

On Inverse of Hexagonal Fuzzy Number Matrices

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Abstract

In the field of science and engineering technology, linear systems have contributed more applications. But In fact, the linear systems occur in uncertain environment. In such situations, the parameter of the system can be represented by fuzzy nature with the help of fuzzy numbers. In this paper, we have to studied fuzzy linear systems with the aid of Hexagonal Fuzzy Numbers. A new procedure namely matrix inversion method is proposed for solving fuzzy linear system (FLS) of equations. Finally, the method is illustrated by solving relevant numerical examples.

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1 Introduction

Linear system of equations are most important topic in applied mathematics, likewise to solve a large proportion of the problems. Linear equations play a major role in different areas such as matrices, statistics, operational research, engineering and social science. Usually , in many applications some of the problems are represented by imprecise number instead of crisp number, and hence it is important to develop the mathematical area and proceed to numerical representation would appropriately handle a general fuzzy linear systems and solve them. The concepts of fuzzy numbers and its arithmetic operations were developed first time by Zadeh [12] and Dubois and Prade [6]. When the inverse of a square and non-singular matrix whose entries are real numbers in concerned in one of the computing methods in the form of linear equation system composing of the product of $n \times n$ matrices where the former denotes the coefficients and latter represents

the unknowns.

In [7] Friedman et.al established a general model for solving a nn fuzzy linear system, whose coefficient matrix is crisp and right hand side column is an arbitrary fuzzy number vectors. To find the solution of the original nn fuzzy linear system (FLS) is replaced with $2n \times 2n$ crisp linear system. Some of the numerical methods can be seen in [5]. They proposed to justify the solution of $2n \times 2n$ crisp linear systems. In [4] Buckley and Qu extended several methods to the linear system of interval equations. In [3] Allahviranloo proposed fuzzy linear systems can be found by solving this interval systems to determine the exact solution of FLS, in which all parameter are triangular fuzzy number.

In [2] Abbasbandy and Jofarin proposed an approximation of the unique solution in fuzzy system of linear equation, applied by the steepest descent method. Abbasbandy et.al [1] proposed LU decomposition method for solving fuzzy system of linear equation. Muruganandam and Abdul Razak [9] studied matrix inversion method for solving fully fuzzy linear systems with triangular fuzzy numbers, Basaran [8] proposed calculating fuzzy inverse matrix using fuzzy linear equation system. Dinagar and Latha [10] proposed invertible on Type-2 Triangular fuzzy matrices. Dinagar and Harinarayanan [11] proposed arithmetic operations on Hexagonal fuzzy number using α -cut method.

This paper is organized as follows, Firstly in section 2, we give some basic supported preliminaries and representation of Triangular and Trapezoidal fuzzy numbers. In section 3, we recall our proposed definitions of Hexagonal Fuzzy Number and its operations. In section 4, we have reviewed the definition of Hexagonal Fuzzy Matrix (HFM) and its operations are presented with the aid of Hexagonal Fuzzy Matrices (HFMs). In section 5, we define a fuzzy linear system and solve the system of equations by matrix inversion method. In section 6, Numerical examples are illustrated by computing the solution of FLS proposed. Finally in section 7, conclusion is presented.

2 Preliminaries

In this section, some basic related definitions are studied and recalled the representations of Fuzzy Numbers.

Definition 1. (Fuzzy Set)

A Fuzzy set is characterized by a membership function mapping the elements of a domain, space or universe of discourse to the unit interval $[0, 1]$.

A fuzzy set A in a universe of discourse X is defined as the following set of pairs

$$A = \{(x, \mu_A(x); x \in X)\}$$

Here $\mu_A : X \rightarrow [0, 1]$ is mapping called the degree of membership function of the fuzzy set A and $\mu_A(x)$ is called the membership value of $x \in X$ in the fuzzy set A . These membership grades are often represented by real ranging from $[0, 1]$.

