda_A(x) \leq 1$, for all $x \in X$. For the sake of simplicity, we shall use the symbol $A = (\lambda_A, \mu_A)$ for the intuitionistic fuzzy set $\{\langle x, \lambda_A(x), \mu_A(x) \rangle : x \in X\}$.

Definition: 2.4. [9] Let τ be a soft fuzzy topology on X . Then, (X, τ) is said to be **soft fuzzy compact**, whenever $\sqcup_{j \in J} (\lambda_j, N_j) = (1, X)$, for $(\lambda_j, N_j) \in \tau, j \in J$, there exists a finite subset F of J such that $\sqcup_{j \in F} (\lambda_j, N_j) = (1, X)$

3 Intuitionistic soft fuzzy topological spaces :

Definition: 3.1. An intuitionistic soft fuzzy set A (inshort. ISFS) in a non-empty set X is defined as $A = \langle X, (\lambda, N), (\mu, M) \rangle$ with $0 \leq \lambda_A(x) + \mu_A(x) \leq 1$, for $x \in X$ and $N \cup M = X$ where $(\lambda, N), (\mu, M)$ are the soft fuzzy sets. The complement of A is $A' = \langle X, (\mu, M), (\lambda, N) \rangle$.

Example: 3.1. Let $X = \{a, b, c\}$ be a non-empty set. Then, the ISFS A is defined as $A = \langle X, (\lambda_1, N_1), (\lambda_2, N_2) \rangle$ where $\lambda_i : X \rightarrow [0, 1]$ (for $i = 1, 2$) such that $\lambda_1(a) = 0; \lambda_1(b) = 0.6; \lambda_1(c) = 0.8; N_1 = \{b\} \subseteq X$ $\lambda_2(a) = 0.5; \lambda_2(b) = 0.2; \lambda_2(c) = 0; N_2 = \{a, c\} \subseteq X$ respectively.

Definition: 3.2. Let X be a non-empty set and the intuitionistic soft fuzzy sets A and B be in the form, $A = \langle X, (\lambda, N), (\mu, M) \rangle$ and $B = \langle X, (\delta, L), (\gamma, K) \rangle$. Then,

- (1) $A \subseteq B \Leftrightarrow (\lambda, N) \sqsubseteq (\delta, L)$ and $(\mu, M) \supseteq (\gamma, K)$.
- (2) $A \cap B \Leftrightarrow (\lambda, N) \sqcap (\delta, L)$ and $(\mu, M) \sqcup (\gamma, K)$.
- (3) $A \cup B \Leftrightarrow (\lambda, N) \sqcup (\delta, L)$ and $(\mu, M) \sqcap (\gamma, K)$.
- (4) $(0, \phi)_{\sim} = \langle X, (0, \phi), (1, X) \rangle$; $(1, X)_{\sim} = \langle X, (1, X), (0, \phi) \rangle$

Definition: 3.3. Let $f : X \rightarrow Y$ be any function and $A = \langle X, (\lambda, N), (\mu, M) \rangle$, $B = \langle Y, (\delta, L), (\gamma, K) \rangle$ be any two intuitionistic soft fuzzy sets of X, Y respectively. Then the pre-image of B is an intuitionistic soft fuzzy set in X is defined by $f^{-1}(B) = \langle X, f^{-1}(\delta, L), f^{-1}(\gamma, K) \rangle$ and the image of A is an intuitionistic soft fuzzy set in Y is defined by $f(A) = \langle Y, f(\lambda, N), f(\mu, M) \rangle$ respectively.

Definition: 3.4. An intuitionistic soft fuzzy topology (inshort. ISFT) on a non-empty set X is a family τ of ISFSs in X satisfying the following axioms:

- (i) $(0, \phi)_{\sim}, (1, X)_{\sim} \in \tau$.
- (ii) $A_1 \cap A_2 \in \tau$, for any $A_1, A_2 \in \tau$.
- (iii) $\cup_i A_i \in \tau$, for any $A_i \in \tau$.

Now the ordered pair (X, τ) is called an intuitionistic soft fuzzy topological space (inshort. ISFTS) and any intuitionistic set in τ is said to be an intuitionistic soft fuzzy open set (inshort. ISFOS) in X . The complement of an ISFOS is an intuitionistic soft fuzzy closed set (inshort. ISFCS) in X .

