Evolutionary Algorithm and Fuzzy C-Means Clustering Based Order Reduction of Discrete Time Interval Systems

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Abstract

This paper deals with a mixed method for model order reduction of linear discrete time interval systems. This method is based on Particle Swarm Optimization, Fuzzy C means Clustering and Kharitonov’s theorem. The reduced order interval denominator coefficients are determined by using Fuzzy C means Clustering technique while Particle Swarm Optimization algorithm is used for obtaining numerator coefficients by minimizing ISE. This method is always generated to a stable reduced order interval system if the original higher order system is stable system. The proposed method has to be applied to a typical numerical example available in the literature. The results are comparing with the results are determined by using some other familiar methods.

Keywords: Interval system, Model order reduction, Particle Swarm Optimization, Fuzzy C means Clustering, Kharitonov’s theorem, Integral Square Error.
1. Introduction

Higher order model systems are complicated to handle due to the computational complexities and they are also very difficult to be used in real-time problems. So it is desirable that the higher model is to be replaced with lower order model. Many number of methods for model order reductions are available in literature [1]. Even though several methods available, no approach gives the best result for all systems. In literature, a very powerful method which involves some simple algebraic calculations i.e. moments matching continued fraction, and Pade approximation [2], [3] are available. In some cases, Pade approximant may turn out to be unstable even though the original system is stable.

Routh approximation based methods are available [2],[3],[6],[8] for continuous-time systems and for discrete-time systems [4],[5],[7]. There are more methods are available for the order reduction of continuous time systems, but very few methods extended to the discrete-time systems. The methods for discrete-time systems are classified into two types. The first one it consists of already existed in continuous-time algorithm, \( G(z) \) into another one \( G_1(w) \), by using bilinear transformation \( z= (1+w)/(1-w) \)[1]. Then one of known techniques for continuous-time systems is applied to determine a reduced approximant \( R_1(w) \) for \( G_1(w) \). Finally the inverse transformation \( w= \phi(z) \) converts \( R_1(w) \) to the required approximant \( R(z) \).

Many systems the coefficients are constants but uncertain within a finite range. Such systems are classified as interval systems. Some methods like [9-15] are proposed for order reduction of interval systems in both continuous and discrete time systems. In [10], a method for interval systems is \( \gamma - \delta \) Routh Approximation is proposed. A modified Routh approximation method is presented for obtaining stable reduced models of continuous-time systems. The reduced models give good approximates of the initial transient response and the steady-state response of the original system. In [11-16] Kharitonov’s theorem is used for reduction technique of linear interval systems Routh Approximation is used to be generates stable reduced interval model.

In this paper, a proposed method for model order reduction of Discrete time interval systems based up on Particle Swarm Optimization (PSO) and Fuzzy C means Clustering technique. The reduced interval model denominator polynomials obtained by Fuzzy C means clustering method
while the numerator coefficients are obtained by using minimization of ISE by PSO. In this method produces a stable reduced model if the original higher order system is to be stable.

The proposed paper is to be organized as, section 2 consists of a problem formulation, section 3 consists of proposed method, section 4 consists of numerical example and simulation results and section 5 consists of conclusion and references.

3. Determination of Reduced Order Denominator polynomials:

For first Kharitonov’s transfer function, Fuzzy C-Means technique is applied to determine reduced order polynomials. Fuzzy clustering method is a powerful unsupervised method for analysis of data and construction models. In many situations, fuzzy c means clustering is natural compared to hard clustering. The boundaries between many classes are not forced to fully belong to one of the classes, but rather then the membership degrees between 0 and 1 indicating the partial membership. Fuzzy c-means algorithm is most widely used. The sum of degrees of belongingness of the data point to all clusters is always equal to unity:

$$\sum_{i=1}^{c} u_{ij} = 1$$  

(1)

The cost function for FCM is generalization is

$$J = \sum_{i=1}^{c} J_i = \sum_{i=1}^{c} \sum_{j=1}^{n} u_{ij}^{m} d_{ij}^{2}$$  

(2)

Where $u_{ij}$ is between 0 and 1, $c_i$ is the cluster center of fuzzy group i; $d_{ij} = \|c_i - x_j\|$ is the Euclidean distance between the $i^{th}$ cluster center and the $j^{th}$ data point; and $m \in [1, \infty]$. The necessary conditions for (9) to reach to its minimum are

$$c_i = \frac{\sum_{j=1}^{n} u_{ij}^{m} x_j}{\sum_{j=1}^{n} u_{ij}^{m}}$$  

(3)

and

$$u_{ij} = \frac{1}{\sum_{k=1}^{c} \left(\frac{\|c_i - x_j\|}{\|c_k - x_j\|}\right)^{2/(m-1)}}$$  

