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MINIMUM DOM STRONG DOMINATING ENERGY OF GRAPH

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Abstract: Let $G = (n, m)$ be a simple graph. The sum of the absolute values of eigen values of its adjacency matrix is called as energy of the graph. The total π-electron energy of conjugated hydro carbon molecules is closely connected with this graph energy. In recent time, various energies are defined with graph matrices. In this paper, new energy Dom strong dominating energy of graph is defined. Dom strong dominating energy is computed for some standard graph.

Key Words: adjacency matrix, dominating set, dom strong dominating set

1. Introduction

Many matrices can be associated to a graph like adjacency matrix, Laplacian matrix, distance matrix. From the spectrum of the graph, some structural properties can be deduced. Through adjacency or laplacian spectrum we can't determine the graph. Energy of graph is related to the spectrum of the graph. In Huckel molecular theory, the total energy of π-electrons is equal to sum of the energies of all π-electrons in the molecule. In 1978, I. Gutman defined energy of graph mathematically. Many mathematical properties of energy of graphs are being investigated. Motivated by this energy of graph, new energy Minimum Dom strong dominating energy of graph is introduced. The properties of Dom

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strong dominated energy of graph are discussed with its bounds. The minimum Dom strong dominating energy of graphs like complete graph and star graph are computed.

2. Preliminaries

Definition 2.1. *Let G be simple graph whose vertex set is V and edge set is E. The adjacency matrix of the graph is given by,*

$$A = \begin{cases} 1 & \textit{if } v_i \textit{ is adjacent to } v_j \\ 0 & \textit{otherwise.} \end{cases}$$

Let $P_G(X)$ be the characteristic polynomial of the adjacency matrix. Let $\lambda_1, \lambda_2, \cdots, \lambda_n$ be the adjacency eigen values of the graph G. The ordinary energy of graph [4] is defined as the sum of the absolute value of eigen values of adjacency matrix A. $E(G) = \sum_{i=1}^{n} |\lambda_i|$ where λ_i is the eigen values of A.

Definition 2.2. Let G be a simple graph with n vertices and m edges. Let $V = \{v_1, v_2, , v_n\}$ be the vertex set. A subset D of V is Dom strong dominating set [7] if every $v \in V - D$, there exist $v_1, v_2 \in D$ such that $v_1v, v_2v \in E(G)$ and $\deg(v_1) \geq \deg(v)$. The Dom strong domination number $\gamma_{DSD}(G)$ is the minimum cardinality taken over all the minimal Dom strong dominating sets of G.

Let DSD be the minimum Dom strong dominating set of graph G of order $n \times n$ is defined as

$$A_{DSD}(G) = \begin{cases} 1 & \text{if } v_iv_j \in E \\ 1 & \text{if} i = j, v_i \in D \\ 0 & \text{otherwise.} \end{cases}$$

The characteristic polynomial is defined as $P(G, \lambda) = \det(\lambda I - A_{DD}(G))$. Let $\lambda_1, \lambda_2, \cdots, \lambda_n$ be the eigen values of $A_{DD}(G)$. The minimal Dom strong dominating energy of G is defined as $E_{DSD}(G) = \sum_{i=1}^{n} |\lambda_i|$.

Example 2.1. Consider the simple graph G with 4 vertices and 5 edges.

The minimum Dom strong dominating set $= \{v_1, v_3\}$ such that $\deg(v_1) \geq \deg(v_4)$ and $\deg(v_2) \geq \deg(v_4)$. The Dom strong dominating matrix is defined

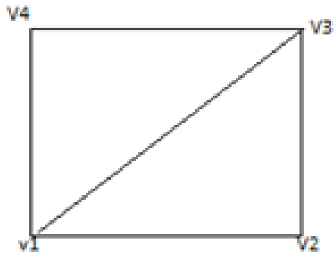
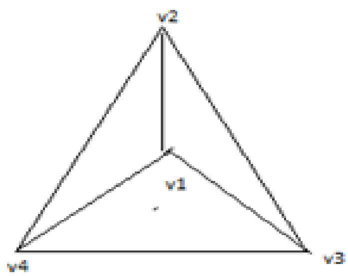


Figure 1: Graph G

by $A_{DSD}(G) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix}$

The characteristic polynomial of $A_{DSD}(G) = x^4 - 2x^3 - 4x^2$.
The eigen values are $-1.2361, 0, 0, 3.2361$
Dom strong dominating energy of graph, $E_{DSD}(G) = 4.4722$.

