

SOLITONS AND EXPONENTIAL SOLUTIONS FOR A NONLINEAR (2+1)dim PDE

S.Padmasekaran¹, S.Rajeswari².

^{1,2}Department of Mathematics,
Periyar University, Salem - 636011,
Tamil Nadu, India.

¹
e-mail: padmasekarans@yahoo. co.in

²
e-mail: rajeswarisab@gmail. com

Abstract

Homogeneous Balance Method is easy to apply and always yield a special exact solutions of nonlinear Partial Differential Equations . Wang showed the Homogeneous Balance Method is powerful for finding analytic solitary wave solutions of Partial Differential Equations. The idea is the highest nonlinear term partially balanced with the highest derivative term. In this paper, the solitary wave solution of nonlinear Partial Differential Equations has been obtained by this method. The exact solutions of (2+1) dimensional Partial Differential Equation is exponential functions besides the solitary wave solution. Based on the backlund transformations, exact solutions are also obtained.

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1. INTRODUCTION

Homogeneous Balance method yields special exact solutions and solitons for nonlinear partial differential equations (Biao Li, Yong Chen and Hongqing Zhang [1], Feng [3], Lei Yang.et.al [4], Wang Mingling [5],[6],[7]). Recently Engui Fan [2] interpreted the transformation used in HBM as an auto-Backlund transformation as it involved first order derivatives. To be precise , any solution u of a pde $L[u] = 0$ is written in term of w and w_x , and a finite set of pdes are derived for w . In the present paper we relate u to w, w_x and w_{xx} and call the relation as a higher order BT.

We consider the $(2 + 1)$ -dimensional equation

$$(u_t + \alpha uu_x + \beta u_{xx} + \gamma u_{xxx})_x + \lambda u_{yy} = 0. \quad (1)$$

2. SOLITONS OF EQUATION (1)

In order that the highest nonlinear term αuu_{xx} partially balance with the forth order derivative term u_{xxxx} , we suppose that the solution of (1) is of the form

$$u(x, t, y) = f''w_x^2 + f'w_{xx} + af'w_x + b, \quad w = w(x, t, y), \quad (2)$$

where the functions f and w as well as the constants a and b are to be determined. It follows from (2) that

$$u_{xt} = f^{IV} w_t w_x^3 + 3f''' w_x^2 w_{xt} + 3[f''' w_t w_x w_{xx} + f'' w_{xt} w_{xx} + f'' w_x w_{xxt}] + f'' w_t w_{xxx} + f' w_{xxx} a f''' w_x^2 w_t + 2a f'' w_x w_{xt} + a f'' w_t w_{xx} + a f' w_{xxt}, \quad (3)$$

$$\alpha u_x^2 = \alpha \left[f'''^2 w_x^6 + 9f''^2 w_x^2 w_{xx}^2 + f'^2 w_{xxx}^2 + a^2 f'^2 w_{xx}^2 + 6f'' f''' w_x^4 w_{xx} + 2f' f'''^2 w_x^3 w_{xxx} + 2a f'' f''' w_x^5 + 2a f' f''' w_x^3 w_{xx} + 6f' f'' w_x w_{xx} w_{xxx} + 6a f''^2 w_x^3 w_{xx} + 6a f' f'' w_x w_{xx}^2 + 2a f' f'' w_x^2 w_{xxx} + 2a f'^2 w_{xx} w_{xxx} + 2a^2 f' f'' w_x^2 w_{xx} \right] \quad (4)$$

