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SINGLE PRODUCT MULTIPLE MANUFACTURES SUPPLY CHAIN MODEL FOR FIXED LIFETIME PRODUCT

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Abstract: In this paper production inventory model is developed for single product with multiple manufacturers. The developed model considered two part coordination scheme (non coordination and coordination). In non coordination scheme Manufacture1 and Manufacture2 produces the same product and shortages are allowed only for Manufacture1. In coordination scheme Manufacture1 stop their production, purchase products from Manufacture2. In order to stimulate sales and reduce inventory Manufacturer2 frequently offers quantity discount to Manufacture1 in coordination scheme. The aim of the study is to determining the optimum multiples of orders which will minimize the total inventory cost. A numerical example is given to illustrate the solution procedure of the model. Finally, based on this example, we conduct a sensitivity analysis of the model.

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1. Introduction

Products with limited shelf period like medicines, food and beverage etc., are manufactured by a large and small scale manufacturers. Sometimes the same product may be manufactured by a large and small scale manufacturer. In this situation, the small manufacturer (Manufacturer1) may decide to purchase the same product from the large scale manufacturer (Manufacturer2) instead of manufacturing by him. At that time, Manufacturer2 frequently offers discounted price to Manufacture1 if a large-enough quantity of an item is purchased. The model proposed in this paper deals with this case.

Fujiwara et al. [1] analyzed optimal and issuing policy for a two-stage inventory system for perishable products. Goyal and Gupta [2] studied integrated inventory models: the buyer-vendor coordination. Kaj - Mikael Bjork [3] analyzed a multi item fuzzy economic production quantity problem with a finite production rate. Kit Nam Francis Leung [4] presented an integrated production inventory system in a multi - stage multi- firm supply chain. Mahdi Tajbakhsh et al. [5] developed an inventory model with random discount offerings. Ravithammal et al. [10] studied deterministic production inventory model for buyer- manufacturer with quantity discount and completely backlogged shortages for fixed life time product. Muniappan et al. [8] developed production inventory model for vendor–buyer coordination with quantity discount, backordering and rework for fixed life time products. Ravithammal et al. [9] developed an integrated production inventory system for perishable items with fixed and linear backorders. Muniappan et al. [6] analyzed EOQ model for deteriorating items with inflation and time value of money considering time-dependent deteriorating rate and delay payments. Muniappan et al. [7] analyzed mathematical technique for computing optimal replenishment polices. Saoussen Krichen, Awatef Laabidi and Fouad Ben Abdelaziz [11] analyzed single supplier multiple cooperative retailer's inventory model with quantity discount and permissible delay in payments. Umamaheswari et al. [13] developed design and analysis of an optimal inventory model for perishable goods with fixed life time. Sharma Vikas et al. [12] studied EOQ models with optimal replenishment policy for perishable items taking account of time value of money.

Yongrui Duan et al. [14] analyzed buyer-vendor inventory coordination with quantity discount incentive for fixed lifetime product. In this model the situation is entirely varied and is applicable in case of inventory decision by two manufacturers. The remainder of this paper is organized as follows: Model Formulation, notation, assumption and are presented in Section 2. In Section 3, the numerical example is provided to illustrate the solution procedure. Section 4 is concluded with remarks.

2. Model Development

The developed model deals, with and without coordination strategy for Manufacturer1 and Manufacturer2. Quantity discount is offered by the Manufacturer2 in the model with coordination.

2.1. Assumptions and Notations

Assumptions

1. Demand is constant and production rate is greater than the demand of an item.
2. Without coordination shortages are permitted and with coordination shortages are not permitted to the Manufacturer1 i.e., There is no shortage for coordination and there is shortage for absence of coordination.
3. During the production run the production of the item is continuous and at a constant rate until production of quantity Q is complete and Manufacturer1 and Manufacturer2 produces a same product.
4. Under coordination strategy Manufacturer1 stop their produce, purchase the product with Manufacturer2.

Notations

D_1, D_2	Annual demand of Manufacturer1 and Manufacturer2
P_1, P_2	Annual production rate for Manufacturer1 and Manufacturer2
L	Life time of product
k_1, k_2	Manufacturer2 and Manufacturer1 setup costs per order, respectively
h_1, h_2	Manufacturer2 and Manufacturer1 holding costs, respectively
p_1, p_2	Delivered unit price paid by the Manufacturer2 and the Manufacturer1 respectively
Q	Manufacturer1 order quantity, $Q = Q_1 + Q_2$
Q_1	Amount remains in the inventory after satisfying

	the shortage demand
Q_2	Amount which is immediately taken to satisfy unfilled demand (Shortage period)
Q_0	Manufacturer2 order quantity
m	Manufacturer2 order multiple in the absence of coordination
n	Manufacturer2 order multiple under coordination
K	Manufacturer1 order multiple under coordination. KQ_0 Manufacture1's new order quantity
$d(K)$	Denotes the per unit dollar discount to the Manufacturer1 if he orders KQ_0 every time
TC_{M1}	Total cost of the Manufacturer1 without coordination
TC'_{M1}	Total cost of the Manufacturer1 with coordination
$TC_{M2}(m)$	Total cost of Manufacturer2 without coordination
$TC_{M2}(n)$	Total cost of Manufacturer2 with coordination.

Case i: Development of model without coordination

In this case Manufacture1 and Manufacture2 produce the same product and shortages are allowed for Manufacture1. The model is formulated as follows:

The total annual cost for the Manufacturer1 is given by

$$TC_{M1} = \frac{D_1 k_2}{Q} + \frac{1}{2Q}\left(\frac{P_1}{P_1 - D_1}\right)\left[h_2\left(Q\left(1 - \frac{D_1}{P_1}\right) - Q_2\right)^2 + Q_2^2 s_2\right].$$

Now $\frac{\partial TC_{M1}}{\partial Q} = 0$ and $\frac{\partial TC_{M1}}{\partial Q_2} = 0$, we get, $Q = \sqrt{\frac{2D_1k_2(h_2+s_2)}{h_2s_2}\left(\frac{P_1}{P_1-D_1}\right)}$ and $Q_2 = Q\left(1 - \frac{D_1}{P_1}\right)\frac{h_2}{h_2+s_2}$. Without coordination strategy, the Manufacturer1 order quantity is

$$Q = \sqrt{\frac{2D_1k_2\left(h_2+s_2\right)}{h_2s_2}\left(\frac{P_1}{P_1-D_1}\right)}$$

and optimum total cost

$$TC_{M1}=\sqrt{\frac{2D_1k_2h_2s_2}{h_2+s_2}\left(1-\frac{D_1}{P_1}\right)}.$$

The Manufacturer2 order is equal to some integer multiple of

$$Q_0 = \sqrt{\frac{2D_2k_1}{h_1}\left(\frac{P_2}{P_2-D_2}\right)},$$

i.e., order size $= mQ_0$ with the fixed intervals

$$t_0=\sqrt{\frac{2k_1}{D_2h_1}\left(\frac{P_2}{P_2-D_2}\right)}.$$

Here, average inventory held up per year of Manufacturer2 is given by

$$=\frac{(m-1)\,Q_0+(m-2)\,Q_0+\ldots.+Q_0+0Q_0}{m}=\frac{(m-1)Q_0}{2}$$

Now the total annual cost for the Manufacturer2 is given by

$$\begin{aligned}TC_{M2}(m)&=\frac{D_2k_1}{mQ_0}+\frac{(m-1)h_1Q_0}{2}\left(1-\frac{D_2}{P_2}\right)\\&=\frac{k_1}{m}\sqrt{\frac{D_2h_1}{2k_1}\left(\frac{P_2-D_2}{P_2}\right)}+(m-1)h_1\sqrt{\frac{D_2k_1}{2h_1}\left(\frac{P_2-D_2}{P_2}\right)}\end{aligned}$$

So without coordination, the Manufacturer2 model can be developed as follows

$$\min TC_{M2}(m)$$
$$s.t\begin{cases} mt_0\leq L,\\ m\geq 1\end{cases} \tag{1}$$

here $mt_0=L$ shows that the product is not overdue before they are sold up by the Manufacturer1.

Theorem 1. *Consider m^* be the optimum of (1), if $L^2=\frac{2k_1}{D_2h_1}\left(\frac{P_2}{P_2-D_2}\right)$, then*

$$m^*=min\left\{\left\lceil\sqrt{\frac{k_1h_2}{k_2h_1}+\frac{1}{4}}-\frac{1}{2}\right\rceil,\left\lceil\frac{L}{\sqrt{\frac{2k_1}{D_2h_1}\left(\frac{P_2}{P_2-D_2}\right)}}\right\rceil\right\}, \tag{2}$$

here $\lceil x\rceil$ is the least integer greater than or equal to x, $L^2\geq\frac{2k_1}{D_2h_1}\left(\frac{P_2}{P_2-D_2}\right)$ is to ensure that $m^*\geq 1$.

Proof. Obviously

$$\frac{d^2TC_{M2}(m)}{dm^2}=\frac{k_1}{m^3}\sqrt{\frac{2D_2h_1}{k_1}\left(\frac{P_2-D_2}{P_2}\right)}>0,$$

$TC_{M2}(m)$ is strictly convex in m. Considered m_1^* be the optimum of $TC_{M2}(m)$, then

$$m_1^* = \max\{\min\{m/TC_{M2}(m) = TC_{M2}(m+1)\}, 1\}$$
$$= \max\{\min\{m/m(m+1) = \frac{2D_2k_1}{Q_0^2\left(1-\frac{D_2}{P_2}\right)h_1}\}, 1\}$$
$$= \left\lceil \sqrt{\frac{h_2k_1}{h_1k_2}+\frac{1}{4}}-\frac{1}{2}\right\rceil = 1.$$

Put the value of $t_0$into the constraints in (1), then we have $m\sqrt{\frac{2k_1}{D_2h_1}\left(\frac{P_2}{P_2-D_2}\right)} = L$. Take $m_2^*=\frac{L}{\sqrt{\frac{2k_1}{D_2h_1}\left(\frac{P_2}{P_2-D_2}\right)}} = 1$, is true since $L^2 = \frac{2k_1}{D_2h_1}\left(\frac{P_2}{P_2-D_2}\right)$. $m^* = m_1^*$ where $m_1^* = m_2^*$, otherwise $m^* = m_2^*$. Therefore $m^* = \min\left\{m_{1>}^*, m_2^*\right\}$, if$L^2 = \frac{2k_2}{D_2h_2}\left(\frac{P_2}{P_2-D_2}\right)$. □

Remark 1. *Without coordination, the Manufacturer1 optimum total cost is TC_{M1}, order size is $\sqrt{\frac{2D_1k_2(h_2+s_2)}{h_2s_2}\left(\frac{P_1}{P_1-D_1}\right)}$ and the Manufacturer2 optimum total cost is $TC_{M2}(m^*)$, order size is $m^*\sqrt{\frac{2D_2k_1}{h_1}\left(\frac{P_2}{P_2-D_2}\right)}$.*

Case ii: Development of model with coordination. In coordination scheme, Manufacturer1 stop their production and purchase the product of Manufacturer2 and no shortages for Manufacturer1. In this strategy, Manufacturer2 given quantity discount with the discount factor $d(K)$, if Manufacture1 change his lot size by KQ_0, $K > 0$. Now the Manufacturer2 lot size is $nKQ_{0,}$where n is a positive integer and $KQ_0$is the Manufacture1 new order quantity. Now, the Manufacture1order quantity $Q_0 = \sqrt{\frac{2D_1k_2}{h_2}\left(\frac{P_1}{P_1-D_1}\right)}$ and optimum total cost $TC'_{M1}=\sqrt{2D_1k_2h_2\left(1-\frac{D_1}{P_1}\right)}$. Therefore, Manufacturer2 total cost

$$TC_{M2}(n) = \frac{D_2k_1}{nKQ_0} + \frac{(n-1)\left(1-\frac{D_2}{P_2}\right)h_1KQ_0}{2} + p_2D_2d(K). \tag{3}$$

In coordination discount strategy, the problem can be developed as follows

$$\min TC_{M2}(n)$$

$$\text{subject to} \begin{cases} nKt_0 \le L, \\ \frac{D_2 k_2}{KQ_0} + \frac{KQ_0\left(1-\frac{D_2}{P_2}\right)h_2}{2} - \sqrt{2D_2k_2h_2\left(1-\frac{D_2}{P_2}\right)} \le p_2 D_2 d(K), \\ n \ge 1, \end{cases} \tag{4}$$

Now$nKt_0 = L$shows's that the product is not overdue before they are sold up by the Manufacturer1. The second constraint shows that the Manufacturer1 cost under coordination cannot exceed that without coordination.