(4)

This algorithm works by assuming the membership to each data point according to each cluster centers. The distance between the cluster center and data point should be more then the data nearer to the cluster center more is its membership towards the particular cluster center. Clearly, combination of membership of each data point should be equal to one.
4 Determination of Reduced Order numerator polynomial:

PSO is an evolutionary algorithm that can be used for solving the nonlinear equations. It is a kind of swarm intelligence that is based on social-psychological principles and provides insights into social behaviour, as well as contributing to engineering applications. In PSO, the velocity and position are randomly chosen for a set of particles. During the start, the initial position is taken as the best position and the velocity is updated. The main purpose of this optimization method is

a) A global optimum for the nonlinear system may be found,
b) It can produce a different number of substitute solutions,
c) There are no mathematical limitations on the formulation of the problem,
d) Comparatively very simple in execution and
e) Numerically strong.

In Table 1, the typical parameters for PSO optimization routines, used in the present study are given.

<table>
<thead>
<tr>
<th>TABLE1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Typical parameters used by PSO</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Name</th>
<th>Value(type)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of generations</td>
<td>100</td>
</tr>
<tr>
<td>Population size</td>
<td>60</td>
</tr>
<tr>
<td>Maximum Particle velocity</td>
<td>2</td>
</tr>
<tr>
<td>Epoch</td>
<td>100</td>
</tr>
<tr>
<td>Termination method</td>
<td>Maximum Generation</td>
</tr>
</tbody>
</table>
5. Numerical Example:

Consider a higher order discrete time interval system

\[
G(z) = \frac{2.6 + 3 + 3.4 + 3.55}{2.2 + 2.25 + 1.3 + 1.6 + 1.1 + 1.2 + 0.85 + 0.95} z^3 + [3.15, 3.45, 2.05, 2.15] z^2 + [6.2, 6.65, 8.45, 10.15, 10.35] z^0
\]

With the help of Kharitonov's theorem, the higher order interval system can be represented as four Kharitonov's higher order transfer functions are given as

\[
G_1^1(z) = \frac{0.85z^5 + 1.12z^4 + 3.45z^3 + 3.4z^2 + 2.6}{10.35z^6 + 6.2z^4 + 1.2z^2 + 1.3z + 2.2}
\]

\[
G_2^1(z) = \frac{0.95z^5 + 1.12z^4 + 3.45z^3 + 3.55z + 2.6}{10.35z^6 + 8.45z^5 + 5.3z^3 + 1.2z^2 + 1.6z + 2.25}
\]

\[
G_3^1(z) = \frac{0.85z^5 + 1.25z^4 + 3.15z^3 + 3.4z^2 + 3.4z + 3}{10.15z^6 + 6.65z^5 + 5.6z^3 + 1.1z^2 + 1.3z + 2.25}
\]

\[
G_4^1(z) = \frac{0.95z^5 + 1.25z^4 + 3.15z^3 + 3.55z + 3}{10.15z^6 + 8.45z^5 + 5.3z^3 + 1.1z^2 + 1.6z + 2.25}
\]

**Step 1:**

Consider the first Kharitonov's transfer function

\[
G_1^1(z) = \frac{0.85z^5 + 1.12z^4 + 3.45z^3 + 3.4z^2 + 2.6}{10.35z^6 + 6.2z^4 + 1.2z^2 + 1.3z + 2.2}
\]

The poles of \(G_1^1(z)\) are

\[
\lambda_1 = -0.0773 + 0.8618i, \quad \lambda_4 = -0.7546 + 0.2622i
\]

\[
\lambda_2 = -0.0773 - 0.8618i, \quad \lambda_5 = 0.4406 + 0.5008i
\]

\[
\lambda_3 = -0.7546 - 0.2622i, \quad \lambda_6 = 0.4406 - 0.5008i
\]

The second order reduced model is desired to be approximated for the system as

\[
G_1(z) = \frac{b_2z^2 + b_1z + b_0}{a_2z^2 + a_1z + a_0}
\]

By using FCM algorithm the poles of the reduced order system are found to be

\[-0.241567 \pm 0.5475i\]

and the denominator of the ROM is obtained as

\[D_2(s) = z^2 + 0.4831z + 0.3581\]