$$\alpha u u_{xx} = \alpha \left[f'' f^{IV} w_x^6 + 6f'' f''' w_x^4 w_{xx} + 3f''^2 w_x^2 w_{xx}^2 + 4f''^2 w_x^3 w_{xxx} + f'' f' w_x^2 w_{xxxx} + a f'' f''' w_x^5 + 3a f''^2 w_x^3 w_{xx} + a f'' f' w_x^2 w_{xxx} + f^{IV} f' w_x^4 w_{xx} + 6f' f''' w_x^2 w_{xx}^2 + 3f' f'' w_{xx}^3 + 4f' f'' w_x w_{xx} w_{xxx} + f'^2 w_{xx} w_{xxxx} + a f' f''' w_x^3 w_{xx} + 3a f' f'' w_x w_{xx}^2 + a f'^2 w_{xx} w_{xxx} + a f^{IV} f' w_x^5 + 6a f' f''' w_x^3 w_{xx} + 3a f' f'' w_x w_{xx}^2 + 4a f' f'' w_x^2 w_{xxx} + a f'^2 w_x w_{xxxx} + a^2 f' f''' w_x^4 + 3a^2 f' f'' w_x^2 w_{xx} + a^2 f'^2 w_x w_{xxx} + b f^{IV} w_x^4 + 6b f''' w_x^2 w_{xx} + 3b f'' w_{xx}^2 + 4b f'' w_x w_{xxx} + b f' w_{xxxx} + a b f''' w_x^3 + 3a b f'' w_x w_{xx} + a b f' w_{xxx} \right], \quad (5)$$

$$\beta u_{xxx} = \beta \left[f^V w_x^5 + 10f^{IV} w_x^3 w_{xx} + 15f''' w_x w_{xx}^2 + 10f''' w_x^2 w_{xxx} + 10f'' w_{xx} w_{xxx} + 5f'' w_x w_{xxxx} + f' w_{xxxx} + a f^{IV} w_x^4 + 6a f''' w_x^2 w_{xx} + 3a f'' w_{xx}^2 + 4a f'' w_x w_{xxx} + a f' w_{xxxx} \right] \quad (6)$$

$$\gamma u_{xxxx} = \left[f^{VI} w_x^6 + 15f^V w_x^4 w_{xx} + 45f^{VI} w_x^2 w_{xx}^2 + 15w_{xx}^3 + 61f''' w_x w_{xx} w_{xxx} + 10f^{VI} w_x^3 w_{xxx} + 14f''' w_x^2 w_{xxxx} + 10f'' w_{xxx}^2 + 15f'' w_{xx} w_{xxxx} + 5f'' w_x w_{xxxx} + f' w_{xxxx} + a f^V w_x^5 + 10a f^{IV} w_x^3 w_{xx} + 15a f''' w_x w_{xx}^2 + 10a f''' w_x^2 w_{xxx} + 10a f'' w_{xx} w_{xxx} + 5a f'' w_x w_{xxxx} + a f' w_{xxxx} \right], \quad (7)$$

$$\lambda u_{yy} = \lambda \left[f^{IV} e_y^2 w_x^2 + f''' w_{yyy} w_x^2 + 4f''' w_y w_x w_{xy} + 2f'' w_{xy}^2 + 2f'' w_{xyy} w_x + f''' w_y^2 w_{xx} + f'' w_{yy} w_{xx} + 2f'' w_{xy} w_y + f' w_{xyy} + a f''' w_y^2 w_x + a f'' w_{yy} w_x + 2a f'' w_{xy} w_y + a f' w_{xyy} \right]. \quad (8)$$

First collecting the terms with w_x^6 in (4), (5) and (7), and setting its coefficient to zero, we obtain an ordinary differential equation for $f(w)$:

$$\alpha f'''^2 + \alpha f'' f^{IV} + \gamma f^{IV} = 0, \quad (9)$$

then the ODE (9) has two solutions

$$f = \pm \frac{12\gamma}{\alpha} \log w, \quad (10)$$

thereby

$$\begin{aligned} f''^2 &= -\frac{2\gamma}{\alpha}f^{IV}, \quad f'^2 - \frac{12\gamma}{\alpha}f'', \quad f''f''' = -\frac{\gamma}{\alpha}f^V, \\ f'f''' &= \frac{4\gamma}{\alpha}f^{IV}, \quad f'f'' = -\frac{6\gamma}{\alpha}f''', \quad f'f^{IV} = -\frac{3\gamma}{\alpha}. \end{aligned} \quad (11)$$

