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TWO-WAREHOUSE SUPPLY CHAIN MODEL FOR DETERIORATING ITEMS WITH RAMP-TYPE DEMAND

M. Ravithammal[1], R. Uthayakumar[2], S. Ganesh[3], P. Muniappan[4]
[1,3,4]Department of Mathematics
Sathyabama University
Chennai, Tamil Nadu, INDIA
[2]Department of Mathematics
Gandhigram Rural Institute
Deemed University
Dindigul, Tamil Nadu, INDIA

Abstract: Inventory control of products with finite lifetime is quite relevant with many business organizations. Since deteriorating items require special storage facilities, the retailer may have a small warehouse providing such facilities and so the need for an additional rented warehouse may arise. In this paper we have considered two warehouses (own warehouse (OW) and rented warehouse (RW)) inventory model with ramp type demand, replenishment rate is infinite, shortages are not allowed and different levels of item deterioration in both warehouses. The goal of the study is to find the optimal replenishment policies for minimizing the total inventory costs and to prevent the deterioration of items. An algorithm is developed to find the optimal solution. Finally, numerical examples are provided to illustrate the proposed model.

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1. Introduction

In day to day life, some products like electronic components, meat, vegetables, fruit and flowers deteriorate with time. For these products, a supplier has to produce more products than the market demand because a part of the products will be deteriorated. This affects the supplier's production planning and inventory management, which in turn influences the retailer's decision. Hence, the inventory model of deteriorating items gradually becomes an interesting and significant subset of supply chain management.

Supplier provides price discount, for wholesale purchases or seasonal products then the retailer tend to purchase goods beyond the level of storage of their own warehouses (OW). The surplus units over the fixed capacity w of the own warehouse are stored in a rented warehouse (RW). Even though the rented warehouse charges higher unit holding cost than the own warehouses, it offers a better preserving facility which gives rise to lower rate of deterioration of the goods. Hence, goods of rented warehouse can be consumed at the earliest which helps to moderate the inventory costs at the most. In own warehouses (OW) goods are store primarily then the surplus items are stored in rented warehouse (RW). Mean while, stock are cleared from rented warehouse (RW) first then own warehouse (OW). Hence, the two warehouses system minimize the deteriorating rate of commodity.

Lee and Hsu [2] presented two-warehouse production model for deteriorating inventory items with time-dependent demands. Jagg et al. [1] developed two-warehouse partial backlogging inventory model for deteriorating items with linear trend in demand under inflationary conditions. Muniappan et al. [4] developed economic lot sizing production model for deteriorating items under two level trade credit.

Teerapabolarn and Khamrod [12] studied inventory models with backorders and defective items derived Algebraically and AGM. Muniappan et al. [5] analyzed economic order quantity model for deteriorating items with inflation and time value of money considering time-dependent deteriorating rate and delay payments. Muniappan et al. [6] developed production inventory model for vendor-buyer coordination with quantity discount, backordering and rework for fixed life time products. Ravithammal et al. [8] studied deterministic production inventory model for buyer- manufacturer with quantity discount and completely backlogged shortages for fixed life time product. Ravithammal et al. [7] concentrated integrated production inventory system for perishable items with fixed and linear backorders.

The time varying demand considered in most of the papers mentioned above assumes demand rate to be either increasing or decreasing throughout with time, while in practice, it stabilizes at the mature stage of the product life cycle once the product has been accepted in the market. This kind of stabilization has been termed as "ramp-type" and has been considered in the literature since Ritchie [10]. Mandal [3] presented an *EOQ* inventory model for Weibull distributed deteriorating items under ramp type demand and shortages. Skouri et al. [11] considered supply chain models for deteriorating products with ramp type demand rate under permissible delay in payments.

Above literature review of the mentioned topic show *EOQ* model is not developed under situation in which ramp type demand, deterioration and two ware houses. So in this paper we want to investigate these issues together and derive a comprehensive model to determine the economic order quantity and minimum total cost. The rest of the paper is organized as follows: In the next section, notations and assumptions are given. Section 3 provides the mathematical formulation of the proposed model. The numerical example is given in Section 4. Finally, the conclusion of the study is summarized.

2. Notations and Assumptions

This study uses following notations and assumptions.