Definition 2. (Convex Fuzzy Set)

A fuzzy set $A = \{(x, \mu_A(x)) \subseteq X$ is called convex set in all A_α are convex set (i.e) for every element $x_1 \in A_\alpha$ and $x_2 \in A_\alpha$ for every $\alpha \in [0, 1]$. $\lambda x_1 + (1 - \lambda) x_2 \in A_\alpha$ for all $\lambda \in [0, 1]$. Otherwise the fuzzy set is called non-convex fuzzy set.

Definition 3. (Fuzzy Number)

A fuzzy set $\tilde{A}$, defined on the set of real numbers R is said to be fuzzy number if its membership function has the following characteristics

1. $\tilde{A}$ is normal.
2. $\tilde{A}$ is convex set.
3. The support of $\tilde{A}$ is closed and bounded then $\tilde{A}$ is called fuzzy number.

Definition 4. (Triangular Fuzzy Number)

A fuzzy number represented with three points as follows $\tilde{A} = (a, b, c)$ is called triangular fuzzy number. This representation is interpreted as the following membership function and pictorially represented in fig 1

$$\mu_{\tilde{A}_h}(x) = \begin{cases} 0, & x \leq a \\ \frac{x-a}{b-a}, & a < x \leq b \\ 1, & x = b \\ \frac{c-x}{c-b}, & b < x \leq c \\ 0, & x \geq c \end{cases}$$

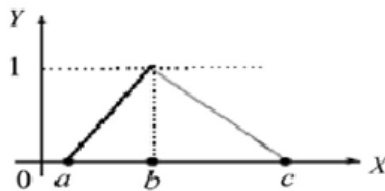


Figure 1: Triangular Fuzzy Number

Definition 5. (Trapezoidal Fuzzy Number)

A fuzzy number represented with four points as follows $\tilde{A} = (a, b, c, d)$ is called trapezoidal fuzzy number. This representation is interpreted as the following membership function and pictorially represented in fig 2

$$\mu_{\tilde{A}_h}(x) = \begin{cases} 0, & x \leq a \\ \frac{x-a}{b-a}, & a < x \leq b \\ 1, & b \leq x \leq c \\ \frac{c-x}{d-c}, & c < x \leq d \\ 0, & x \geq d \end{cases}$$

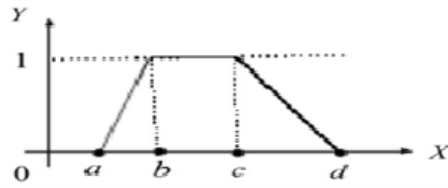


Figure 2: Trapezoidal Fuzzy Number

However, real-life problems are sometimes concerned with more than four parameters. To resolve those problems, we propose the another fuzzy number namely Hexagonal Fuzzy Number (HFN). We discuss the hexagonal fuzzy number in the next section.

3 Hexagonal Fuzzy Number

In general, the human life span can be classified by different stages. One such interesting classification is given as follows:

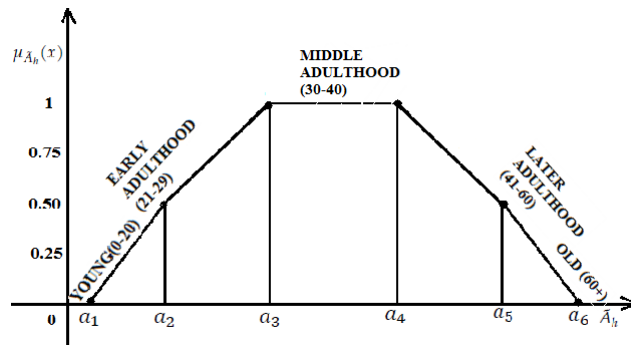
Young (around 10 years), Early Adulthood (around 25 years), Middle Adulthood (around 35 years), Later Adulthood (around 50 years) and Old (around 60 years).

Suppose a problem like the following is occurred that would be solved with the aid of fuzzy theory. The problem of the study of humanbeings whose economic and energy status are maximum, with their human life span, is purely fuzzy in nature. It can be clearly noted that the humanbeing to middle Adulthood be more suit for the above said problem. This situation is clearly in nature and it can be represented by a fuzzy number called Hexagonal Fuzzy Number.