Definition: 3.5. Let (X, τ) be an ISFTS and $A = \langle X, (\lambda, N), (\mu, M) \rangle$ be an ISFS in X . Then, the interior and closure of A are defined by $\text{int}(A) = \cup \{G : G \text{ is an ISFOS in } X \text{ and } G \subseteq A\}$ and $\text{cl}(A) = \cap \{K : K \text{ is an ISFCS in } X \text{ and } K \supseteq A\}$ respectively.

Proposition: 3.1. Let (X, τ) be an intuitionistic soft fuzzy topological space. Let A, B be any two intuitionistic soft fuzzy sets. Then, the following properties hold.

- (1) $\text{int}(A) \subseteq A \subseteq \text{cl}(A)$
- (2) $A \subseteq B \Rightarrow \text{int}(A) \subseteq \text{int}(B)$; $A \subseteq B \Rightarrow \text{cl}(A) \subseteq \text{cl}(B)$
- (3) $\text{int}(\text{int}(A)) = \text{int}(A)$; $\text{cl}(\text{cl}(A)) = \text{cl}(A)$
- (4) $\text{int}(A \cap B) = \text{int}(A) \cap \text{int}(B)$
- (5) $\text{cl}(A \cup B) = \text{cl}(A) \cup \text{cl}(B)$
- (6) $\text{int}((1, X)_{\sim}) = (1, X)_{\sim}$
- (7) $\text{cl}((0, \phi)_{\sim}) = (0, \phi)_{\sim}$
- (8) $\text{cl}(A') = (\text{int}(A))'$; $\text{int}(A') = (\text{cl}(A))'$

Definition: 3.6. Let (X, τ) , (Y, σ) be any two ISFTSs and $f : X \rightarrow Y$ be any function. Then f is said to be an intuitionistic soft fuzzy continuous function, if the inverse image of every intuitionistic soft fuzzy open set is intuitionistic soft fuzzy open.

Example: 3.2. Let $X = \{a, b, c\}$; $Y = \{p, q\}$; $M_j \subseteq Y$ and $N_i \subseteq X$ $i = 1, 2, 3, 4$, $j = 1, 2$. Define $\tau = \{(0, \phi)_{\sim}, (1, X)_{\sim}, \langle X, (\lambda_1, N_1), (\lambda_2, N_2) \rangle, \langle X, (\lambda_3, N_3), (\lambda_2, N_2) \rangle, \langle X, (\lambda_4, N_4), (\lambda_2, N_2) \rangle\}$; $\sigma = \{(0, \phi)_{\sim}, (1, Y)_{\sim}, \langle Y, (\mu_1, M_1), (\mu_2, M_2) \rangle\}$ where $\lambda_i : X \rightarrow [0, 1]$ (for $i = 1, 2, \dots, 6$) and $\mu_i : Y \rightarrow [0, 1]$ (for $i = 1, 2$) such that $\lambda_1(a) = 0, \lambda_1(b) = 0, \lambda_1(c) = 0.2, N_1 = \{\phi\}$; $\lambda_2(a) = 0.1, \lambda_2(b) = 0, \lambda_2(c) = 0, N_2 = \{a\}$; $\lambda_3(a) = 0.1, \lambda_3(b) = 0, \lambda_3(c) = 0.2, N_3 = \{b, c\}$; $\lambda_4(a) = 0.1, \lambda_4(b) = 0.2, \lambda_4(c) = 0.2, N_4 = \{b, c\}$; $\mu_1(p) = 0.2, \mu_1(q) = 0.1, M_1 = \{p\}$; $\mu_2(p) = 0, \mu_2(q) = 0.1, M_2 = \{q\}$ respectively. Let $f : X \rightarrow Y$ be defined as $f(a) = q; f(b) = f(c) = p$. Now f is an intuitionistic soft fuzzy continuous function.

