

Free Convection Flow Past from Horizontally Inclined Plate in Thermally Stratified Medium

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Abstract

A finite-difference analysis of unsteady laminar free convection flow from an incompressible viscous fluid past a horizontally inclined plate immersed in a stable stratified medium is considered here. The dimensionless governing equations of the flow that are unsteady, coupled and non-linear integro-partial differential equations are solved by an efficient, accurate and unconditionally stable finite difference scheme of Crank-Nicolson type. The velocity and temperature profiles are obtained for different values of Prandtl number and the thermal stratified parameter and are analyzed graphically.

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Key Words: Free convection, Finite difference, inclined plate, Stratified Medium.

1 Introduction

Though many applications in engineering devices indulge with natural convection along an inclined plate, this does not receive much attention than the study on horizontal and vertical plates. Thus the

Heat transfer plays an important role due to its diverse application in electronic cooling, heat exchangers and thermal systems etc.

Natural convection flows arise in a thermally-stratified environment. When the thermal energy in an enclosed fluid region is input, either discharge of hot fluid or removal of heat from heated bodies occurs. Further it entails in generating stable thermal stratification and increasing temperature according to height in respect of the fluid; the fluid on increase of temperature reduces its density. As regards a stably stratified environment in which the natural convection flow raised from heated surfaces is immersed, the stratification gives much importance in both the heat rejection aspects and energy storage processes.

When thermal stratification in reservoirs could reduce the vertical mixing of oxygen, to certain point, the bottom water becomes anoxic through the action of biological processes. Stokes [1] first resolved an exact, convergent and stable solution to the Navier-stokes equation which relates to the flow of a viscous incompressible fluid past an impulsively started semi-infinite horizontal plate by finite difference method of a mixed explicit-implicit type. Following Stokes analysis, Soundalgekar [2] was the first to arrive an exact solution to the flow of a viscous fluid past an impulsively started infinite isothermal vertical plate by the Laplace transform technique. In seigals [3] view, analytical solution was derived using an integral method in the problem of free convective flow past a semi infinite vertical isothermal plate. Rich [4] was the first and foremost person to study natural convection over an inclined plate. Soundalgekar and Ganesan [5] used an implicit finite difference scheme of crank-Nicolson type to investigate the problem of transient free convective flow over an isothermal vertical plate. Ganesan and Ekambavanan [6] choosing an implicit finite difference method studied the problem of free convective flow of an incompressible viscous fluid past a semi-infinite isothermal inclined plate.

The effect of the natural convection heat transfer out of a non-isothermal vertical flat plate while immersed in a temperature stratified environment was studied by Yang et al. [8]. By using a finite difference method Venkatachala and nath [9], solved the problem of an isothermal wall in a linearly stratified atmosphere. The ambient temperature varies as $\frac{x_1}{3}$ in a thermally stratified darcian fluid; in this fluid a vertical flat plate of uniform heat flux surface

is embedded; the development of the flow caused in this process has been studied by Pop & Takhar [10]. Non-darcy effects including boundary and inertia effect on the natural convection boundary layer flow along an isothermal vertical surface embedded in a stable thermally stratified high porosity medium are considered and solved by chen and lin [11]. A mathematical model governing free convection boundary layer flow over an isothermal inclined plate embedded in a thermally stratified porous medium in the presence of a non-uniform inverse transverse magnetic field being a power law function of the distance along the plate is developed and studied by Chamkha [12]. Magyari et al. [13] studied the unsteady free convection along an infinite vertical flat plate embedded in a stably stratified fluid saturated porous medium by the similarity transformation method. The effect of linear thermal stratification in stable stationary ambient fluid on free convective flow of a viscous incompressible fluid along a plane wall is investigated numerically with wide range of stratification parameter for different kinds of fluids (air, water and oil) with different grashof number and different inclination angle by Ishan and Ali. [14].

2 Mathematical Analysis

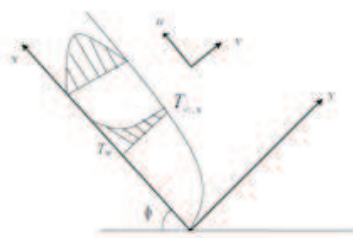


Figure 1: Physical and Co-ordinate System

A transient laminar, unsteady natural convection flow of a viscous and incompressible fluid past a semi-infinite inclined plate which

is placed in a thermally stratified medium with magnetic field has been considered. The angle of inclination of the plate with the horizontal is assumed to be ϕ . Initially, at time $t' \leq 0$, the fluid and the plate are assumed to be at the same temperature $T_{\infty,x}$. Then at time $t' > 0$, the temperature of the plate is raised to T_w . In the ambient, the temperature increases linearly with height, where $T_{\infty,0}$ being its value at $x=0$ respectively. The fluid considered here is limited to an optically thick fluid. Introducing the co-ordinate system (Shown in Figure 1) such that the x -axis is taken along the plate in the vertically upward direction and the y -axis is taken normal to the plate.

