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A NOVEL METHOD OF COMPUTING THE VALUE OF COMBINATIONS

Shyama S,
Parameswaran R.,
Department of
Mathematics,

School of Arts and Sciences

Amrita University, Kochi.

Abstract—**The discrete mathematics forms the backbone for the development of complicated algorithms used in the programming languages. Combinatorics is a branch of discrete mathematics which plays a vital role. In the modern era of technology we always plan to minimize the memory requirement to solve a problem. In this paper we present a technique to find the value of n_{C_r} with less number of mathematical operations using binomial theorem.**

Keywords—Binomial Theorem, Combinations.

1. Introduction [1]

FOR an integer n ≥ 0, *n factorial* (denoted n!) is defined by

$$0! = 1$$
$$n! = (n)(n-1)(n-2)\ldots\ldots\ldots(3)(2)(1);\, for\; n \geq 1.$$

Permutation of a collection is defined as any (linear) arrangement of the collection of n different objects.
By the rule of product, the number of permutations of size r for n distinct objects is

$$n_{P_r} = \frac{n!}{(n-r)!}.$$

where r is an integer, with 1 ≤ r ≤ n.
Combinations is the another name for the subset chosen from the super set with n distinct objects. That is if we start with n different objects, each selection or combination, of r of these objects, with no mention to order, correspond to r! permutations of size r from the n objects. Thus the number of combinations of size r from a collection of size n is

$$n_{C_r} = \frac{n_{P_r}}{r!} = \frac{n!}{r!\,(n-r)!}\ , 0 \leq r \leq n$$

A. *Pascal's Triangle [2]*

The set of binomial coefficients n_{C_r}'s can be appropriately arranged in a trilateral form from topmost to nethermost and from left to right in an increasing order of the values of n and r respectively.

$$\binom{0}{0}$$
$$\binom{1}{0} \quad \binom{1}{1}$$
$$\binom{2}{0} \quad \binom{2}{1} \quad \binom{2}{2}$$
$$\binom{3}{0} \quad \binom{3}{1} \quad \binom{3}{2} \quad \binom{3}{3}$$
$$\binom{4}{0} \quad \binom{4}{1} \quad \binom{4}{2} \quad \binom{4}{3} \quad \binom{4}{4}$$

Figure 1

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

Figure 2

This diagram, called the *Pascal's triangle*, is one of the most significant number patterns in the History of mathematics, after the distinguished French Mathematician Blaise Pascal who discovered it and made influential contributions to the understanding of it in 1653.This treatise was one of the first works on Probability Theory made it significant. It also appeared in the treatise Szu-yuan Yu-chien ("The Precious Mirror of the Four Elements") by a mathematician of China Shih-Chieh Chu in 1303, where they were said to be an old invention. The initial known thorough discussion of binomial coefficients is in a 10^{th} century annotation, due to Halayudha, on an earliest Hindu classic, Pingala's Chandah-sutra. In around 1150 the Hindu Mathematician Bhaskara Acharya gave a very clear explanation of binomial coefficients in his manuscript Lilavati. The notation n_{C_r} was introduced by Andreas von Ettingshausen in his book Die Combinatorische Analysis.

B. Binomial Theorem [2]

The discovery of the binomial theorem was announced by Isaac Newton in letters to Oldenburg on June 13 and October 24, 1676. At that time the necessity of the proof was not fully realized and he had no real proof of the formula. Although the effort was incomplete, L. Euler had given the first attempted proof in 1774. Finally in 1812 the first actual proof was given by C.F. Gauss.

One of our principal tools is the *Binomial Theorem*, for n ≥ 0,

$$(x+y)^n = \sum_r n_{C_r}\ x^r y^{n-r}$$

$\sum_r$. $\sum_r$. is used instead of $\sum_{r=0}^{n}$. . If there is no restriction on r, we are summing over all integers, $-\infty < r < +\infty$. The two notations are equivalent in this case, because when r < 0 or r > n, the terms in the above equation will become zero. We prefer the simpler form $\sum_r$. because when the conditions of summation are simpler; all manipulations with sums are simpler. If we do not need to keep the lower or upper limits of summation, leave the limits unspecified, whenever possible. This notation has one another advantage, i.e, if n is not a non-negative integer, and then the equation becomes an infinite sum.

It is important to be noted that when y=1, the equation become

$$(x+1)^n = \sum_r n_{C_r}\ x^r$$

The binomial theorem is also expressed as

$$(x+y)^n = n_{C_0}x^n y^0 + n_{C_1}x^{n-1}y^1 + \cdots \ldots\ldots + n_{C_{n-1}}x^1 y^{n-1} + n_{C_n}x^0 y^n$$

where each n_{C_r} is a precise positive integer, which is defined as the *Binomial Coefficient.*

II. Objective

To reduce the number of mathematical operations in finding n_{C_r}.

In this paper we give a novel method to compute the different possible combinations using the binomial theorem. For any natural number n, we find a sufficiently larger p and find $(p+1)^n$ from which we will get all possible values of n_{C_r} for different n. This p will be different for different values of n.

III. Illustration

To find the different values of 4_{C_r} we choose a natural number (say p) and find$(p+1)^4$, and then divide it successively by p. The remainders form the binomial coefficients. We choose minimum such p.

$(7+1)^4$ = 4096		
	Quotient	Remainder
$\frac{4096}{7}$	585	1
$\frac{585}{7}$	83	4
$\frac{83}{7}$	11	6
$\frac{11}{7}$	1	4
$\frac{1}{7}$	0	1

$(12+1)^4$=28561		
	Quotient	Remainder
$\frac{28561}{12}$	2380	1
$\frac{2380}{12}$	198	4
$\frac{198}{12}$	16	6
$\frac{16}{12}$	1	4
$\frac{1}{12}$	0	1

This method of computations of 4_{C_r} , for the different values of r = 0,1,2,3,4 altogether involves mathematical operations less than the usual traditional method of applying the formula. For any number greater than 7 and taking the fourth power with less mathematical operations we can find all the values of 4_{C_r}.The same technique can be used to find the value of n_{C_r}, for any numbers, the minimum value of p must be fixed as per the number 'n' , the value of p is different for different values of 'n'. For the number 5 it is 11 , for 6 it is 21 and so on .

IV. Concllusion

The number of mathematical operations involved in computation of n_{C_r} for different values of r can be drastically reduced in this method and this helps a lot in saving the memory allocation during programming. So the same can be applied in developing algorithms which involves the calculation of Combinations with less memory storage. The generalization for the lower bound for the value of 'p' in $(p+1)^n$for any 'n' used to find all possible values of n_{C_r} is in progress.

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