Example 2.2. Consider the complete graph K_4.



The minimum Dom strong dominating set $= \{v_1, v_3\}$.

The Dom strong dominating matrix $A_{DD}(G) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$

The characteristic polynomial is $x^4 - 2x^3 - 5x^2 - 2x$.
The eigen values are $-1, -0.5616, 0, 3.5616$
Dom strong dominating energy, $E_{DSD}(G) = 5.1232$.

3. Property of Dom Strong Dominating Energy

Theorem 3.1. *Let $G = (V, E)$ be a simple graph. Let DSD be the minimum Dom strong dominating set of G. Let $g_n(G, \lambda) = c_0\lambda^n + c_1\lambda^{n-1} + c_2\lambda^{n-2} + ... + c_n$ be characteristic polynomial of G. Then*

(i) $C_0 = 1$

(ii) $C_1 = -|DSD|$

Proof. (i) From the definition of $g_n(G, \lambda)$, it follows that $C_0 = 1$.

(ii) Since the sum of the diagonal elements of $A_{DD}(G)$ is equal to $|DSD|$, the sum of the determinants of all 1×1 principal sub matrices of $A_{DSD}(G)$ which is equal to $|DSD|$.

Thus $(-1)^1 C_1 = |DSD|$.

Theorem 3.2. *Let $\lambda_1, \lambda_2, \cdots, \lambda_n$ be the eigen values of $A_{DSD}(G)$, then* $\sum_{i=1}^{n} \lambda_i^2 = 2|E| + |DSD|$.

Proof. The sum of squares of the eigenvalues of $A_{DSD}(G)$ is equal to $A_{DSD}(G)^2$. Therefore,

$$\begin{aligned}\sum_{i=1}^{n} \lambda_i^2 &= \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}a_{ji} \\ &= \sum_{i<j} (a_{ij})^2 + \sum_{i=1}^{n} a_{ii}^2 \\ &= 2|E| + |DSD|.\end{aligned}$$

Theorem 3.3. *Let G be a simple graph with n vertices and m edges. Let DSD be the minimum Dom strong dominating set of G. Then* $E_{DSD}(G) \leq \sqrt{n(2m + |DSD|)}$.

Proof. Let $\lambda_1, \lambda_2, \cdots, \lambda_n$ be the eigenvalues of $A_{DSD}(G)$. By Cauchy-Schwartz inequality,

$$\left(\sum_{i=1}^{n} a_i b_i\right)^2 \leq \left(\sum_{i=1}^{n} a_i^2\right)\left(\sum_{i=1}^{n} b_i^2\right)$$

We choose $a_i = 1$ and $b_i = |\lambda_i|$.

By theorem 3.2, $E_{DSD}(G)^2 = \left(\sum_{i=1}^{n} |\lambda_i|\right)^2 \leq n\left(\sum_{i=1}^{n} |\lambda_i|\right) = n\sum_{i=1}^{n} \lambda_i^2 = n(2m + |DSD|)$.

Hence the proof.

Theorem 3.4. *Let G be a simple graph with n vertices and m edges and let DSD be the minimum Dom strong dominating set of G. If* $D^{'} = |\det(A_{DSD}(G))|$, *then* $E_{DSD}(G) \geq \sqrt{2m + |DSD| + n(n-1)(D^{'})^{2/n}}$.

Proof.

$$\begin{aligned} E_{DSD}(G)^2 &= \left(\sum_{i=1}^{n} |\lambda_i|\right)^2 \\ &= \left(\sum_{i=1}^{n} |\lambda_i|\right)\left(\sum_{j=1}^{n} |\lambda_j|\right) \\ &= \sum_{i=1}^{n} |\lambda_i|^2 + \sum_{i\neq j} |\lambda_i||\lambda_j| \end{aligned}$$

Using the relation between arithmetic and geometric mean, we obtain

$$\frac{1}{n(n-1)} \sum_{i\neq j} |\lambda_i||\lambda_j| \geq \left(\prod_{i\neq j} |\lambda_i||\lambda_j|\right)^{1/n(n-1)}.$$