Next substituting (3)-(8) into the left hand side of (1) and using (11), we obtain

$$\begin{aligned} u_{xt} + \alpha u_x^2 + \alpha u u_{xx} \\ + \beta u_{xxx} + \gamma u_{xxxx} + \lambda u_{yy} &= \left[-\frac{5\alpha A}{4}w_x^4 w_{xx} - \frac{\alpha a A}{2}w_x^5 + \beta w_x^5 + 15\gamma w_x^4 w_{xx} + a\gamma w_x^5 \right] f^V \\ &+ \left[w_t w_x^3 - \frac{23}{6}A\alpha w_x^2 w_{xx}^2 - \frac{\alpha A}{2}a^2 w_x^4 - \frac{4}{3}\alpha A w_x^3 w_{xxx} - \frac{9}{2}\alpha A a w_x^3 w_{xx} \right. \\ &+ b w_x^4 + 10\beta w_x^3 w_{xx} + a w_x^4 + 45\gamma w_x^2 w_{xx}^2 + 20\gamma w_x^3 w_{xxx} + 10a\gamma w_x^3 w_{xx} \\ &+ \lambda w_y^2 w_x^2 \left. \right] f^{IV} + [3w_x^2 w_{xt} + 3w_x w_t w_{xx} + a w_t w_x^2 - 5A\alpha w_x w_{xx} w_{xxx} \\ &- 6aA\alpha w_x w_{xx}^2 - \frac{7}{2}aA\alpha w_x^2 w_{xxx} - \frac{3}{2}\alpha A w_{xx}^3 - \frac{3}{2}a^2\alpha A w_x^2 w_{xx} \\ &+ 6b\alpha w_x^2 w_{xx} + ab\alpha w_x^3 + 15\beta w_x w_{xx}^2 + 10\beta w_x^2 w_{xxx} + 6a\beta w_x^2 w_{xx} + 15\gamma w_{xx}^3 \\ &+ 61\gamma w_x w_{xx} w_{xxx} + 14\gamma w_x^2 w_{xxxx} + 15a\gamma w_x w_{xx}^2 + 10a\gamma w_x^2 w_{xxx} \\ &+ \lambda w_{yy} w_x^2 + 4\lambda w_x w_y w_{xy} + \lambda w_y^2 w_{xx} + a\lambda w_y^2 w_x \left. \right] f''' \\ &+ [3w_{xt} w_{xx} + 3w_x w_{xxt} + w_t w_{xxx} + 2a w_x w_{xt} + a w_t w_{xx} - \alpha A w_{xxx}^2 \\ &- a^2\alpha A w_{xx}^2 - 3aA\alpha w_{xx} w_{xxx} - \alpha A w_{xx} w_{xxxx} - \alpha A w_x w_{xxxx} \\ &- \alpha a^2 A w_x w_{xxx} + 3b\alpha w_{xx}^2 + 4b\alpha w_x w_{xxx} + 3ab\alpha w_x w_{xx} + 10\beta w_{xx} w_{xxx} \\ &+ 5\beta w_x w_{xxxx} + 3a\beta w_{xx}^2 + 4a\beta w_x w_{xxx} + 9\gamma w_{xxx}^2 + 13\gamma w_{xx} w_{xxxx} \\ &+ 5\gamma w_x w_{xxxxx} + 10a\gamma w_{xx} w_{xxx} + 5a\gamma w_x w_{xxxx} + 2\lambda w_{xy}^2 + 2\lambda w_x w_{xyy} \\ &+ \lambda w_{yy} w_{xx} + 2\lambda w_y w_{xy} + a\lambda w_{yy} w_x + 2a\lambda w_y w_{xy} \left. \right] f'' \\ &+ [w_{xxxt} + a w_{xxt} + b\alpha w_{xxxx} + ab\alpha w_{xxx} + a\beta w_{xxxx} \gamma w_{xxxxx} \\ &+ a\gamma w_{xxxxx} + \lambda w_{xyy} + a\lambda w_{xyy} \left. \right] f' \end{aligned} \quad (12)$$