Proposition: 3.2. Let (X, τ) and (Y, σ) be any two ISFTSs and $f : X \rightarrow Y$ be any function. Then, the following statements are equivalent:

- (1) f is an intuitionistic soft fuzzy continuous.
- (2) Inverse image of every ISFCS in (Y, σ) is an intuitionistic soft fuzzy closed set in (X, τ) .
- (3) For each ISFS A in (X, τ) , $f(int(A)) \supseteq int(f(A))$.
- (4) For each ISFS A in (X, τ) , $f(cl(A)) \subseteq cl(f(A))$.

4 Intuitionistic soft fuzzy compact spaces :

Definition: 4.1. An intuitionistic soft fuzzy topological space, (X, τ) is said to be

- (i) **intuitionistic soft fuzzy compact**, whenever $\bigcup_{j \in J} A_j = (1, X)_{\sim}$, for $A_j \in \tau, j \in J$, there exists a finite subset F of J such that $\bigcup_{j \in F} A_j = (1, X)_{\sim}$.
- (ii) **intuitionistic soft fuzzy almost compact**, whenever $\bigcup_{j \in J} A_j = (1, X)_{\sim}$, for $A_j \in \tau, j \in J$, there exists a finite subset F of J such that $\bigcup_{j \in F} cl(A_j) = (1, X)_{\sim}$.
- (iii) **intuitionistic soft fuzzy nearly compact**, whenever $\bigcup_{j \in J} A_j = (1, X)_{\sim}$, for $A_j \in \tau, j \in J$, there exists a finite subset F of J such that $\bigcup_{j \in F} int(cl(A_j)) = (1, X)_{\sim}$.

Proposition: 4.1.

- (i) Every intuitionistic soft fuzzy compact space is intuitionistic soft fuzzy nearly compact.
- (ii) Every intuitionistic soft fuzzy nearly compact space is intuitionistic soft fuzzy almost compact.

Proof. (i) Let (X, τ) be an intuitionistic soft fuzzy compact space. Then, for every family $\{A_i : i \in J\}$ of ISFOSs such that $\bigcup_{i \in J} A_i = (1, X)_{\sim}$ has a finite subfamily of ISFOSs such that $\bigcup_{j \in F} A_j = (1, X)_{\sim}$. Now, by the Proposition:3.1., $(1, X)_{\sim} = \bigcup_{j \in F} A_j = \bigcup_{j \in F} int(A_j) \subseteq \bigcup_{j \in F} int(cl(A_j))$. Thus, $\bigcup_{j \in F} int(cl(A_j)) = (1, X)_{\sim}$. Hence, (X, τ) is an intuitionistic soft fuzzy nearly compact space.

(ii) Proof is similar.

Proposition: 4.2.

- (i) Continuous image of an intuitionistic soft fuzzy compact space is an intuitionistic soft fuzzy compact space.
- (ii) Continuous image of an intuitionistic soft fuzzy compact space is an intuitionistic soft fuzzy almost compact space.

Example: 4.1. Let $X = [a, b]$ and N_i, M_i, L be the subsets of X , for $i = 1, 2, 3, \dots, \infty$. Define $\tau = \{(0, \phi)_{\sim}, (1, X)_{\sim}, \langle X, (\lambda_i, N_i), (\mu_i, M_i) \rangle, \langle X, (\delta_1, L_1), (\delta_2, L_2) \rangle, \mathbb{M}_i \langle X, (\lambda_i, N_i), (\mu_i, M_i) \rangle, \langle X, (\lambda_i, N_i), (\mu_i, M_i) \rangle \mathbb{M} \langle X, (\delta_1, L_1), (\delta_2, L_2) \rangle\}$ where $\lambda_i, \mu_i : X \rightarrow [0, 1]$ (for $i = 1, 2, \dots, \infty$) such that

$$\lambda_i(x) = \begin{cases} 0.9, & \text{if } x = a \\ 1, & \text{if } a < x < \frac{a+b}{2} \\ \frac{1}{i+2}, & \text{if } x = \frac{a+b}{2} \\ 1, & \text{if } \frac{a+b}{2} < x < b \\ \frac{i+4}{2(i+2)}, & \text{if } x = b \end{cases}$$