Therefore,

$$\begin{aligned} (E_{DSD}(G))^2 &\geq \sum_{i=1}^{n} |\lambda_i|^2 + n(n-1)\left(\prod_{i\neq j} |\lambda_i||\lambda_j|\right)^{1/n(n-1)} \\ &\geq \sum_{i=1}^{n} |\lambda_i|^2 + n(n-1)\left(\prod_{i\neq j} |\lambda_i|^{2(n-1)}\right)^{1/n(n-1)} \\ &= \sum_{i=1}^{n} |\lambda_i|^2 + n(n-1)|\prod_{i=1}^{n} \lambda_i|^{2/n} \\ &= 2m + |DSD| + n(n-1)(D^{'})^{2/n}. \end{aligned}$$

Hence the theorem holds true.

4. Minimum Dom Strong Dominating Energies of Some Families of Graphs

Theorem 4.1. *For $n \geq 2$, the minimum Dom strong dominating energy of complete graph K_n is $2n-1$.*

Proof. For complete graph K_n, the minimum Dom strong dominating set $= \{v_1, v_3\}$.

$$A_{DD}(K_n) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & \cdots & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & \cdots & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & \cdots & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & \cdots & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & \cdots & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & \cdots & 1 & 1 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \ddots & \cdots & \cdots \\ 1 & 1 & 1 & 1 & 1 & 1 & \cdots & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & \cdots & 1 & 0 \end{pmatrix}.$$

The characteristic polynomial of $A_{DD}(K_n)$ is

$$g_n(K_n, \lambda) = \begin{pmatrix} \lambda-1 & -1 & -1 & -1 & -1 & -1 & \cdots & -1 & -1 \\ -1 & \lambda & -1 & -1 & -1 & -1 & \cdots & -1 & -1 \\ -1 & -1 & \lambda-1 & -1 & -1 & -1 & \cdots & -1 & -1 \\ -1 & -1 & -1 & \lambda & -1 & -1 & \cdots & -1 & -1 \\ -1 & -1 & -1 & -1 & \lambda & -1 & \cdots & -1 & -1 \\ -1 & -1 & -1 & -1 & -1 & \lambda & \cdots & -1 & -1 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \ddots & \cdots & \cdots \\ -1 & -1 & -1 & -1 & -1 & -1 & \cdots & \lambda & -1 \\ -1 & -1 & -1 & -1 & -1 & -1 & \cdots & -1 & \lambda \end{pmatrix},$$

$R_i \to R_i - R_2,\ i = 3, 4, \cdots, n.$

$$\sim \begin{pmatrix} \lambda-1 & -1 & -1 & -1 & -1 & -1 & \cdots & -1 & -1 \\ -1 & \lambda & -1 & -1 & -1 & -1 & \cdots & -1 & -1 \\ 0 & -(\lambda+1) & \lambda & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & -(\lambda+1) & 0 & \lambda+1 & 1 & 1 & \cdots & 1 & 1 \\ 0 & -(\lambda+1) & 0 & 0 & \lambda+1 & 1 & \cdots & 1 & 1 \\ 0 & -(\lambda+1) & 0 & 0 & 0 & \lambda+1 & \cdots & 1 & 1 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \ddots & \cdots & \cdots \\ 0 & -(\lambda+1) & 0 & 0 & 0 & 0 & \cdots & \lambda+1 & 1 \\ 0 & -(\lambda+1) & 0 & 0 & 0 & 0 & \cdots & 0 & \lambda+1 \end{pmatrix}$$

$C_2 \to C_2 + \sum_{i=3}^{n} C_i.$

$$\sim \begin{pmatrix} \lambda-1 & -1 & -1 & -1 & -1 & -1 & \cdots & -1 & -1 \\ -1 & \lambda-(n-2) & -1 & -1 & -1 & -1 & \cdots & -1 & -1 \\ 0 & 1 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \lambda+1 & 1 & 1 & \cdots & 1 & 1 \\ 0 & 0 & 0 & 0 & \lambda+1 & 1 & \cdots & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & \lambda+1 & \cdots & 1 & 1 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \ddots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & \lambda+1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & \lambda+1 \end{pmatrix}$$

$= (\lambda+1)^{n-3}\left(\lambda^3 - \lambda^2(n-1) + \lambda(n-2)\right).$

Therefore, $\lambda = -1[(n-3)]\text{times}, \lambda = 0(1 \text{ time}), \lambda = n+2(1\text{time})$ Dom strong dominating energy of complete graph, $K_n = 2n-1$.