Setting the coefficients of f^V , f^{IV} , f''' , f'' and f' in (12) to zero respectively, we obtain the

following five equations for the determination of $w(x, t)$:

$$-\frac{5\alpha A}{4}w_x^4w_{xx} - \frac{\alpha a A}{2}w_x^5 + \beta w_x^5 + 15\gamma w_x^4w_{xx} + a\gamma w_x^5 = 0, \quad (13)$$

$$\begin{aligned} w_tw_x^3 - \frac{23}{6}A\alpha w_x^2w_{xx}^2 - \frac{\alpha A}{2}a^2w_x^4 - \frac{4}{3}\alpha Aw_x^3w_{xxx} - \frac{9}{2}\alpha Aaw_x^3w_{xx} \\ + bw_x^4 + 10\beta w_x^3w_{xx} + aw_x^4 + 45\gamma w_x^2w_{xx}^2 + 20\gamma w_x^3w_{xxx} + 10a\gamma w_x^3w_{xx} \\ + \lambda w_y^2w_x^2 = 0, \end{aligned} \quad (14)$$

$$\begin{aligned} 3w_x^2w_{xt} + 3w_xw_tw_{xx} + aw_tw_x^2 - 5A\alpha w_xw_{xx}w_{xxx} \\ - 6A\alpha w_xw_{xx}^2 - \frac{7}{2}aA\alpha w_x^2w_{xxx} - \frac{3}{2}\alpha Aw_{xx}^3 - \frac{3}{2}a^2\alpha Aw_x^2w_{xx} + 6\alpha w_x^2w_{xx} \\ + a\alpha w_x^3 + 15\beta w_xw_{xx}^2 + 10\beta w_x^2w_{xxx} + 6a\beta w_x^2w_{xx} + 15\gamma w_{xx}^3 \\ + 59\gamma w_xw_{xx}w_{xxx} + 14\gamma w_x^2w_{xxxx} + 15a\gamma w_xw_{xx}^2 + 10a\gamma w_x^2w_{xxx} \\ + \lambda w_{yy}w_x^2 + 4\lambda w_xw_yw_{xy} + \lambda w_y^2w_{xx} + a\lambda w_y^2w_x = 0, \end{aligned} \quad (15)$$

$$\begin{aligned} 3w_{xt}w_{xx} + 3w_xw_{xxt} + w_tw_{xxx} + 2aw_xw_{xt} + aw_tw_{xx} - \alpha Aw_{xxx}^2 \\ - a^2\alpha Aw_{xx}^2 - 3aA\alpha w_{xx}w_{xxx} - \alpha Aw_{xx}w_{xxxx} - \alpha Aw_xw_{xxxx} \\ - \alpha a^2Aw_xw_{xxx} + 3b\alpha w_{xx}^2 + 4b\alpha w_xw_{xxx} + 3ab\alpha w_xw_{xx} + 9\beta w_{xx}w_{xxx} \\ + 4\beta w_xw_{xxxx} + 3a\beta w_{xx}^2 + 4a\beta w_xw_{xxx} + 9\gamma w_{xxx}^2 + 13\gamma w_{xx}w_{xxxx} \\ + 5\gamma w_xw_{xxxxx} + 10a\gamma w_{xx}w_{xxx} + 5a\gamma w_xw_{xxxx} + 2\lambda w_{xy}^2 + 2\lambda w_xw_{xyy} \\ + \lambda w_{yy}w_{xx} + 2\lambda w_yw_{xyy} + a\lambda w_{yy}w_x + 2a\lambda w_yw_{xy} = 0, \end{aligned} \quad (16)$$

$$\begin{aligned} w_{xxxt} + aw_{xxt} + b\alpha w_{xxxx} + ab\alpha w_{xxx} + a\beta w_{xxxx}\gamma w_{xxxxx} \\ + a\gamma w_{xxxxx} + \lambda w_{xyyy} + a\lambda w_{xyy} = 0. \end{aligned} \quad (17)$$