All the fluid physical properties are assumed to be constant except density variations, which includes buoyancy forces in the momentum equation. The governing boundary layer equations of Continuity, Momentum and Energy under Boussinesq approximation (Herman Schlichting [15]) are as follows.

Basic Equations

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$\begin{aligned} \frac{\partial u}{\partial t'} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = g\beta \cos \phi \frac{\partial}{\partial x} \int_y^\infty (T' - T_{\infty,x}) \\ + g\beta \sin \phi (T' - T_{\infty,x}) + \nu \frac{\partial^2 u}{\partial y^2} \end{aligned} \quad (2)$$

$$\frac{\partial T'}{\partial t'} + u \frac{\partial T'}{\partial x} + v \frac{\partial T'}{\partial y} = \alpha \frac{\partial^2 T'}{\partial y^2} \quad (3)$$

Boundary Conditions

$$\left. \begin{aligned} t' \leq 0 : u = 0, v = 0, T' = T_{\infty,x} \text{ for all } x \text{ and } y \\ t' > 0 : u = 0, v = 0, T' = T'_w \text{ at } y = 0 \\ u = 0, v = 0, T' = T_{\infty,0} \text{ at } x = 0 \\ u \rightarrow 0, T' \rightarrow T_{\infty,x} \text{ as } y \rightarrow \infty \end{aligned} \right\} \quad (4)$$

Introducing the non-dimensional quantities,

$$\left. \begin{aligned} X &= \frac{x}{L}, Y = \frac{y}{L}(Gr_L)^{\frac{1}{4}}, U = \frac{uL}{\nu}(Gr_L)^{-\frac{1}{2}}, V = \frac{vL}{\nu}(Gr_L)^{-\frac{1}{4}} \\ t &= \frac{vt'}{L^2}(Gr_L)^{\frac{1}{2}}, T = \frac{T' - T_{\infty,x}}{T_w - T_{\infty,0}}, Gr_L = \frac{g\beta L^3(T_w - T_{\infty,0})}{\nu^2}, \\ S_T &= \frac{\frac{dT_{\infty,x}}{dX}}{T_w - T_{\infty,0}}, Pr = \frac{\nu}{\alpha} \end{aligned} \right\} \quad (5)$$

Equations are reduced to the Non Dimensional form as,

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (6)$$

$$\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = (Gr_L)^{-\frac{1}{4}} \cos \phi \frac{\partial}{\partial X} \int_Y^\infty T dY + T \sin \phi + \frac{\partial^2 U}{\partial Y^2} \quad (7)$$

$$\frac{\partial T}{\partial t} + U \frac{\partial T}{\partial X} + V \frac{\partial T}{\partial Y} + S_T U = \frac{1}{Pr} \frac{\partial^2 T}{\partial Y^2} \quad (8)$$

The boundary condition for T in non-dimensional form is given by

$$T = 1 - \frac{T_{\infty,x} - T_{\infty,0}}{T_w - T_{\infty,0}}$$

As $T_{\infty,x}$ are linear functions of x .

$$T = 1 - \frac{1}{\Delta t_0} \frac{T'_{\infty,x}}{dX} \text{ where } \Delta t_0 = T_w - T_{\infty,0}$$

$$T = 1 - S_T X$$

$$\text{where } S_T = \frac{1}{\Delta t_0} \frac{T'_{\infty,x}}{dX}$$

The boundary Condition for temperature along the wall in non-dimensional form are obtained as follows

$$\left. \begin{aligned} t \leq 0 : U &= 0, V = 0, T = 0 \\ t > 0 : U &= 1, V = 0, T = 1 - S_T X \text{ at } Y = 0 \\ U &= 0, V = 0, T = 0 \text{ at } X = 0 \\ U &\rightarrow 0, T \rightarrow 0 \text{ as } Y \rightarrow \infty \end{aligned} \right\} \quad (9)$$

3 Solution Procedure:

In order to solve these unsteady, non-linear coupled Equations (6) to (8) under the conditions (10), an implicit finite difference scheme

of Crank-Nicolson type have been employed. This method has been extensively developed in recent years and remains one of the most reliable procedures for solving partial differential equation systems. It is unconditionally stable. It utilizes a central differencing procedure for space and is an implicit method. The partial differential terms are converted to difference equations and the resulting algebraic problem is solved using a tri diagonal matrix algorithm. The integrals in this equation are evaluated by the Newton-Cotes closed integration.