Theorem 4.2. *For $n > 2$, the minimum Dom strong dominating energy of Star graph S_n is $(n-2) + \sqrt{n-1}$.*

Proof. For Star graph, $S_{1,n-1}$ whose vertices are $\{v_1, v_2, \cdots, v_n\}$.

The minimum Dom strong dominating set $= \{v_1, v_2, \cdots, v_n\}$.

$$A_{DD}(S_n) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & \cdots & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & \cdots & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \ddots & \cdots & \cdots & \\ 1 & 0 & 0 & 0 & 0 & 0 & \cdots & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 1 \end{pmatrix}.$$

The characteristic polynomial of $A_{DD}(S_n)$ is

$$g_n(K_n,\lambda)=\begin{pmatrix}\lambda-1 & -1 & -1 & -1 & -1 & -1 & \cdots & -1 & -1\\ -1 & \lambda-1 & 0 & 0 & 0 & 0 & \cdots & 0 & 0\\ -1 & 0 & \lambda-1 & 0 & 0 & 0 & \cdots & 0 & 0\\ -1 & 0 & 0 & \lambda-1 & 0 & 0 & \cdots & 0 & 0\\ -1 & 0 & 0 & 0 & \lambda-1 & 0 & \cdots & 0 & 0\\ -1 & 0 & 0 & 0 & 0 & \lambda-1 & \cdots & 0 & 0\\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \ddots & \cdots & \cdots\\ -1 & 0 & 0 & 0 & 0 & 0 & \cdots & \lambda-1 & 0\\ -1 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & \lambda-1\end{pmatrix},$$

$R_i\to R_i-R_2,\ i=3,4,\cdots,n.$

$$\sim\begin{pmatrix}\lambda-1 & -1 & -1 & -1 & -1 & -1 & \cdots & -1 & -1\\ -1 & \lambda-1 & 0 & 0 & 0 & 0 & \cdots & 0 & 0\\ 0 & -(\lambda-1) & \lambda-1 & 0 & 0 & 0 & \cdots & 0 & 0\\ 0 & -(\lambda-1) & 0 & \lambda-1 & 0 & 0 & \cdots & 0 & 0\\ 0 & -(\lambda-1) & 0 & 0 & \lambda-1 & 0 & \cdots & 0 & 0\\ 0 & -(\lambda-1) & 0 & 0 & 0 & \lambda-1 & \cdots & 0 & 0\\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \ddots & \cdots & \cdots\\ 0 & -(\lambda-1) & 0 & 0 & 0 & 0 & \cdots & \lambda-1 & 0\\ 0 & -(\lambda-1) & 0 & 0 & 0 & 0 & \cdots & 0 & \lambda-1\end{pmatrix}$$

$C_2\to C_2+\sum_{i=3}^{n}C_i.$

$$\sim\begin{pmatrix}\lambda-1 & -(n-1) & -1 & -1 & -1 & -1 & \cdots & -1 & -1\\ -1 & \lambda-1 & 0 & 0 & 0 & 0 & \cdots & 0 & 0\\ 0 & 0 & \lambda-1 & 0 & 0 & 0 & \cdots & 0 & 0\\ 0 & 0 & 0 & \lambda-1 & 0 & 0 & \cdots & 0 & 0\\ 0 & 0 & 0 & 0 & \lambda-1 & 0 & \cdots & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & \lambda-1 & \cdots & 0 & 0\\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \ddots & \cdots & \cdots\\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & \lambda-1 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & \lambda-1\end{pmatrix}.$$

The characteristic equation is $(\lambda-1)^{n-2}\left(\lambda^2-2\lambda-n+2\right)=0$, $\lambda=1(n-2 \text{ times})$, $\lambda=\frac{2\pm2\sqrt{n-1}}{2}$.

The minimum Dom strong dominating energy of Star graph $S_n=(n-2)+\sqrt{n-1}$.

5. Conclusion

In this paper, new energy Dom strong dominating energy is introduced. The properties of Dom strong energy is discussed with their bounds. Dom strong energy is calculated for some standard graphs. Similarly, Dom strong energy can be calculated for various family of graphs.

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