$$\mu_i(x) = \begin{cases} 0.1, & \text{if } x = a \\ 0, & \text{if } a < x < \frac{a+b}{2} \\ \frac{i+1}{i+2}, & \text{if } x = \frac{a+b}{2} \\ 0, & \text{if } \frac{a+b}{2} < x < b \\ \frac{i}{2(i+2)}, & \text{if } x = b \end{cases}$$

$$\delta_1(x) = \begin{cases} 1, & \text{if } x = a \\ 0, & \text{if } a < x < \frac{a+b}{2} \\ 1, & \text{otherwise} \end{cases}$$

$$\delta_2(x) = \begin{cases} 0, & \text{if } a < x < \frac{a+b}{2} \\ 1, & \text{otherwise} \end{cases}$$

for all $x \in X$, and $N_i = [a, \frac{a+b}{2}] = L_1$; $M_i = [\frac{a+b}{2}, b] = L_2$. Now, (X, τ) is an intuitionistic soft fuzzy almost and nearly compact space yet it is not an intuitionistic soft fuzzy compact space.

5 Intuitionistic soft fuzzy filters :

Definition: 5.1. Let X be a non-empty set. An *intuitionistic soft fuzzy filter*, $\mathfrak{F}$ is a non-empty collection of intuitionistic soft fuzzy sets in X provided:

- (1) $(0, \phi)_{\sim} \notin \mathfrak{F}$.
- (2) $A, B \in \mathfrak{F} \Rightarrow A \mathbb{M} B \in \mathfrak{F}$.
- (3) $A \in \mathfrak{F}$ and $A \subset B \Rightarrow B \in \mathfrak{F}$.

Definition: 5.2. An intuitionistic soft fuzzy filter $\mathfrak{F}$ is said to be an *intuitionistic soft fuzzy prime filter*, whenever $A \sqcup B \in \mathfrak{F} \Rightarrow B \in \mathfrak{F}$ or $A \in \mathfrak{F}$.

Proposition: 5.1. For any intuitionistic soft fuzzy topological space (X, τ) , the following statements are equivalent:

- (i) (X, τ) is an intuitionistic soft fuzzy compact.
- (ii) Every intuitionistic soft fuzzy filter $\mathfrak{F}$ satisfies $\bigcap_{A \in \mathfrak{F}} A \neq (0, \phi)_{\sim}$.
- (ii) Every intuitionistic soft fuzzy prime filter $\mathfrak{P}$ satisfies $\bigcap_{A \in \mathfrak{P}} A \neq (0, \phi)_{\sim}$.
- (ii) Every intuitionistic soft fuzzy ultrafilter $\mathfrak{U}$ satisfies $\bigcap_{A \in \mathfrak{U}} A \neq (0, \phi)_{\sim}$.

References

- [1] K.Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems, 20, (1986), pp.87 - 96.
- [2] A.S.Bin Shahna, On fuzzy compactness and fuzzy Lindelofness, Bull. Cal. Math. Soc., 83,(1991), 146-150.
- [3] C.L.Chang, Fuzzy topological spaces, J. Math. Anal. Appl., 24, (1968), pp.182-190.
- [4] D.Coker, A note on intuitionistic sets and intuitionistic points, Turkish J. Math., 20, (1996), pp. 343-351.
- [5] J.Dugundji, Topology, Prentice Hall of India Private Limited, New Delhi (1975).
- [6] A.Haydar Es, Almost compactness and near compactness in fuzzy topological spaces,Fuzzy Set and Systems, 22, (1987), pp.289-295.
- [7] Ismail. U.Tiryaki, Fuzzy sets over the poset I, Haccettepe J. of Math. And Stat., 37 (2), (2008), pp.143-166.
- [8] V.Visalakshi, M. K. Uma, E. Roja, On soft fuzzy G_{δ} pre continuity in soft fuzzy topological space, Annals of Fuzzy Mathematics and Informatics, Vol. 8, No. 6(2014), 921-939..
- [9] T.Yogalakshmi, E. Roja, M.K.Uma, Compactification of a soft fuzzy product C-space,Italian Journal of Pure and Applied Mathematics, 31, (2013), pp. 111-124.
- [10] L.A.Zadeh, Fuzzy sets, Inform and Control, 8, (1965), pp. 338-353.