Definition 6. (Hexagonal Fuzzy Number)

A fuzzy number with six points as follows $\tilde{A}_h = (a_1, a_2, a_3, a_4, a_5, a_6)$ is called Hexagonal fuzzy number, where $(a_1 \leq a_2 \leq a_3 \leq a_4 \leq a_5 \leq a_6)$. It is denoted by $\tilde{A}_h$, which are real number satisfying $a_2 - a_1 \leq a_3 - a_2$ and $a_5 - a_4 \geq a_6 - a_5$ and its membership function $\mu_{\tilde{A}_h}(x)$ is given by

$$\mu_{\tilde{A}_h}(x) = \begin{cases} 0 & ; x < a_1 \\ \frac{1}{2} \left(\frac{x-a_1}{a_2-a_1} \right) & ; a_1 \leq x \leq a_2 \\ \frac{1}{2} + \frac{1}{2} \left(\frac{x-a_2}{a_3-a_2} \right) & ; a_2 \leq x \leq a_3 \\ 1 & ; a_3 \leq x \leq a_4 \\ 1 - \frac{1}{2} \left(\frac{x-a_4}{a_5-a_4} \right) & ; a_4 \leq x \leq a_5 \\ \frac{1}{2} \left(\frac{a_6-x}{a_6-a_5} \right) & ; a_5 \leq x \leq a_6 \\ 0 & ; x \geq a_6. \end{cases}$$

Figure 3: Hexagonal Fuzzy Number $\tilde{A}_h$

3.1 Arithmetic Operations On Hexagonal Fuzzy Numbers(HFNs)

The arithmetic operations between hexagonal fuzzy numbers (HFNs) are proposed given below.

Let us consider $\tilde{A}_h = (a_1, a_2, a_3, a_4, a_5, a_6)$ and $\tilde{B}_h = (b_1, b_2, b_3, b_4, b_5, b_6)$ be two hexagonal fuzzy numbers then,

(i) **Addition :**

$$\tilde{A}_h (+) \tilde{B}_h = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5, a_6 + b_6).$$

(ii) **Subtraction :**

$$\tilde{A}_h (-) \tilde{B}_h = (a_1 - b_6, a_2 - b_5, a_3 - b_4, a_4 - b_3, a_5 - b_2, a_6 - b_1).$$

(iii) **Multiplication :**

$$\tilde{A}_h (\times) \tilde{B}_h = \left(\frac{a_1}{6} \sigma_b, \frac{a_2}{6} \sigma_b, \frac{a_3}{6} \sigma_b, \frac{a_4}{6} \sigma_b, \frac{a_5}{6} \sigma_b, \frac{a_6}{6} \sigma_b \right).$$

$$\text{Where } \sigma_b = (b_1 + b_2 + b_3 + b_4 + b_5 + b_6).$$

(iv) **Division :**

$$\tilde{A}_h (\div) \tilde{B}_h = \left(\frac{6a_1}{\sigma_b}, \frac{6a_2}{\sigma_b}, \frac{6a_3}{\sigma_b}, \frac{6a_4}{\sigma_b}, \frac{6a_5}{\sigma_b}, \frac{6a_6}{\sigma_b} \right).$$

$$\text{Where } \sigma_b = (b_1 + b_2 + b_3 + b_4 + b_5 + b_6).$$

(v) **Scalar Multiplication :**

If $k \neq 0$ is scalar $k\tilde{A}_h$ is defined as

$$k\tilde{A}_h = \begin{cases} (ka_1, ka_2, ka_3, ka_4, ka_5, ka_6) & \text{if } k \geq 0 \\ (ka_6, ka_5, ka_4, ka_3, ka_2, ka_1) & \text{if } k < 0 \end{cases}$$

Definition 7. (Ranking Function)

We define a ranking function $\check{R} : F(R) \rightarrow R$ which maps each fuzzy numbers to real line $F(R)$ represented the set of all hexagonal fuzzy numbers. If R be any linear ranking functions.

$$\check{R}(\tilde{A}_h) = \left(\frac{a_1 + a_2 + a_3 + a_4 + a_5 + a_6}{6} \right).$$