The region of integration is considered as a rectangle with sides $X_{max}(= 1)$ and $Y_{max}(= 14)$, where Y_{max} corresponds to $Y = \infty$, which lies very well outside the momentum and energy boundary layers. The maximum of Y was chosen as 14 after some preliminary investigations, so that the last two of the boundary conditions (9) are satisfied within the tolerance limit 10^{-5} . Hence the stability and compatibility ensure the convergence.

4 Results and Discussion

Transient velocity and temperature profiles at $X = 1.0$ for different values of Pr and S shown in figures 2 - 5. The steady state temperature is denoted by *. Transient velocity and temperature profiles for different values of Pr are shown in figures 2 & 3. From figures, we observed that increase in Pr reduces the velocity and temperature profiles. Also, the thermal and momentum boundary layer thickness reduces with increasing Pr .

Transient velocity and temperature profiles for different values of S_T are shown in Figure 4 & 5. From figures, we observed that increase in S_T reduces the temperature and velocity profiles. Also, as S_T increases, the velocity and temperature reduce throughout the transient period.

For $S_T < 1$, there is a positive buoyancy at $X = 1$, but negative values of the non-dimensional temperature profiles are still observed in the outer region. For high values of S_T , the temperature in the ambient medium increases rapidly with the height and the fluid coming up will be much cooler in the outer region of the boundary layer. Hence, negative values of non-dimensional temperature appear in the wings of profile. This occurrence was referred

by many researchers as temperature Defect. When Gr increases, the velocity value decreases. When Gr increases, Temperature also increases. When angle increases, Velocity and Temperature also increases. Due to brevity the results (Local and Average Skin friction, Nusselt numbers effects) are not presented graphically in this article.

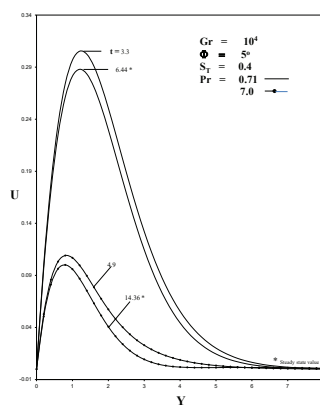


Fig. 2. Transient velocity profiles at $X = 1.0$ for different values of Pr

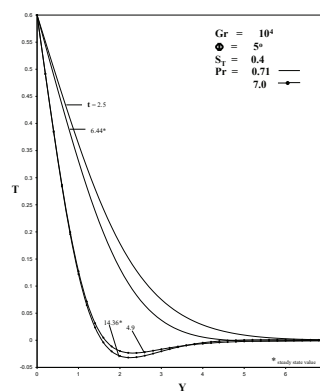


Fig. 3. Transient temperature profiles at $X = 1.0$ for different values of Pr

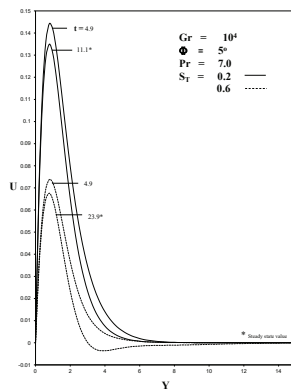


Fig. 4. Transient velocity profiles at $X = 1.0$ for different values of S

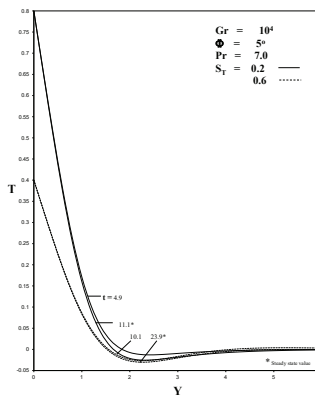


Fig. 5. Transient temperature profiles at $X = 1.0$ for different values of S

5 Conclusion

1. Velocity increases for lower values of Pr and the stratification parameter S_T .
2. The temperature decreases when the values of Pr and the stratification parameter S_T increases.
3. When the values of Gr increases, velocity decreases and reverse effect is found in temperature.

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