The solution of equations (13)-(17) is of the form

$$w(x, t) = w_0 + e^{k_1x + k_2t + k_3y + k_4}, \quad (18)$$

where w_0, k_4 are arbitrary constants and k_1, k_2, k_3 are constants to be determined by the following equations

$$\gamma k_1^5 + \beta k_1^4 + a\gamma k_1^4 + k_1^3(b\alpha + a\beta) + k_1^2(k_2 + ab\alpha) + k_1(ak_2 + \lambda k_3^2) + a\lambda k_3^2 = 0, \quad (19)$$

$$\begin{aligned} 7k_1^3k_2 + 3ak_1^2k_2 + k_1^6(31\gamma - 2\alpha A) + k_1^5(15\beta + 15a\gamma - 4a\alpha A) \\ + k_1^4(7a\beta + 7b\alpha - 2\alpha a^2A) + 3ab\alpha k_1^3 + 7\lambda k_3^2k_1^2 + 3a\lambda k_1k_3^2 = 0, \end{aligned} \quad (20)$$

$$\begin{aligned} 6k_1^3k_2 + k_1^2(ak_2 + 6\lambda k_3^2) + k_1^6(-7\alpha A + 90\gamma) + k_1^5\left(-\frac{19}{2}\alpha aA + 25\beta + 25a\gamma\right) \\ + k_1^4\left(-\frac{5}{2}\alpha a^2A + 6\alpha + 6a\beta\right) + a\alpha k_1^3 + a\lambda k_1k_3^2 = 0, \end{aligned} \quad (21)$$

$$k_1^3 k_2 + k_1^6 \left(-\frac{16}{3} \alpha A a^2 + 65 \gamma \right) + k_1^5 \left(-\frac{9}{2} \alpha a A + 10 \beta + 10 a \gamma \right) - \frac{1}{2} \alpha A a^2 k_1^4 + \lambda k_1^2 k_3^2 = 0, \quad (22)$$

$$15 \gamma - \frac{5}{4} \alpha A = 0, \quad (23)$$

and the constants a, b are determined by

$$a = -\frac{\beta}{\gamma - \frac{\alpha A}{2}}, \quad (24)$$

$$b = -\frac{\beta}{\gamma - \frac{\alpha A}{2}} \left(\frac{\alpha \beta A}{2(\gamma - \frac{\alpha A}{2})} - 1 \right). \quad (25)$$

Substituting (18) and (10) in (2), we obtain a exact solution of equation (1)

$$u(x, y, t) = -\frac{12\gamma k_1^2}{\alpha} \frac{e^{2(k_1 x + k_2 t + k_3 y + k_4)}}{[w_0 + e^{k_1 x + k_2 t + k_3 y + k_4}]^2} + \frac{12\gamma k_1^2}{\alpha} \frac{e^{(k_1 x + k_2 t + k_3 y + k_4)}}{[w_0 + e^{k_1 x + k_2 t + k_3 y + k_4}]} + \frac{12a\gamma k_1}{\alpha} \frac{e^{(k_1 x + k_2 t + k_3 y + k_4)}}{[w_0 + e^{k_1 x + k_2 t + k_3 y + k_4}]} + b. \quad (26)$$

Consider the (2+1) dimensional equation

$$u_t + u^p u_x + \alpha u + \beta u^q - u_{xx} + \gamma u_{xxx} + u_{yy} = 0. \quad (27)$$

3. EXPONENTIAL SOLUTIONS OF EQUATION (27)

Case 1: $p = q = 1$ Equation (27) reduces to

$$u_t + uu_x + \alpha u + \beta u - u_{xx} + \gamma u_{xxx} + u_{yy} = 0. \quad (28)$$

In order that the highest nonlinear term uu_x partially balance with the third order derivative term u_{xxx} , we suppose that the solution of (28) is of the form

$$u(x, t, y) = f'' w_x^2 + f' w_{xx} + a f' w_x + b, \quad w = w(x, t, y), \quad (29)$$

where the functions f and w as well as the constants a and b are to be determined. It follows

from (29) that

$$u_t = f'''w_t w_x^2 + 2f''w_{xt}w_x + f''w_t w_{xx} + f'w_{xxt} + af''w_t w_x + af'w_{xt}, \quad (30)$$

$$\begin{aligned} uu_x = & f''f'''w_x^5 + 3f''^2w_x^3w_{xx} + f'f''w_x^2w_{xxx} + af''^2w_x^4 + 5af'f''w_x^2w_{xx} \\ & + f'f'''w_x^3w_{xx} + 3f'f''w_xw_{xx}^2 + f'^2w_{xx}w_{xxx} + af'^2w_{xx}^2 \\ & + af'f'''w_x^4 + af'^2w_xw_{xxx} + a^2f'^2w_xw_{xx} + a^2f'f''w_x^3 + bf'''w_x^3 \\ & + 3bf''w_xw_{xx} + bf'w_{xxx} + abf''w_x^2 + abf'w_{xx}, \end{aligned} \quad (31)$$

$$\begin{aligned} -u_{xx} = & -[f^{IV}w_x^4 + 6f'''w_x^2w_{xx} + 3f''w_{xx}^2 + 4f''w_xw_{xxx} + f'w_{xxxx} \\ & + af'w_{xxx} + af'''w_x^3 + 3af''w_xw_{xx}], \end{aligned} \quad (32)$$

$$\begin{aligned} \gamma u_{xxx} = & \gamma[f^Vw_x^5 + 10f^{IV}w_x^3w_{xx} + 15f'''w_xw_{xx}^2 + 10f'''w_x^2w_{xxx} + 10f''w_{xx}w_{xxx} + 5f''w_xw_{xxxx} \\ & + f'w_{xxxxx} + af^{IV}w_x^4 + 6af'''w_x^2w_{xx} + 3af''w_{xx}^2 + 4af''w_xw_{xxx} + af'w_{xxxx}] \end{aligned} \quad (33)$$

$$\begin{aligned} u_{yy} = & f^{IV}e_y^2w_x^2 + f'''w_{yyy}w_x^2 + 4f'''w_yw_xw_{xy} + 2f''w_{xy}^2 + 2f''w_{xyy}w_x + f'''w_y^2w_{xx} \\ & + f''w_{yy}w_{xx} + 2f''w_{xxy}w_y + f'w_{xxyy} + af'''w_y^2w_x + af''w_{yy}w_x \\ & + 2af''w_{xy}w_y + af'w_{xyy}. \end{aligned} \quad (34)$$

First collecting the terms with w_x^5 in (31), and (33), and setting its coefficient to zero, we obtain an ordinary differential equation for $f(w)$:

$$f''f''' + \gamma f^V = 0, \quad (35)$$

then the ODE (35) has two solutions

$$f = \pm 12\gamma \log w, \quad (36)$$

thereby

$$\begin{aligned} f''^2 = & -\frac{2\gamma}{\alpha}f^{IV}, \quad f'^2 - \frac{12\gamma}{\alpha}f'', \quad f''f''' = -\frac{\gamma}{\alpha}f^V, \\ f'f''' = & \frac{4\gamma}{\alpha}f^{IV}, \quad f'f'' = -\frac{6\gamma}{\alpha}f'''. \end{aligned} \quad (37)$$

Next substituting(30)-(34) into the left hand side of (28) and using (37), we obtain

$$\begin{aligned}
 u_t + uu_x + \alpha u + \beta u \\
 -u_{xx} + \gamma u_{xxx} + u_{yy} = & [w_{xxt} + aw_{xt} + bw_{xxx} + abw_{xx} + (\alpha + \beta)(w_{xx} + aw_x) \\
 & -w_{xxxx} - aw_{xxx} + \gamma(w_{xxxxx} + aw_{xxx}) + w_{xyy} + aw_{xyy}] f' \\
 & + [2w_x w_{xt} + w_t w_{xx} + aw_t w_x - Aw_{xx} w_{xxx} - aAw_{xx}^2 - aAw_x w_{xxx} \\
 & -a^2 Aw_x w_{xx} + 3bw_x w_{xx} + abw_x^2 + (\alpha + \beta)w_x^2 - 3w_{xx}^2 - 4w_x w_{xxx} \\
 & -3aw_x w_{xx} + \gamma(10w_{xx} w_{xxx} + 5w_x w_{xxxx} + 3aw_{xx}^2 + 4aw_x w_{xxx}) \\
 & + 2w_{xy}^2 + 2w_x w_{xyy} + w_{yy} w_{xx} + 2w_y w_{xy} + aw_{yy} w_x \\
 & + 2aw_y w_{xy}] f'' + \left[w_t w_x^2 - \frac{A}{2} w_x^2 w_{xxx} - \frac{5A}{2} w_x w_{xx}^2 - \frac{a^2 A}{2} w_x^3 \right. \\
 & -bw_x^3 - 6w_x^2 w_{xx} - aw_x^3 + \gamma(10w_x^2 w_{xxx} + 15w_x w_{xx}^2 + 6aw_x^2 w_{xx}) \\
 & + w_{yy} w_x^2 + 4w_x w_y w_{xy} + w_y^2 w_{xx} + aw_y^2 w_x] f''' + \left[-\frac{5A}{6} w_x^3 w_{xx} \right. \\
 & \left. - (1 + \frac{aA}{2}) w_x^4 + \gamma(10w_x^3 w_{xx} + aw_x^4) + w_y^2 w_x^2 \right] f^{IV} \\
 & + \left[-\frac{A}{12} w_x^5 + \gamma w_x^5 \right] f^V + (\alpha + \beta)b, \tag{38}
 \end{aligned}$$

Setting the coefficients of f^V , f^{IV} , f''' , f'' and f' in (38) to zero respectively, we obtain the following five equations for the determination of $w(x, t)$:

$$\begin{aligned}
 w_{xxt} + aw_{xt} + bw_{xxx} + abw_{xx} + (\alpha + \beta)(w_{xx} + aw_x) - w_{xxxx} - aw_{xxx} \\
 + \gamma(w_{xxxxx} + aw_{xxx}) + w_{xyy} + aw_{xyy} = 0, \tag{39}
 \end{aligned}$$

$$\begin{aligned}
 2w_x w_{xt} + w_t w_{xx} + aw_t w_x - Aw_{xx} w_{xxx} - aAw_{xx}^2 - aAw_x w_{xxx} \\
 -a^2 Aw_x w_{xx} + 3bw_x w_{xx} + abw_x^2 + (\alpha + \beta)w_x^2 - 3w_{xx}^2 - 4w_x w_{xxx} \\
 -3aw_x w_{xx} + \gamma(10w_{xx} w_{xxx} + 5w_x w_{xxxx} + 3aw_{xx}^2 + 4aw_x w_{xxx}) \\
 + 2w_{xy}^2 + 2w_x w_{xyy} + w_{yy} w_{xx} + 2w_y w_{xy} + aw_{yy} w_x + 2aw_y w_{xy} = 0, \tag{40}
 \end{aligned}$$

$$\begin{aligned}
 w_t w_x^2 - \frac{A}{2} w_x^2 w_{xxx} - \frac{5A}{2} w_x w_{xx}^2 - \frac{a^2 A}{2} w_x^3 - bw_x^3 - 6w_x^2 w_{xx} - aw_x^3 + aw_y^2 w_x \\
 + \gamma(10w_x^2 w_{xxx} + 15w_x w_{xx}^2 + 6aw_x^2 w_{xx}) + w_{yy} w_x^2 + 4w_x w_y w_{xy} + w_y^2 w_{xx} = 0, \tag{41}
 \end{aligned}$$