Also we define the order on $F(R)$ by

$$\check{R}(\tilde{A}_h) \geq \check{R}(\tilde{B}_h) \text{ if and only if } \tilde{A}_h \underset{\check{R}}{\geq} \tilde{B}_h$$

$$\check{R}(\tilde{A}_h) \leq \check{R}(\tilde{B}_h) \text{ if and only if } \tilde{A}_h \underset{\check{R}}{\leq} \tilde{B}_h$$

$$\check{R}(\tilde{A}_h) = \check{R}(\tilde{B}_h) \text{ if and only if } \tilde{A}_h \underset{\check{R}}{=} \tilde{B}_h$$

Definition 8. (Zero Hexagonal Fuzzy Number)

If $\tilde{A}_h = (0, 0, 0, 0, 0, 0)$ then $\tilde{A}_h$ is said to be zero hexagonal fuzzy number. It is denoted by 0.

Definition 9. (Zero-Equivalent Hexagonal Fuzzy Number)

A hexagonal fuzzy number $\tilde{A}_h$ is said to be zero-equivalent hexagonal fuzzy number if $\tilde{R}(\tilde{A}_h)=0$. It is denoted by $\tilde{0}$.

Definition 10. (Unit Hexagonal Fuzzy Number)

If $\tilde{A}_h = (1, 1, 1, 1, 1, 1)$ then $\tilde{A}_h$ is said to be unit hexagonal fuzzy number. It is denoted by 1.

Definition 11. (Unit-Equivalent Hexagonal Fuzzy Number)

A hexagonal fuzzy number $\tilde{A}_h$ is said to be unit-equivalent hexagonal fuzzy number if $\tilde{R}(\tilde{A}_h)=1$. It is denoted by $\tilde{1}$.

Definition 12. (Inverse Hexagonal Fuzzy Number)

If $\tilde{a}_h$ is hexagonal fuzzy number and $\tilde{a}_h \neq 0$ then we define $\tilde{a}_h^{-1} = \frac{\tilde{1}}{\tilde{a}}$.

4 Hexagonal Fuzzy Matrices(HFMs)

In this section, we proposed new definition of Hexagonal Fuzzy Matrix and corresponding its matrix operations.

Definition 13. (Hexagonal Fuzzy Matrix)

A fuzzy matrix $\hat{A} = (a_{hij})_{m \times n}$ of order $m \times n$ is called a Hexagonal fuzzy matrix if the elements of the matrix are Hexagonal fuzzy numbers, i.e., of the form $(a_{ij1}, a_{ij2}, a_{ij3}, a_{ij4}, a_{ij5}, a_{ij6})$.

4.1 Operations on Hexagonal Fuzzy Matrices(HFMs)

Through classical matrix algebra, we achieve some algebraic operations of HFM. Let $\hat{A} = (\tilde{a}_{hij})_{m \times n}$ and $\hat{B} = (\tilde{b}_{hij})_{m \times n}$ be two HFMs of same order. Then we have the following

1. $\hat{A} + \hat{B} = (\tilde{a}_{hij} + \tilde{b}_{hij})$
2. $\hat{A} - \hat{B} = (\tilde{a}_{hij} - \tilde{b}_{hij})$
3. For $\hat{A} = (\tilde{a}_{hij})_{m \times n}$ and $\hat{B} = (\tilde{b}_{hij})_{n \times k}$ then $\hat{A}\hat{B} = (\tilde{c}_{hij})_{m \times k}$ where $(\tilde{c}_{hij})_{m \times k} = \sum_{p=1}^n \tilde{a}_{hip} \tilde{b}_{hpj}$, $i=1, 2, \dots, m$ and $j=1, 2, \dots, k$
4. $\hat{A}^T \text{ or } \hat{A}' = (\tilde{a}_{hji})$
5. $k\hat{A} = (k\tilde{a}_{hij})$, where k is scalar.

A Hexagonal Fuzzy Matrix(HFM) is said to be a zero HFM if all its entries are 0 and it is denoted by $\hat{O}$.

The square HFM is said to be a unit HFM if the diagonal elements are 1 and the rest of elements are 0. It is denoted by $\hat{I}$

5 Matrix Inversion Method For Fuzzy Linear System (FLS) Using Hexagonal Fuzzy Number

In this section, we define the concept of fuzzy linear system is justify in matrix inversion method with the aid of hexagonal fuzzy numbers and the relevant definitions are recalled in nature.

