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BUYER-VENDOR MODEL FOR DETERIORATING ITEMS INVOLVING BACK ORDERS, SCREENING PROCESS AND TRANSPORTATION COST

A. Sugirtha Veda Selvi[1], M. Ravithammal[2 §],
R. Uthayakumar[3], P. Muniappan[4]
[1,2]Department of Mathematics
Vels University
Chennai, 600117, Tamil Nadu, INDIA
[3]Department of Mathematics
Gandhigram Rural Institute - Deemed University
Dindigul, 624302, Tamil Nadu, INDIA
[4]Department of Mathematics
Sathyabam University
Chennai, 600117, Tamil Nadu, INDIA

Abstract: This paper deals with the buyer - vendor inventory replenishment policy for deteriorating items. The inventory situation with backorders, screening process and transportation cost is also analyzed in this paper. In addition, we develop total system cost for equal benefits of both the buyer and the vendor to minimizing annual total relevant cost. Finally, a numerical example is given to illustrate these results and managerial insights are provided.

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Key Words: inventory, order quantity, transportation cost, screening cost

1. Introduction

In every organization inventory plays a vital role which helps to increase the

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§Correspondence author

production and distribution process which are linked by inventory. In inventory management quality decision is the major component which affects customer's demand, customer satisfaction and inventory costs. So industries to face the challenging task like, maintaining the huge number of quantity and screening process. The objective is to increase the customer satisfaction which leads to higher profits.

Ravithammal et al. [7] analyzed buyer - vendor inventory model with fixed lifetime product with fixed and linear back orders. Ravithammal et al. [9] studied production inventory model for perishable items with fixed and linear backorders. Samak-Kulkarni and Rajhans [10] developed determination of optimum inventory model for minimizing total inventory Cost. Muniappan and Uthayakumar [4] developed mathematical model for computing optimal replenishment polices. Muniappan et al. [6] studied inventory model for a deteriorating item with time - dependent quadratic demand and delay in payment for two warehouses. Ravithammal et al. [8] considered production inventory model for buyer- manufacturer with quantity discount and completely backlogged shortages for fixed life time product.

J.-T. Hsu and L.-F. Hsu [1] developed EOQ model with imperfect quality items, inspection errors, shortage backordering, and sales returns. Muniappan et al. [5] studied production inventory model for vendor-buyer coordination with quantity discount, backordering and rework for fixed life time products. Khan et al. [2] developed supply chain model with errors in quality inspection and learning in production. Teerapabolarn and Khamrod [11] developed inventory models with backorders and defective items derived algebraically and AGM. Mohamad Y. Jaber and Ahmed M.A. El Saadany [3] analyzed economic production and remanufacturing model with learning effects. Yuan-Shyi Peter Chiu et al. [12] developed mathematical modeling for determining the replenishment policy for EMQ model with rework and multiple shipments.

The detailed description of the paper is as follows. In Section 2, assumption and notations are given. In Section 3, formulation of the model is given. In Section 4, numerical examples and sensitivity analysis illustrate the developed model. Finally conclusion and summary are presented.

2. Assumptions and Notations

The model use the following assumptions and notations.

Assumptions

(i) Demand rate is uniform and constant.

(ii) Shortages are allowed for buyer only.

(iii) Buyer screened the damaged items for resale.

(iv) For benefits of buyer and vendor the system cost is formed. The system cost can be written as $TC_S = TC_B + TC_V$

Notations:

A Vendor's unit setup cost per order
K Buyer's unit ordering cost per order
D Demand rate per time unit
B Backorders level
b Vendor's unit backorder cost per order per unit
Q Economic Order quantity
H_1Buyer's unit holding cost per order per unit
H_2Vendor's unit holding cost per order per unit
c_s Vendor's unit screening cost per order per unit
F Vendor's fixed transportation cost per delivery
U Vendor's unit variable cost for order handling and receiving
n Vendor's multiples of order
TC_B Buyer's total cost
TC_V Vendor's total cost
TC_S System cost

3. Formulation of the Model

The total cost for buyer has four components:

(i) The ordering cost which is equal to $\dfrac{KD}{Q}$

(ii) The holding cost which is equal to $\dfrac{H_1 B^2}{2Q}$

(iii) The back order cost which is equal to $\dfrac{b(Q-B)^2}{2Q}$

(iv) The screening cost which is equal to $\dfrac{c_s Q}{2}$

Hence, total cost for buyer can be written as

$$TC_B = \text{Ordering cost} + \text{Holding cost} + \text{Back Order Cost} \quad (3.1)$$
$$+ \text{Screening Cost} \quad (3.2)$$
$$= \frac{KD}{Q} + \frac{H_1 B^2}{2Q} + \frac{b(Q-B)^2}{2Q} + \frac{c_s Q}{2} \quad (3.3)$$

The total cost for vendor has three components:

(i) The setup cost which is equal to $\frac{AD}{nQ}$

(ii) The holding cost which is equal to $\frac{H_2(n-1)Q}{2}$

(iii) The transportation cost which is equal to $n(F+UQ)$

Hence, total cost for vendor can be written as

$$TC_V = \text{Setup cost} + \text{Holding cost} + \text{Transportaion Cost} \quad (3.4)$$
$$= \frac{AD}{nQ} + \frac{H_2(n-1)Q}{2} + n(F+UQ) \quad (3.5)$$

Now the system cost can be written as

$$TC_S = TC_B + TC_V \quad (3.6)$$
$$= \frac{KD}{Q} + \frac{AD}{nQ} + \frac{H_1 B^2}{2Q} + \frac{H_2(n-1)Q}{2} \quad (3.7)$$
$$+ \frac{b(Q-B)^2}{2Q} + \frac{c_s Q}{2} + n(F+UQ) \quad (3.8)$$

Therefore

$$\frac{\partial TC_S}{\partial B} = \frac{H_1 B}{Q} + \frac{-b(Q-B)}{Q} \quad (3.9)$$
$$\frac{\partial^2 TC_B}{\partial B^2} = \frac{H_1}{Q} + \frac{b}{Q} \quad (3.10)$$
$$\frac{\partial TC_S}{\partial Q} = \frac{-KD}{Q^2} - \frac{AD}{nQ^2} - \frac{H_1 B^2}{2Q^2} + \frac{(n-1)H_2}{2} \quad (3.11)$$
$$+ \frac{4Qb(Q-B) - 2b(Q-B)^2}{4Q^2} + \frac{Cs}{2} + nU \quad (3.12)$$
$$\frac{\partial^2 TC_S}{\partial Q^2} = \frac{2KD}{Q^3} + \frac{2AD}{nQ^3} + \frac{H_1 B^2}{Q^3} + \frac{bB^2}{Q^3} \quad (3.13)$$

For optimality $\dfrac{\partial TC_S}{\partial B} = 0$ and $\dfrac{\partial^2 TC_s}{\partial B^2} > 0$ we get,

$$B^* = \frac{bQ}{H_1 + b} \tag{3.14}$$

For optimality $\dfrac{\partial TC_S}{\partial Q} = 0$ and $\dfrac{\partial^2 TC_s}{\partial Q^2} > 0$ we get,

$$Q^* = \sqrt{\frac{2D[K + \frac{A}{n}]}{(H_1 + b)((n-1)H_2 + C_s + 2nU) + bH_1}} \tag{3.15}$$

Also, the profit of buyer and vendor is calculated as follows:
Profit of the buyer $P_B = \dfrac{TC_B}{TC_S} \times 100$
Profit of the vendor $P_V = \dfrac{TC_V}{TC_S} \times 100$

4. Numerical Example

Let $D = 10000$ units per year, $K = 1000\$$ per order, $A = 3000\$$ per order, $H_1 = 5\$$, $H_2 = 30\$$, $n = 3$, $C_s = 0.5$, $b = 25\$$, $F = 10$, $U = 0.2$.

The optimal solutions are $Q = 71.44$, $B = 59.53$, $TC_B = 1.4014 \times 10^5$, $TC_V = 1.4219 \times 10^5$, $TC_S = 2.8234 \times 10^5$, $P_B = 49.64\%$, $P_V = 50.36\%$.