Notations

Q : The order quantity per replenishment

$D(t)$: Demand rate at any time $t \geq 0$

r : The replenishment cost per order

T : The length of replenishment cycle

O_w : The owned warehouse

R_w : The rented warehouse

w : The storage capacity of owned warehouse

p_1 : The selling price per unit item

p : Purchase cost per unit item

h_0 : The holding cost per unit per unit time in OW

h_r : The holding cost per unit per unit time in RW and $h_r \geq h_0$

θ_1 : The deterioration rate in OW, where $\theta_1 < 0$

θ_2 : The deterioration rate in RW, where $\theta_2 < 0$ and $\theta_1 > \theta_2$

$I_1(t)$: The inventory level in OW at time t

$I_2(t)$: The inventory level in RW at time t

T_w : The time at which the inventory level reaches zero in RW

t_1 : The time at which the inventory level reaches zero in OW

Assumptions

(1) The demand rate is a ramp function of time. i.e., $D(t) = a[t - (t - \mu)H(t-\mu)]$ where a and μ are constants such that $\mu > 0$ and $H(t-\mu)$ is a Heaviside unit function of time defined as

$$H(t-\mu) = \begin{cases} 1, & t \geq \mu \\ 0, & t < \mu \end{cases}.$$

Also, a stands for the initial demand rate and μ is a fixed point in time.

(2) Shortages are not permitted

(3) The own warehouse (OW) has limited capacity of w units

(4) The rented warehouse (RW) has unlimited capacity

(5) The items of RW are consumed first and next the items of OW

(6) Transportation cost from RW to OW and transportation time are negligible

3. Mathematical Formulation

According to the notations and assumptions mentioned above, the model of this paper to be discussed is how the retailer knows whether or not to rent the RW to hold more items under the affects of the ramp demand function. Therefore, when the order quantity $Q \leq w$, the inventory level depletes at the time because of ramp type demand and deterioration at OW. At the beginning stage, the inventory level reaches its maximum I_0 units of item at time $t = 0$. During the time interval $[0, t_1]$, the inventory depletes due mainly to demand and partly to deterioration. At time $t = \mu < t_1$, the inventory level depletes to d_1 units. Therefore, the differential equation representing the inventory level at any time t are governed by the differential equations

$$\frac{dI_1(t)}{dt} + \theta_1 I_1(t) = -at, \quad 0 \le t \le \mu \tag{3.1}$$

$$\frac{dI_1(t)}{dt} + \theta_1 I_1(t) = -a\mu, \quad \mu \le t \le t_1 \tag{3.2}$$

with the boundary condition $I_1(0) = I_0$, $I_1(\mu) = d_1$ and $I(t_1) = 0$. Using the boundary conditions in equations (3.1) and (3.2), we have

$$I_1(t) = \frac{a}{\theta_1^2}(\theta_1\mu - 1)\left(e^{\theta_1(\mu - t)} - 1\right) + d_1 e^{\theta_1(\mu - t)}, \quad 0 \le t \le \mu \tag{3.3}$$

$$I_1(t) = \frac{a\mu}{\theta_1}\left(e^{\theta_1(t_1 - t)} - 1\right), \quad \mu \le t \le t_1 \tag{3.4}$$

The order quantity for each cycle in OW is

$$\begin{aligned} Q_1 &= I_1(0) \text{ at } 0 \le t \le \mu + I_1(0) \text{ and } \mu \le t \le t_1 \\ &= \frac{a}{\theta_1^2}(a\mu - 1)\left(e^{\theta_1\mu} - 1\right) + d_1 e^{\theta_1\mu} + \frac{a\mu}{\theta_1^2}\left(e^{\theta_1 t_1} - 1\right) \end{aligned} \tag{3.5}$$

On the other hand, when the order quantity $Q > w$, the inventory level at RW reduces due to deterioration and demand for a time T_w until reaching zero. By that time a portion of the inventory level at OW is depleted due to deterioration. During the time interval (T_w, T), the inventory level at OW becomes depleted as a result of the combined effect of ramp demand and deterioration until time T.

As described above, the variation of $I_2(t)$ with respect to time is governed by the following differential equation

$$\frac{dI_2(t)}{dt} + \theta_2 I_2(t) = -at, \quad 0 \le t \le \mu \tag{3.6}$$

$$\frac{dI_2(t)}{dt} + \theta_2 I_2(t) = -a\mu, \quad \mu \le t \le T_w \tag{3.7}$$

with the boundary condition $I_2(0) = I_0$, $I_2(\mu) = d_1$ and $I(T_w) = 0$. Using the boundary conditions in equations (3.6) and (3.7), we have

$$I_2(t) = \frac{a}{\theta_2^2}(\theta_2\mu - 1)\left(e^{\theta_2(\mu-t)} - 1\right) + d_1 e^{\theta_2(\mu-t)}, \quad 0 \le t \le \mu \tag{3.8}$$

$$I_1(t) = \frac{a\mu}{\theta_2}\left(e^{\theta_2(T_w-t)} - 1\right), \quad \mu \le t \le T_w \tag{3.9}$$

During the time interval $0 \le t \le T_w$, the variation of $I_{01}(t)$ with respect to time is governed by the following differential equation

$$\frac{dI_{01}(t)}{dt} = -\theta_1 I_{01}(t), \quad 0 \le t \le T_w \tag{3.10}$$

with initial condition $I_{01}(t) = w$. Using the initial condition in equation (3.10), we have

$$I_{01}(t) = we^{-\theta_1 t}, \quad 0 \le t \le T_w \tag{3.11}$$

The order quantity for each cycle in RW is

$$\begin{aligned} Q_2 &= I_2(0) \;\; 0 \le t \le \mu + I_2(0) \text{ and } \mu \le t \le t_1 + I_{01}(0) \\ &= \frac{a}{\theta_2^2}(\theta_2\mu - 1)\left(e^{\theta_2\mu} - 1\right) + d_1 e^{\theta_2\mu} + \frac{a\mu}{\theta_2}\left(e^{\theta_2 T_w} - 1\right) + w \end{aligned} \tag{3.12}$$

The total inventory cost per unit time consists of the following components

(i) Annual ordering cost is $\frac{r}{T}$

(ii) Deterioration cost (DC) in the cycle $[0, t_1]$

$$\begin{aligned} DC =& \frac{p}{T}\left\{t_0 - \left(\int_0^{\mu} atdt + \int_{\mu}^{t_1} a\mu dt\right)\right\} \\ =& \frac{p}{T}\left\{\frac{a}{\theta_1^2}\left[(a\mu - 1)\left(e^{\theta_1\mu} - 1\right) + \mu\left(e^{\theta_1\mu} - 1\right)\right] - a\mu\left(t_1 - \frac{\mu}{2}\right)\right. \\ & \left. + d_1 e^{\theta_1\mu}\right\}. \end{aligned}$$

(iii) Annual stock holding cost in the RW is

$$\begin{aligned} HC_R =& \frac{h_r}{T}\left\{\int_0^{\mu} I_2(t)dt + \int_{\mu}^{T_w} I_2(t)dt\right\} \\ =& \frac{h_r}{T}\left\{\frac{a}{\theta_2^3}(\theta_2\mu - 1)\left(e^{\theta_2\mu} - \theta_2\mu - 1\right)\right. \\ & \left. + \frac{a\mu}{\theta_2^2}\left[e^{\theta_2(T_w-\mu)} + \theta_2(\mu - T_w) - 1\right] + \frac{d_1}{\theta_2}\left(e^{\theta_2\mu} - 1\right)\right\} \end{aligned}$$

(iv) Annual stock holding cost in the OW is

$$HC_0 = \frac{h_0}{T}\left\{\int_0^{\mu} I_1(t)dt + \int_{\mu}^{t_1} I_1(t)dt + \int_0^{T_w} I_{01}(t)dt\right\}$$
$$= \frac{h_0}{T}\left\{\frac{a}{\theta_1^3}(\theta_1\mu - 1)\left(e^{\theta_1\mu} - \theta_1\mu - 1\right) + \frac{a\mu}{\theta_1^2}\left[e^{\theta_1(t_1-\mu)} + \theta_1(\mu - t_1) - 1\right]\right.$$
$$\left. + \frac{d_1}{\theta_1}\left(e^{\theta_1\mu} - 1\right) - \frac{w}{\theta_1}\left(e^{-\theta_1 T_w} - 1\right)\right\}$$

(v) Annual purchasing cost in OW is $PC_O = \frac{pQ_1}{T}$ and annual purchasing cost in RW is $PC_R = \frac{pQ_2}{T}$

Therefore, total purchasing cost in both warehouse is

$$PC = \frac{p(Q_1 + Q_2)}{T}$$
$$= \frac{p}{T}\left\{\frac{a}{\theta_1^2}(a\mu - 1)\left(e^{\theta_1\mu} - 1\right) + \mu\left(e^{\theta_1 t_1} - 1\right) + \frac{a}{\theta_2^2}(\theta_2\mu - 1)\left(e^{\theta_2\mu} - 1\right)\right.$$
$$\left. + \frac{a\mu}{\theta_2}\left(e^{\theta_2 T_w} - 1\right) + d_1\left(e^{\theta_2\mu} + e^{\theta_1\mu}\right) + w\right\}$$

Hence, the total inventory cost per unit time is given by

$$TC(T, t_1, T_w) = \frac{1}{T}(OC + DC + HC_R + HC_O + PC)$$
$$= \frac{1}{T}\left\{r + p\left\{\frac{a}{\theta_1^2}\left[(a\mu - 1)\left(e^{\theta_1\mu} - 1\right) + \mu\left(e^{\theta_1\mu} - 1\right)\right]\right.\right.$$
$$\left. - a\mu\left(t_1 - \frac{\mu}{2}\right) + d_1 e^{\theta_1\mu}\right\} + h_r\left\{\frac{a}{\theta_2^3}(\theta_2\mu - 1)\left(e^{\theta_2\mu} - \theta_2\mu - 1\right)\right.$$
$$\left. + \frac{a\mu}{\theta_2^2}\left[e^{\theta_2(T_w-\mu)} + \theta_2(\mu - T_w) - 1\right] + \frac{d_1}{\theta_2}\left(e^{\theta_2\mu} - 1\right)\right\}$$
$$+ h_0\left\{\frac{a}{\theta_1^3}(\theta_1\mu - 1)\left(e^{\theta_1\mu} - \theta_1\mu - 1\right) + \frac{a\mu}{\theta_1^2}\left[e^{\theta_1(t_1-\mu)} + \theta_1(\mu - t_1) - 1\right]\right.$$
$$\left. + \frac{d_1}{\theta_1}\left(e^{\theta_1\mu} - 1\right) - \frac{w}{\theta_1}\left(e^{-\theta_1 T_w} - 1\right)\right\} + p\left\{\frac{a}{\theta_1^2}(a\mu - 1)\left(e^{\theta_1\mu} - 1\right)\right.$$
$$+ \mu\left(e^{\theta_1 t_1} - 1\right) + \frac{a}{\theta_2^2}(\theta_2\mu - 1)\left(e^{\theta_2\mu} - 1\right)$$

$$+\frac{a\mu}{\theta_2}\left(e^{\theta_2 T_w}-1\right)+d_1\left(e^{\theta_2\mu}+e^{\theta_1\mu}\right)+w\Big\}\Big\}$$

We now proceed to determine T, η_1 and η_2 optimality by treating them as decision variables. The inventory cost per unit time, $TC(T,\eta_1,\eta_2)$ being a function of three variables T, η_1 and η_2 has to be partially differentiated with respect to T, η_1 and η_2 separately and then put equal to zero. This given the necessary condition for minimizing the total inventory cost per time $TC(T^*,\eta_1^*,\eta_2^*)$ and minimum order quantity in each warehouses is Q_1^*, Q_2^*. We assumes

$$t_1=\eta_1 T,\quad 0<\eta_1<1 \text{ and } T_w=\eta_2 T,\quad 0<\eta_2<1, \tag{3.13}$$

$$\begin{aligned}TC(T,t_1,T_w)=&\frac{1}{T}\Big\{r+\varepsilon_7+\frac{h_r a\mu}{\theta_2^2}\left[e^{\theta_2(\eta_2 T-\mu)}+\theta_2(\mu-\eta_2 T)-1\right]\\&+\frac{h_0 a\mu}{\theta_1^2}\left[e^{\theta_1(\eta_1 T-\mu)}+\theta_1(\mu-\eta_1 T)-1\right]-\frac{wh_0}{\theta_1}\left(e^{-\theta_1\eta_2 T}\right)\\&+2p\mu\left(e^{\theta_1\eta_1 T}-1\right)\\&+\frac{pa\mu}{\theta_2}\left(e^{\theta_2\eta_2 T}-1\right)-a\mu\left(\eta_1 T-\frac{\mu}{2}\right)\Big\}\end{aligned} \tag{3.14}$$