Consider the system of n fuzzy linear non- homogeneous HFN equations in n unknown HFN vectors $x_{h1}, x_{h2}, \dots, x_{hn}$.

where $\tilde{a}_{hij}, x_{hi}, b_{hi}$ hexagonal fuzzy numbers.

$$\begin{array}{l} \tilde{a}_{h11}x_{h1} \oplus \tilde{a}_{h12}x_{h2} \oplus \dots \dots \dots \oplus \tilde{a}_{h1n}x_{hn} = b_{h1} \\ \tilde{a}_{h21}x_{h1} \oplus \tilde{a}_{h22}x_{h2} \oplus \dots \dots \dots \oplus \tilde{a}_{h2n}x_{hn} = b_{h2} \\ \vdots \qquad \qquad \qquad \vdots \qquad \qquad \qquad \qquad \qquad \qquad \vdots \qquad \qquad \qquad \vdots \\ \tilde{a}_{h n1}x_{h1} \oplus \tilde{a}_{h n2}x_{h2} \oplus \dots \dots \dots \oplus \tilde{a}_{h n n}x_{h n} = b_{h n} \end{array}$$

The above linear system is represented in the form is given by

$$\hat{A}x_h = b_h \quad (5.1)$$

Where $\hat{A} = (\tilde{a}_{hij})$, $1 \leq i, j \leq n$ is n times n hexagonal fuzzy matrix and $\tilde{a}_{hij} \in F(R)$ and $(x_{hi}, b_{hi}) \in F(R)$, for all $i=1,2,\dots,n$ and $j=1,2,\dots,n$. This system is called fuzzy linear system.

If the coefficient Hexagonal fuzzy matrix $\hat{A}$ is non singular, then $\hat{A}^{-1}$ we get

$$\begin{aligned}\hat{A}^{-1}(\hat{A}x_h) &= \hat{A}^{-1}b_h \\ (\hat{A}^{-1}\hat{A})x_h &= \hat{A}^{-1}b_h \\ x_h &= \hat{A}^{-1}b_h\end{aligned}$$

The solution of FLS will be represented by

$$x_h = \hat{A}^{-1}b_h$$

The determinant of $n \times n$ HFM $\hat{A} = (\tilde{a}_{hij})$ is denoted by $|\hat{A}|$ or $\det(\hat{A})$ and is defined as

follows

$$\begin{aligned} |\hat{A}| &= \sum_{q \in s_n} \text{sgn } q \prod_{i=1}^n \tilde{a}_{hiq(i)} \\ &= \sum_{q \in s_n} \text{sgn } q \tilde{a}_{h1q(1)} \cdot \tilde{a}_{h2q(2)} \cdots \tilde{a}_{hmq(n)} \end{aligned}$$

where $\tilde{a}_{hiq(i)} = (a_{iq(i)1}, a_{iq(i)2}, a_{iq(i)3}, a_{iq(i)4}, a_{iq(i)5}, a_{iq(i)6})$ are hexagonal fuzzy number (HFN) and s_n denotes the symmetric group of all permutations of the indices $\{1, 2, \dots, n\}$ and $\text{sgn } q = 1$ or -1 according as the permutation $q = \begin{pmatrix} 1 & 2 & \cdots & n \\ q(1) & q(2) & \cdots & q(n) \end{pmatrix}$ is even or odd respectively.

Definition 17. (Adjoint of HFM)

Let $\hat{A} = (\tilde{a}_{hij})$ be a square HFM of order n . Find the cofactor $\tilde{A}_{hij}$ of every element $\tilde{a}_{hij}$ in $\hat{A}$ and replace every $\tilde{a}_{hij}$ by its cofactor $\tilde{A}_{hij}$ in $\hat{A}$ and let it be $\hat{B}$. i.e., $\hat{B} = (\tilde{A}_{hij})$. Then the transpose of $\hat{B}$ is called the adjoint or adjugate of $\hat{A}$ and is denoted by $\text{adj } \hat{A}$. I.e., $\hat{B}^T = \hat{A}_{hji} = \text{adj } \hat{A}$.