$$\begin{aligned}
 -\frac{5A}{6} w_x^3 w_{xx} - (1 + \frac{aA}{2}) w_x^4 + \gamma(10w_x^3 w_{xx} + aw_x^4) + w_y^2 w_x^2 = 0, \tag{42}
 \end{aligned}$$

$$(\alpha + \beta)b = 0. \tag{43}$$

The solution of equations (39)-(43) is of the form

$$w(x, t) = w_0 + e^{k_1 x + k_2 t + k_3 y + k_4}, \quad (44)$$

where w_0, k_4 are arbitrary constants and k_1, k_2, k_3 are constants to be determined by the following equations

$$-\frac{5}{12}k_1^5 - \left(\frac{aA}{2} + 1\right)k_1^4 + \gamma(10k_1^5 + ak_1^4) + k_3^2 k_1^2 = 0, \quad (45)$$

$$-2Ak_1^5 - \left(\frac{5aA}{2} + 6\right)k_1^4 - \frac{a^2 A}{2}k_1^3 - ak_1^3 + \gamma(10k_1^5 + 15k_1^3 + 6ak_1^4) + 6k_3^2 k_1^2 + ak_1 k_3^2 = 0, \quad (46)$$

$$3k_1^2 k_2 + k_1 k_2 - Ak_1^5 - 2aAk_1^4 - 7k_1^4 - a^2 Ak_1^3 + (\alpha + \beta)k_1^2 - 3ak_1^3 + \gamma(15k_1^5 + 7ak_1^4) + 7k_1^2 k_3^2 + 3ak_1 k_3^2 = 0, \quad (47)$$

$$k_1^2 k_2 + ak_1 k_2 + (\alpha + \beta)(k_1^2 + ak_1) - k_1^4 - ak_1^3 + \gamma(k_1^5 + ak_1^4) + (1 + a)k_1^2 k_3^2 = 0, \quad (48)$$

$$b = 0. \quad (49)$$

Substituting (44) and (36) in (29), we obtain a exact solution of equation (27)

$$u(x, y, t) = 12\gamma e^{(k_1 x + k_2 t + k_3 y + k_4)} \left[-\frac{k_1^2}{[w_0 + e^{k_1 x + k_2 t + k_3 y + k_4}]^2} + \frac{k_1^2}{[w_0 + e^{k_1 x + k_2 t + k_3 y + k_4}]} + \frac{ak_1}{[w_0 + e^{k_1 x + k_2 t + k_3 y + k_4}]} \right]. \quad (50)$$

4. RESULTS AND CONCLUTIONS

In this paper, we obtain the exact solution of (1) and (27) are

$$u(x, y, t) = -\frac{12\gamma k_1^2}{\alpha} \frac{e^{2(k_1 x + k_2 t + k_3 y + k_4)}}{[w_0 + e^{k_1 x + k_2 t + k_3 y + k_4}]^2} + \frac{12\gamma k_1^2}{\alpha} \frac{e^{(k_1 x + k_2 t + k_3 y + k_4)}}{[w_0 + e^{k_1 x + k_2 t + k_3 y + k_4}]} + \frac{12a\gamma k_1}{\alpha} \frac{e^{(k_1 x + k_2 t + k_3 y + k_4)}}{[w_0 + e^{k_1 x + k_2 t + k_3 y + k_4}]} + b \quad (51)$$

$$u(x, y, t) = 12\gamma e^{(k_1 x + k_2 t + k_3 y + k_4)} \left[-\frac{k_1^2}{[w_0 + e^{k_1 x + k_2 t + k_3 y + k_4}]^2} + \frac{k_1^2}{[w_0 + e^{k_1 x + k_2 t + k_3 y + k_4}]} + \frac{ak_1}{[w_0 + e^{k_1 x + k_2 t + k_3 y + k_4}]} \right]. \quad (52)$$

The exact solution of (1) and (27) are in terms of exponential functions.

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