Theorem 2. *$TC_{M2}(n^*) = TC_{M2}(m^*)$ is true, if $m^*$is optimum of* (1) *and n^* be the optimum of* (4).

Proof. If the second constraint must be an equation ,then $p_2D_2d(K)$ takes smallest value and $TC_{M2}(n)$ is optimized.

i.e., $\frac{D_2k_2}{KQ_0}+\frac{KQ_0\left(1-\frac{D_2}{P_2}\right)h_2}{2}-\sqrt{2D_2k_2h_2\left(1-\frac{D_2}{P_2}\right)}=p_2D_2d(K)$

$$d(K) = \frac{\frac{D_2k_2}{KQ_0}+\frac{KQ_0\left(1-\frac{D_2}{P_2}\right)h_2}{2}-\sqrt{2D_2k_2h_2\left(1-\frac{D_2}{P_2}\right)}}{p_2D_2} \tag{5}$$

If $K = 1$, then $d(1) = \frac{\sqrt{2D_2k_2h_1\left(1-\frac{D_2}{P_2}\right)}-\sqrt{2D_2k_2h_1\left(1-\frac{D_2}{P_2}\right)}}{p_2D_2} = 0$. So if $K = 1$, then (4) is equivalent to (1). Therefore, $TC_{M2}(n^*) \le TC_{M2}(m^*)$ is true. □

Remark 2. *Theorem* (2)*, ensures that Manufacturer2 will get more benefit to compare with Manufacturer1if the manufacturer1 order size is $KQ_0, K > 0$ because optimum total cost under coordination is less than without coordination.*

Put equation (5) into equation (3), we have

$$TC_{M2}(n) = \frac{D_2k_1}{nKQ_0}+\frac{(n-1)\left(1-\frac{D_2}{P_2}\right)h_1KQ_0}{2} + p_2D_2\left(\frac{\frac{D_2k_2}{KQ_0}+\frac{KQ_0\left(1-\frac{D_2}{P_2}\right)h_2}{2}-\sqrt{2D_2k_2h_2\left(1-\frac{D_2}{P_2}\right)}}{p_2D_2}\right). \tag{6}$$

Let K^* be the optimum of $TC_{M2}(n)$, we have

$$K^*(n)=\frac{1}{Q_0}\sqrt{\frac{2D_2(\frac{k_1}{n}+k_2)}{\left(1-\frac{D_2}{P_2}\right)[(n-1)h_1+h_2]}} \tag{7}$$

From first constraint of (4), we have

$$\left(\frac{k_1}{n}+k_2\right)n^2=\frac{L^2Q_0^2h_2}{4k_2}\left(1-\frac{D_2}{P_2}\right)^2((n-1)h_1+h_2).$$

Take

$$g(n)=-k_2n^2+\left(\frac{D_2L^2}{2}\left(\frac{P_2-D_2}{P_2}\right)h_1-k_1\right)n + \frac{D_2L^2}{2}\left(\frac{P_2-D_2}{P_2}\right)(h_2-h_1). \tag{8}$$

Substituting (7) and $t_0=\sqrt{\frac{2k_2}{D_2h_2}\left(\frac{P_2}{P_2-D_2}\right)}$ into (3), we have

$$TC_{M2}(n)=\left[2D_2\left[k_1\left(1-\frac{D_2}{P_2}\right)h_1+\frac{k_1\left(1-\frac{D_2}{P_2}\right)[h_2-h_1]}{n} + nk_2\left(1-\frac{D_2}{P_2}\right)h_1+\left(1-\frac{D_2}{P_2}\right)k_2[h_2-h_1]\right]-2Dh_2k_2\left(1-\frac{D_2}{P_2}\right)\right]^{1/2}. \tag{9}$$

Therefore, (4) becomes

$$\min TC_{M2}(n)$$

$$\text{subject to}\ \begin{cases} g(n)=0, \\ n=1, \end{cases} \tag{10}$$

for $x\geq 0$, $\sqrt{x}$ is a strictly increasing so the above equation is equivalent to

$$\min\widetilde{TC_{M2}}(n)=D_2\left[k_1\left(1-\frac{D_2}{P_2}\right)h_1+\frac{k_1\left(1-\frac{D_2}{P_2}\right)[h_2-h_1]}{n}+nk_2\left(1-\frac{D_2}{P_2}\right)h_1 + \left(1-\frac{D_2}{P_2}\right)k_2[h_2-h_1]\right]$$

$$\text{subject to} \begin{cases} g(n) \geq 0, \\ n \geq 1 \end{cases} \tag{11}$$

Here, $\widetilde{TC_{M2}}(n)$ is convex when $h_2 = h_1$, since

$$\widetilde{TC_{M2}}''(n) = \frac{2D_2k_1\left(1-\frac{D_2}{P_2}\right)[h_2 - h_1]}{n^3} > 0,$$

otherwise it is concave. $g(n)$ is strictly concave because $g''(n) = -2k_2 < 0$.

Lemma 1. *Let n_1^* be the optimum of $\widetilde{TC_{M2}}(n)$for $n = 1$, then*

$$n_1^* = \begin{cases} \left\lceil \sqrt{\frac{k_1[h_2-h_1]}{k_2h_1} + \frac{1}{4}} - \frac{1}{2} \right\rceil, & \frac{k_1[h_2-h_1]}{k_2h_1} = 2 \\ 1, & \textit{otherwise} \end{cases} \tag{12}$$

Proof. $\widetilde{TC_{M2}}(n_1^*) = \min\left\{\widetilde{TC_{M2}}(n_1^*-1), \widetilde{TC_{M2}}(n_1^*+1)\right\}$ because n_1^* is the minimum of $\widetilde{TC_{M2}}(n)$,$n \geq 1$. Now

$$\widetilde{TC_{M2}}(n_1^*) - \widetilde{TC_{M2}}(n_1^*-1) = \frac{-D_2k_1\left(1-\frac{D_2}{P_2}\right)[h_2 - h_1]}{n_1^*(n_1^*-1)} + D_2k_2\left(1-\frac{D_2}{P_2}\right)h_1 = 0,$$

$$\left(n_1^*-\frac{1}{2}\right)^2 = \frac{k_1[h_2 - h_1]}{k_2h_1} + \frac{1}{4}. \tag{13}$$

Similarly, by $\widetilde{TC_{M2}}(n_1^*) - \widetilde{TC_{M2}}(n_1^*+1) = 0$, we have

$$\left(n_1^*+\frac{1}{2}\right)^2 = \frac{k_1[h_2 - h_1]}{k_2h_1} + \frac{1}{4} \tag{14}$$

Hence, if $\frac{k_1[h_2-h_1]}{k_2h_1} + \frac{1}{4} < 0$, $\widetilde{TC_{M2}}(n_1^*) = \widetilde{TC_{M2}}(n_1^*+1)$ for any given n, then $n_1^* = 1$. If $\frac{k_1[h_2-h_1]}{k_2h_1} + \frac{1}{4} \geq 0$ by (13) & (14), $\sqrt{\frac{k_1[h_2-h_1]}{k_2h_1} + \frac{1}{4}} - \frac{1}{2} \leq n_1^* \leq \sqrt{\frac{k_1[h_2-h_1]}{k_2h_1} + \frac{1}{4}} + \frac{1}{2}$. So $n_1^* = \left\lceil \sqrt{\frac{k_1[h_2-h_1]}{k_2h_1} + \frac{1}{4}} - \frac{1}{2} \right\rceil$. Also note that, if $0 < \frac{k_1[h_2-h_1]}{k_2h_1} < 2$ then $n_1^* = 1$, so (12) holds. □

3. Numerical Example

In this section, numerical examples are presented to illustrate the performance of developed model.

Example 1. Given k_1 = 300\$ per order, k_2 = 100\$ per order, h_1= 10\$ per year, h_2 = 5\$ per year, s_2 = 25\$ per year, P_1 = 8000 units per year, D_1 = 5000 units per year, P_2 = 15000 units per year, D_2 = 10000 units per year, p_2 = 30\$ per unit, $\alpha = 0.5$, L = 0.5 year.