Definition 18. (Singular HFM)

Let $\hat{A} = (\tilde{a}_{hij})$ be a square HFM of order n , then it is said to be singular HFM if $|\hat{A}| = \tilde{0}$.

Definition 19. (Non-Singular HFM)

Let $\hat{A} = (\tilde{a}_{hij})$ be a square HFM of order n , then it is said to be non-singular HFM if $|\hat{A}| \neq \tilde{0}$.

Definition 20. (Inverse of HFM)

A non-singular HFM $\hat{A} = (\tilde{a}_{hij})$ of order n is said to be invertible if there exist a HFM $\hat{B}$ of order n such that $\hat{A}\hat{B} = \hat{I} = \hat{B}\hat{A}$.

Then $\hat{B}$ is called the inverse of $\hat{A}$ and is denoted by $\hat{A}^{-1}$. Thus $\hat{A}\hat{A}^{-1} = \hat{I} = \hat{A}^{-1}\hat{A}$. Also $\hat{A}^{-1} = \frac{1}{R(|\hat{A}|)} \text{adj } \hat{A}$.

6 Numerical Example

In this section two simple examples are given in order to illustrate the proposed method.

Example 21. Consider the following fuzzy linear system and solve by matrix inversion method.

$$\begin{aligned} (-2, 0, 1, 2, 5, 6)x_{h1} \oplus (-2, -1, 0, 1, 2, 6)x_{h2} &= (-1, 0, 9, 10, 11, 13) \\ (-4, 0, 3, 4, 7, 8)x_{h1} \oplus (-2, 0, 2, 3, 4, 5)x_{h2} &= (0, 10, 11, 12, 13, 20). \end{aligned}$$

Solution:

The given linear system may be written as

$$\begin{pmatrix} (-2, 0, 1, 2, 5, 6) & (-2, -1, 0, 1, 2, 6) \\ (-4, 0, 3, 4, 7, 8) & (-2, 0, 2, 3, 4, 5) \end{pmatrix} \begin{pmatrix} x_{h1} \\ x_{h2} \end{pmatrix} = \begin{pmatrix} (-1, 0, 9, 10, 11, 13) \\ (0, 10, 11, 12, 13, 20) \end{pmatrix}$$

$$x_h = \hat{A}^{-1}b_h$$

Now, $|\hat{A}| = (-10, -7, -2, 1, 10, 16)$.

Then, $\tilde{R}(|\hat{A}|) = 1 \neq \tilde{0}$.

Since $\hat{A}$ is non-singular, then $\hat{A}^{-1}$ exists,

$$\hat{A}^{-1} = \frac{1}{\tilde{R}(|\hat{A}|)} \text{adj} \hat{A}.$$

$$\text{adj} \hat{A} = \begin{pmatrix} (-2, 0, 2, 3, 4, 5) & (-6, -2, -1, 0, 1, 2) \\ (-8, -7, -4, -3, 0, 4) & (-2, 0, 1, 2, 5, 6) \end{pmatrix}.$$

$$\hat{A}^{-1} = \begin{pmatrix} (-2, 0, 2, 3, 4, 5) & (-6, -2, -1, 0, 1, 2) \\ (-8, -7, -4, -3, 0, 4) & (-2, 0, 1, 2, 5, 6) \end{pmatrix}.$$

The solution is, $x_h = \hat{A}^{-1}b_h$

$$\begin{pmatrix} x_{h1} \\ x_{h2} \end{pmatrix} = \begin{pmatrix} (-80, -22, 3, 21, 39, 57) \\ (-78, -49, -17, 1, 55, 94) \end{pmatrix}$$

The solution is, $x_{h1} = (-80, -22, 3, 21, 39, 57)$,

$x_{h2} = (-78, -49, -17, 1, 55, 94)$.