where $\varepsilon_1=\frac{h_r a}{\theta_2^3}(\theta_2\mu-1)\left(e^{\theta_2\mu}-\theta_2\mu-1\right)$, $\varepsilon_2=\frac{h_r d_1}{\theta_2}\left(e^{\theta_2\mu}-1\right)$, $\varepsilon_3=\frac{h_0 a}{\theta_1^3}(\theta_1\mu-1)\left(e^{\theta_1\mu}-\theta_1\mu-1\right)$, $\varepsilon_4=\frac{h_r d_1}{\theta_2}\left(e^{\theta_2\mu}-1\right)$, $\varepsilon_5=\frac{2pa}{\theta_1^2}(a\mu-1)\left(e^{\theta_1\mu}-1\right)$, $\varepsilon_6=p\left\{\frac{a}{\theta_2^2}(\theta_2\mu-1)\left(e^{\theta_2\mu}-1\right)+d_1\left(2e^{\theta_2\mu}+e^{\theta_1\mu}\right)+w\right\}$, $\varepsilon_7=\varepsilon_1+\varepsilon_2+\varepsilon_3+\varepsilon_4+\varepsilon_5+\varepsilon_6$. For optimality, $\frac{\partial TC}{\partial T}=0$, $\frac{\partial TC}{\partial \eta_1}=0$ and $\frac{\partial TC}{\partial \eta_2}=0$.

Now

$$\begin{aligned}\frac{\partial TC}{\partial T}=&\frac{1}{-T^2}\Big\{r+\varepsilon_7+\frac{h_r a\mu}{\theta_2^2}\left[e^{\theta_2(\eta_2 T-\mu)}+\theta_2(\mu-\eta_2 T)-1\right]\\&+\frac{h_0 a\mu}{\theta_1^2}\left[e^{\theta_1(\eta_1 T-\mu)}+\theta_1(\mu-\eta_1 T)-1\right]-\frac{wh_0}{\theta_1}\left(e^{-\theta_1\eta_2 T}-1\right)\\&+2p\mu\left(e^{\theta_1\eta_1 T}-1\right)\\&+\frac{pa\mu}{\theta_2}\left(e^{\theta_2\eta_2 T}-1\right)-a\mu\left(\eta_1 T-\frac{\mu}{2}\right)\Big\}+\frac{1}{T}\Big\{\frac{h_r a\mu}{\theta_2^2}\left[\theta_2\eta_2 e^{\theta_2(\eta_2 T-\mu)}+\theta_2\eta_2\right]\\&+\frac{h_0 a\mu}{\theta_1^2}\left[\theta_1\eta_1 e^{\theta_1(\eta_1 T-\mu)}-\theta_1\eta_1\right]-\frac{wh_0}{\theta_1}\left(\theta_1\eta_2 e^{-\theta_1\eta_2 T}\right)+2p\mu\left(\theta_1\eta_1 e^{\theta_1\eta_1 T}\right)\end{aligned}$$

$$+\frac{pa\mu}{\theta_2}\left(\theta_2\eta_2 e^{\theta_2\eta_2 T}\right) - a\mu\eta_1\Big\}$$

$$\Rightarrow \left\{-a\mu\left[h_r\eta_2^2 + h_0\eta_1^2\right] + h_0 w\theta_1\eta_2^2 - p\mu\left[2\theta_1^2\eta_1^2 + a\theta_2\eta_1^2\right]\right\} T^2$$

$$+a\mu^2\left[h_r\eta_2 + h_0\eta_1\right] T + \frac{a\mu^2}{2} + r + \varepsilon_7 = 0 \tag{3.15}$$

$$\frac{\partial TC}{\partial \eta_1} = 0 \Rightarrow \left\{h_0 a\mu T + 2p\mu\theta_1^2 T\right\}\eta_1 + \frac{h_0 a\mu}{\theta_1} - h_0 a\mu^2 + 2p\mu\theta_1 - \theta_1 - a\mu = 0$$