4.1. Sensitivity Analysis

We now study the effects of changes in the value of parameters D, H_1, H_2, b, n,C_s on Q,b,TC_B,TC_V and TC_B of the Example 1. i.e., the sensitivity analysis is performed by changing one parameter by +50%, +25%, −25% and −50% and keeping the remaining parameters unchanged. The results are shown in Table 1.

		Q	B	TC_B	TC_V	TC_s	P_B	P_V
D	+50	87.50	72.91	1.7164×10^5	1.7414×10^5	3.4579×10^5	49.64	50.36
	+25	79.87	66.56	1.5569×10^5	1.5897×10^5	3.1566×10^5	49.64	50.36
	-25	61.87	51.58	1.2137×10^5	$1.2315X10^5$	$2.4452X10^5$	49.64	50.36
	-50	5.052	42.10	$9.9.97 \times 10^4$	1.0055×10^5	1.9965×10^5	49.63	50.37
h_1	+50	58.33	44.90	1.7162×10^5	1.7325×10^5	3.4487×10^5	49.76	50.24
	+25	63.90	51.12	1.5668×10^5	1.5849×10^5	3.1516×10^5	49.71	50.29
	-25	82.49	71.73	1.2138×10^5	1.2378×10^5	2.4516×10^5	49.51	50.49
	-50	101.03	91.85	9.9119×10^4	1.0210×10^5	2.0122×10^5	49.26	50.74
h_2	+50	58.75	48.96	1.7034×10^5	1.7291×10^5	3.4325×10^5	49.63	50.37
	+25	64.17	53.48	1.5597×10^5	1.5830×10^5	3.1427×10^5	49.63	5.037
	-25	81.91	68.26	1.2228×10^5	1.2401×10^5	2.4630×10^5	49.65	50.35
	-50	98.92	82.44	1.0132×10^5	1.0266×10^5	2.0398×10^5	49.67	50.38
c_s	+50	71.30	59.42	1.4043×10^5	1.4247×10^5	2.8290×10^5	49.64	50.36
	+25	71.37	59.47	1.4029×10^5	1.4233×10^5	2.8262×10^5	49.64	50.36
	-25	71.51	59.59	1.4000×10^5	1.4206×10^5	2.8206×10^5	49.64	50.36
	-50	71.58	59.65	1.3986×10^5	1.4192×10^5	2.8178×10^5	49.63	50.37
b	+50	58.33	51.47	1.7158×10^5	1.7352×10^5	3.4483×10^5	49.76	50.24
	+25	63.90	55.08	1.5665×10^5	1.5849×10^5	3.1514×10^5	49.71	52.29
	-25	80.38	64.14	1.2460×10^5	1.2690×10^5	2.5150×10^5	49.54	50.46
	-50	101.03	72.17	9.9185×10^4	1.0210×10^5	2.0129×10^5	49.28	50.72
n	+50	49.62	41.35	2.0164×10^5	1.3705×10^5	3.3869×10^5	59.54	40.46
	+25	58.04	48.37	1.7244×10^5	1.4032×10^5	3.1275×10^5	55.14	44.86
	-25	96.73	80.61	1.0361×10^5	1.3972×10^5	2.4333×10^5	42.58	57.42
	-50	161.54	139.62	6.0078×10^4	1.2070×10^5	1.8077×10^5	33.26	66.77

The profit of both vendor P_V and buyer P_B are slightly sensitive if changing the value of D, H_1, H_2, C_s, b and moderately sensitive if changing the value of n.

5. Conclusion

This paper develops buyer - vendor optimal inventory replenishment policy for a deteriorating item considering backorders, screening process and transportation cost. System cost is developed for minimizing the total relevant cost. It is also provides the equal benefits of buyer and vendor. The model is solved analytically to obtain the optimal solution of the problem. It is then illustrated with the help of numerical examples. The proposed model can further extended by taking more realistic assumptions such as quantity discounts, stochastic demand, finite replenishment rate, multi products and others.

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