Example 22. Consider the following fuzzy linear system and solve by matrix inversion method.

$$(-1, 1, 3, 5, 7, 9)x_{h1} \oplus (-1, 0, 1, 2, 4, 6)x_{h2} \oplus (-2 - 1, 0, 1, 2, 6)x_{h3} = (0, 2, 4, 6, 8, 10)$$

$$(-1, 0, 1, 3, 6, 9)x_{h1} \oplus (-1, 0, 1, 2, 4, 6)x_{h2} \oplus (-2 - 1, 0, 1, 2, 6)x_{h3} = (-1, 0, 1, 3, 6, 9)$$

$$(-2 - 1, 0, 1, 2, 6)x_{h1} \oplus (-2 - 1, 0, 1, 2, 6)x_{h2} \oplus (-2 - 1, 0, 1, 2, 6)x_{h3} = (-1, 1, 3, 5, 7, 9).$$

Solution:

The given linear system may be written as

$$\begin{pmatrix} (-1, 1, 3, 5, 7, 9) & (-1, 0, 1, 2, 4, 6) & (-2 - 1, 0, 1, 2, 6) \\ (-1, 0, 1, 3, 6, 9) & (-1, 0, 1, 2, 4, 6) & (-2 - 1, 0, 1, 2, 6) \\ (-2 - 1, 0, 1, 2, 6) & (-2 - 1, 0, 1, 2, 6) & (-2 - 1, 0, 1, 2, 6) \end{pmatrix} \begin{pmatrix} x_{h1} \\ x_{h2} \\ x_{h3} \end{pmatrix} = \begin{pmatrix} (0, 2, 4, 6, 8, 10) \\ (-1, 0, 1, 3, 6, 9) \\ (-1, 1, 3, 5, 7, 9) \end{pmatrix}$$

$$x_h = \hat{A}^{-1}b_h$$

Now, $|\hat{A}| = (-15, -8, -1, 4, 9, 17)$.

Then, $\tilde{R}(|\hat{A}|) = 1 \neq \tilde{0}$.

Since $\hat{A}$ is non-singular, then $\hat{A}^{-1}$ exists,

$$\hat{A}^{-1} = \frac{1}{\tilde{R}(|\hat{A}|)} \text{adj} \hat{A}.$$

$$\text{adj} \hat{A} = \begin{pmatrix} (-7, -2, 0, 2, 5, 8) & (-8, -5, -2, 0, 2, 7) & (-7, -4, -1, 1, 4, 7) \\ (-6, -4, -2, -1, 0, 1) & (-7, -1, 2, 5, 8, 11) & (-10, -7, -4, 0, 5, 10) \\ (-13, -4, -1, 3, 8, 13) & (-13, -9, -5, -1, 3, 13) & (-20, -10, 0, 8, 14, 20) \end{pmatrix}.$$

$$\hat{A}^{-1} = \begin{pmatrix} (-7, -2, 0, 2, 5, 8) & (-8, -5, -2, 0, 2, 7) & (-7, -4, -1, 1, 4, 7) \\ (-6, -4, -2, -1, 0, 1) & (-7, -1, 2, 5, 8, 11) & (-10, -7, -4, 0, 5, 10) \\ (-13, -4, -1, 3, 8, 13) & (-13, -9, -5, -1, 3, 13) & (-20, -10, 0, 8, 14, 20) \end{pmatrix}.$$

The solution is, $x_h = \hat{A}^{-1}b_h$

$$\begin{pmatrix} x_{h1} \\ x_{h2} \\ x_{h3} \end{pmatrix} = \begin{pmatrix} (-87, -41, -10, 14, 47, 89) \\ (-91, -51, -20, 10, 44, 78) \\ (-184, -87, -20, 44, 56, 184) \end{pmatrix}$$

The solution is,

$$x_{h1} = (-87, -41, -10, 14, 47, 89),$$

$$x_{h2} = (-91, -51, -20, 10, 44, 78),$$

$$x_{h3} = (-184, -87, -20, 44, 56, 184).$$

7 Conclusion

In this article, we investigated the fuzzy linear system of equations with fuzzy coefficients involving in fuzzy variables. The matrix inversion method is applied to solve the system of equations. This method is illustrated with numerical examples. The notion of fuzzy linear system can be applying in Cramers rule and LU Decomposition method by this proposed method in future.

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