The computational result shows the following optimal values k^*= 3.0000, $d(k^*)$ = 0.0015, $TC_{M2}(m^*)$= 2236.1, $TC_{M2}(n^*)$ = 1825.7, TC_{M1}= 1250, TC'_{M1}= 1369.3.

Example 2. Given k_1 = 400\$ per order, k_2 = 200\$ per order, h_1= 15\$ per year, h_2 = 10\$ per year, s_2 = 55\$ per year, P_1 = 12000 units per year, D_1 = 10000 units per year, P_2 = 20000 units per year, D_2 = 15000 units per year, p_2 = 30\$ per unit, $\alpha = 0.5$, $L = 0.5$ year. The computational result shows the following optimal values k^*= 1.7321, $d(k^*)$ = 0.0013, $TC_{M2}(m^*)$= 3354.1, $TC_{M2}(n^*)$ = 2835.2, TC_{M1}= 2375, TC'_{M1}= 2582.

3.1. Sensitivity Analysis

We now study the effects of changes in the value of system parameters h_1, h_2, k_1, k_2, s_2 on the Manufacturer1 and Manufacture2 minimum total relevant cost per unit time TC^*_{M1},TC'^*_{M1}, $TC^*_{M2}(m)$ and $TC^*_{M2}(n)$ of the Example 1. The sensitivity analysis is performed by taking one parameter at a time and keeping the remaining parameters unchanged. The results are shown in Table 1.
Computational result indicates that

1. The optimum total cost of Manufacturer2 under coordination is less than that without coordination. i.e., Manufacturer2 is comparatively highly benefited than Manufacturer1 in spite of giving quantity discount.

2. An increase in holding cost for Manufacturer2, the optimal total cost of Manufacturer1 and Manufacturer2 remain same or increase.

3. A reduce value of holding cost for Manufacturer1 tends to reduce in total cost for Manufacturer1 and Manufacturer2.

4. The set up cost for Manufacturer1 and Manufacturer2 decrease, automatically the total cost of Manufacturer1and Manufacturer2 gets decreased.

Table 1: Effect of changes in the parameters of the inventory

Parameter		k^*	$d(k^*)$	$TC_{M2}(m^*)$	$TC_{M2}(n^*)$	TC_{M1}	TC'_{M1}
	400	3.3541	0.0021	2582.0	2256.7	1250	1369.3
k_1	450	3.5178	0.0023	2738.6	2456.0	1250	1369.3
	500	3.6742	0.0026	2886.8	2646.8	1250	1369.3
	50	3.9686	0.0022	2236.1	2124.7	885	965.25
k_2	100	3.0000	0.0015	2236.1	1825.7	125	1369.3
	150	2.5981	0.0012	2236.1	1636.9	1531	1677.1
	35	3.0000	0.0015	2236.1	1825.7	1281	1369.3
s_2	45	3.0000	0.0015	2236.1	1825.7	1299	1369.3
	55	3.0000	0.0015	2236.1	1825.9	1311	1369.3
	12	3.0000	0.0015	2449.5	1825.7	1250	1369.3
h_1	14	3.0000	0.0015	2645.8	1825.7	1250	1369.3
	16	3.0000	0.0015	2828.4	1825.37	1250	1369.3
	6	3.0000	0.0017	2236.1	2000.0	1347	1500.0
h_2	7	3.0000	0.0018	2236.1	2160.2	1432	1620.2
	8	3.0000	0.0019	2236.1	2309.4	1508	1732.1

4. Conclusion

This study develops single product and multiple manufacturers for fixed life time products. This model assumes coordination and non coordination scheme. In absence of coordination, Manufacture1 and Manufacture2 produce the same product and shortages allowed for Manufacture1. Manufacture1 more benefited under absence of coordination. Under coordination scheme Manufacturer1 stop their produce and purchase items from Manufacture2 with quantity discount. This paper concludes coordination scheme is more benefited to Manufacture 2 even though he offers quantity discount to Manufacture1. It is proved that the quantity discount is the best strategy to achieve system optimization and win – win outcome of Manufacture2. The goal of this paper is to minimize the total relevant cost to determined optimal decision variables. Numerical examples are also provided to illustrate the proposed model. Sensitivity analysis on the parameter changes is also performed. The proposed model can be extended by considering factors like multiple products, random discount offering, credit periods etc,

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