$$\eta_1 = \frac{-(\frac{h_0 a\mu}{\theta_1} - h_0 a\mu^2 + 2p\mu\theta_1 - \theta_1 - a\mu)}{h_0 a\mu T + 2p\mu\theta_1^2 T} \tag{3.16}$$

$$\frac{\partial TC}{\partial \eta_2} = 0 \Rightarrow \left\{h_r a\mu T - h_0 w\theta_1 T + p\mu a\theta_2 T\right\}\eta_2$$

$$+\frac{h_r a\mu}{\theta_2} - h_r a\mu^2 + p\mu a - \theta_2 - h_0 w = 0$$

$$\eta_2 = \frac{-(\frac{h_r a\mu}{\theta_2} - h_r a\mu^2 + p\mu a - \theta_2 - h_0 w)}{h_r a\mu T - h_0 w\theta_1 T + p\mu a\theta_2 T} \tag{3.17}$$

By using following algorithm we have to find the total minimum inventory cost for both warehouses $TC^*(T^*, t_1^*, T_w^*)$, $t_1^* = T^*\eta_1^*$, $T_w^* = T^*\eta_2^*$ and the order quantity for both warehouses $Q^* = Q_1^* + Q_2^*$.

Algorithm

Step 1. Input the values

Step 2. Substituting the values into equation (3.15) and find T_1

Step 3. Using T_1, determine $\eta_{1(1)}$, $\eta_{2(1)}$ from equations (3.16) and (3.17)

Step 4. Using equation (3.13), determine $t_{1(1)}$, $T_{w(1)}$

Step 5. Using equation (3.14), determine $TC(T_{(1)}, t_{1(1)}, T_{w(1)})$

Step 6. Repeat step 2 to step 5 until

$$TC(T_{(n)-1}, t_{1(n)-1}, T_{w(n)-1}) \le TC(T_{(n)}, t_{1(n)}, T_{w(n)}).$$

Set $T^* = T_{(n)-1}, t_1^* = t_{1(n)-1}, T_w^* = T_{w(n)-1}$ and go to step 7

Step 7. Determine $Q^* = Q_1^* + Q_2^*$ and $TC^*(T^*, t_1^*, T_w^*)$.

4. Numerical Examples

Example 1. In order to illustrate the model, we consider an inventory situation with following data:
$r = 100$, $a = 100$, $h_r = 0.3$, $h_o = 0.2$, $\theta_1 = 0.6$, $\theta_2 = 0.3$, $\eta_1 = 0.3$, $\eta_2 = 0.2$, $\mu = 0.1$, $w = 50$, $p = 0.2$, $d_1 = 20$. The optimal solutions are found to be $T^* = 28.4498$, $t_1^* = 3.6947$, $T_w^* = 8.9167$, $Q^* = 891$, $TC^*(T^*, t_1^*, T_w^*) = 12.9911$.

Example 2. Let $r = 200$, $a = 150$, $h_r = 0.5$, $h_o = 0.3$, $\theta_1 = 0.8$, $\theta_2 = 0.5$, $\eta_1 = 0.5$, $\eta_2 = 0.1$, $\mu = 0.1$, $w = 100$, $p = 0.2$, $d_1 = 50$. The optimal solutions are found to be $T^* = 11.3027$, $t_1^* = 2.3407$, $T_w^* = 3.1167$, $Q^* = 692$, $TC^*(T^*, t_1^*, T_w^*) = 104.5328$.

5. Conclusion

In this paper, we study an inventory model for deteriorating items with ramp-type demand and two warehouse systems depending on the model parameters. Our adoption of ramp-type demand reflects a real market demand for newly launched product. The aim of the paper is to minimize the expected total annual cost by optimizing the period length T^*, t_1^* and T_w^*. An algorithm is developed to obtain the overall optimal replenishment policy, which would enable the manager to decide upon the feasibility of renting a warehouse. The dynamics of the model and application of the algorithm are demonstrated through numerical examples. For future research, the model can be modified to contain shortages as well. Moreover, the model can be extended for time dependent deterioration rate, transportation cost from RW to OW, deterioration of items during transportation etc.

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