The reduced order model is:

\[
G_1^2(z) = \frac{b_{12}z^2 + b_{11}z + b_{10}}{z^2 + 0.4831z + 0.3581}
\]

Repeat the FCM algorithm for remaining Kharitonov’s transfer function the reduced order models are:

\[
G_2^2(z) = \frac{b_{22}z^2 + b_{21}z + b_{20}}{z^2 + 0.5412z + 0.3692}; G_3^2(z) = \frac{b_{32}z^2 + b_{31}z + b_{30}}{z^2 + 0.4936z + 0.3689}; G_4^2(z) = \frac{b_{42}z^2 + b_{41}z + b_{40}}{z^2 + 0.5484z + 0.3793}
\]
Step 2:
The reduced order numerator coefficients are determined by the minimization of ISE using PSO algorithm for 4 Kharitonov’s transfer functions and there given as

\[ N_1^2(z) = 0.05534 z^2 + 0.3731z + 0.2846; \quad N_2^2(z) = 0.05348 z^2 + 0.4874z + 0.2012 \]

\[ N_3^2(z) = 0.0212 z^2 + 0.4095z + 0.2012; \quad N_4^2(z) = 0.0211z^2 + 0.4157z + 0.3191 \]

The four reduced order Kharitonov’s transfer functions are:

\[ G_1^2(z) = \frac{0.05534 z^2 + 0.3731z + 0.2846}{z^2 + 0.4031z + 0.3581}; \quad G_2^2(z) = \frac{0.05348 z^2 + 0.4874z + 0.2012}{z^2 + 0.5412z + 0.3692} \]

\[ G_3^2(z) = \frac{0.0212 z^2 + 0.4095z + 0.2012}{z^2 + 0.4936z + 0.3689}; \quad G_4^2(z) = \frac{0.0211z^2 + 0.4157z + 0.3191}{z^2 + 0.5484z + 0.3793} \]

Therefore, the reduced order interval model is obtained as

\[ G_2(z) = \frac{[0.0211, 0.05534]z^2 + [0.3731, 0.4913]z}{[1, 1]z^2 + [0.4831, 0.5484]z + [0.3581, 0.3793]} \]

Comparing the proposed model with reduced order model obtained by the method given in [9]

\[ G_2(z) = \frac{[0.02285, 0.0791]z^2 + [0.3665, 0.4956]z}{[1, 1]z^2 + [0.4286, 0.712]z + [0.2478, 0.399]} \]

Fig. 3: Step Response comparison for 1st Kharitonov’s TF
Fig. 4: Step Response comparison for 2\textsuperscript{nd} Kharitonov’s TF

Fig. 5: Step Response comparison for 3\textsuperscript{rd} Kharitonov’s TF

Fig. 6: Step Response comparison for 3\textsuperscript{rd} Kharitonov’s TF

The step responses of the four reduced order Kharitonov’s models are compared with the method given in [9]. It has been observed...
from figure 3 to figure 6 that the step response of four Kharitonov's transfer functions of higher order time interval system and the respective reduced models obtained by the present method are closely matching than the method given in [9]. This method also retains the stability of the reduced order interval model since the original system is stable. It is observed that the reduced order model obtained by the present method has better matching of transient and steady state response than the method in [9]. From Table 1 it is observed that the ISEs of lower and upper bounds of reduced interval model determined by the present method are less than the method given in [9].

<table>
<thead>
<tr>
<th>MOR</th>
<th>ISE for lower bound</th>
<th>ISE for upper bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed Method</td>
<td>0.292</td>
<td>0.3061</td>
</tr>
<tr>
<td>Pade &amp; Pole clustering Method[11]</td>
<td>0.4052</td>
<td>0.3545</td>
</tr>
</tbody>
</table>

5. Conclusion

In this proposed paper, the model order reduction for a linear discrete time interval system is done by combining the pole clustering technique and an evolutionary algorithm based soft computing technique i.e. PSO algorithm. The reduced model is determined by using the Kharitonov's theorem and Fuzzy C-means Clustering algorithm for denominator coefficients, while numerator coefficients are determined by using PSO algorithm. The main objective to find the numerator is minimization of ISE. The use of interval arithmetic has the possibility of producing an unstable reduced model. To avoid this problem, Kharitonov's theorem is used to make reduced interval models robustly stable. It is observed that, since the original discrete time interval system is stable the reduced order polynomials are also stable, has better matching response. Therefore, the original higher order interval system is reduced to second order interval model by minimizing the ISE.

References:


