Semi Markov Model for Optimizing Availability of Testable Systems

1V. Mariappan, 2N. Dhawalikar Mahesh, 3P.K. Srividhya and 4Kurtikar Vishal

1Agnel Institute of Technology & Design, Assagao, Bardez, Goa, India.
2Department of Mechanical Engineering, Goa College of Engineering, Farmagudi, Ponda, Goa, India.
3Department of Mechanical Engineering, Periyar Maniammai University, Vallam, Thanjavur, Tamilnadu, India.
4Department of Mechanical Engineering, Gvt. Polytechnic Panaji, Altinoh, Panaji, Goa, India.

Abstract

In this era of cut throat competition, availability of systems is crucial in majority of industrial organizations. Periodic testing is an important measure of ensuring system reliability enabling timely discover and repair of possible hidden failures. However, during the test and repair, the equipment cannot perform its intended function; leading to decrease in the availability of the system. It is therefore necessary to identify the optimal test interval to maximize availability. Mechanical and electrical systems often involve non-exponential transition rates for which failure interactions can be addressed by Semi-Markov modeling using method of stages. This paper presents models using method of stages which can be conveniently used to find the optimal test rate for maximizing availability of the standby and working systems. These models can be widely used for optimizing availability of testable systems involving non-exponential transition rates, which is very common in Mechanical and Electrical Engineering.

Key Words: Optimal inspection rate, optimal test interval, semi-markov model, availability analysis.
1. Introduction

Most of the machines and equipments in industry are used regularly. However there are some machines operated rarely only when need arises. Therefore the machinery can be classified into working systems and standby systems. The status of the system can be good, degraded or failed. The machine can fail due to both deterioration and random failure. If the machine has failed either due to deterioration or random failure, maintenance is essential to bring it back to operable state using Corrective Maintenance. If the machine is in degraded state, Predictive Maintenance can be done to find the condition of the machine and decide the maintenance actions required. The above maintenance activities can lead to production delays which can be avoided by performing periodic Preventive Maintenance. In standby systems, failures if present are noticed either when test is conducted on the machine or on next activation. During test if degraded failure is found, maintenance is required to be performed to reduce critical failure rate. Therefore tests must be conducted periodically to find hidden failures. It then becomes important to decide the frequency of test interval. If test is carried out frequently it will occupy the system, leading to increased unavailability. Frequent tests also involve increased cost. On the other hand if the test is carried out after too long a period, it may lead to failure of the system by deterioration. Therefore an optimum inspection rate and test interval for working and standby systems needs to be obtained for addressing this problem.

2. Literature Review

Hokstad and Frovig (1996) presented the failure mechanism of a standby component showing the dependence of degraded and critical failures assuming exponential failure. The authors carried out Markov analysis to obtain the state probabilities. The effect of preventive maintenance is shown on the rate of occurrence of critical failures.

Papazoglou (2000) developed Semi-Markov models for systems undergoing periodic test and maintenance. Problems addressed by the models are those concerned with optimum assessment of test intervals, and allowable outage times.

Pievatolo and Valade (2003) presented a study of reliability performance of uninterrupted power supply. A Semi-Markov model with non-exponential life and repair time distributions is developed. Reliability analysis is performed to evaluate the mean time between failures and mean time to restoration of load supply.

Droguett (2008) carried out a study of pressure temperature optical monitoring system in intelligent oil field. The author developed an availability assessment model in which the system dynamics is described through continuous time Semi-Markovian process in terms of probabilities. This model is integrated with a Bayesian belief network characterizing the cause effect relationship among factors influencing the repairmen error probability during maintenance.
Lo et al (2009) presented an approach to address the issues in randomized clinical trials when many endpoints are collected over time. The technique combines saddle point approximation and likelihood theory to fit a semi-Markov model, specified in terms of its transition probabilities and moment generating function. A key feature of this approach is that the proportional hazard assumption commonly assumed for all transition time distribution is no longer needed.

Moura and Droguett (2009) proposed use of homogeneous semi-Markov processes (NHSMP) over classical method which requires a considerable computational effort. The authors have proposed a more efficient mathematical formulation and numerical treatment, which are based on transition frequency densities and general quadrature methods respectively, for NHSMPs.

Soszynska (2010) proposed a semi-Markov model for multistate component system to find its Reliability and risk characteristics. The pipeline system is described and its operation process unknown parameters are identified on the basis of real statistical data. The mean values of the pipeline system operation process unconditional sojourn times in particular operation states are found and applied to determining this process transient probabilities in these states.

D’Amico (2011) presented a non-homogenous age usage Semi-Markov model with measurable state space. The analysis of age usage model leads to the study of Markov renewal equations that are indexed by usage stochastic process. Sufficient conditions to assure the existence and uniqueness of their solutions are provided. The numerical analysis of these equations is executed through the construction of process discrete in time and space, which is shown to converge with the continuous one in the Skorohod topology. The author has presented an algorithm useful for solving the discretised system of equations by using matrix representation.

Gupta and Dharmaraja (2011) proposed an analytical dependability model for voice over internet protocol. The study is focused on analyzing combined effects of resource degradation and security breaches on the quality of service of voice over internet protocol. Software rejuvenation is adopted to prevent or postpone software failures which cause resource degradation as a preventive maintenance policy. The state transition model is analyzed under Semi-Markov process which captures the dependence of systems behavior on time spent in each state. The steady state and time dependent analysis of model are presented and validated via simulation. The dependability attributers like availability, reliability and confidentiality is also obtained.

Li and Yun (2011) studied reliability analysis for a two dissimilar units warm standby repairable system with single repairmen and unreliable transfer switch. The whole system fails immediately as the switch fails. Both units after repair are “as good as new” and first unit is given priority in use. It is assumed that the failure and the repair time distributions of the two units are both exponential. Markov approach and Laplace transform are used to derive some important reliability indexes and some steady state system indexes.
Mathew (2011), presented reliability modeling and analysis of a two-unit system of a continuous casting plant where the two-unit system operates in parallel, on full-reduced installed capacity. Upon the unit failure, an inspection is carried out to decide on the type of maintenance activity to be performed. Optimized reliability indices of the plant are obtained. Semi-Markov processes and regenerative point techniques are used for the analysis.

Veeramany and Pandey (2011) carried out reliability analysis of nuclear component cooling water (NCCW) system with repairable and non-repairable components. The failure time of components is assumed to have Weibull distribution. Poly-Weibull distribution and Semi-Markov process model is used to determine system failure probability. This concept can be further used as an initiating event probability in probabilistic safety assessment projects.

Wang and Cui (2011) developed an aggregated Semi-Markov repairable model with history dependent up and down states. It is used to model the processes in which the system states can be up or down and their status of being up or down depends upon the recent system evolution history. The instantaneous and steady reliability indices, the mean time to first failure and the frequency of failures are obtained.

Moghaddass and Zuo (2012) conducted study on the overall performance of mechanical device with multiple discrete states according its degree of deterioration. The degradation states of condition-monitored device are not directly observable and only incomplete information is available through condition monitoring. After modeling this multi-state device, an unsupervised parameter estimation method is developed, which employs historical condition monitoring information. This helps to estimate the unknown characteristic parameters of the degradation process and the observation process.

Saminu (2013) performed reliability analysis on two unit active parallel non-maintained system. The deterioration of these units follows Weibull distribution. The expression for Mean time to failure of the system is obtained analytically and the effect of the shape parameter (from Weibull Distribution) on the system reliability is depicted.

3. Availability Analysis of Testable Systems

Testable systems are the ones which can be inspected for their conditions. After inspection, corrective actions like preventive maintenance, corrective maintenance, overhauling etc can be undertaken as per the requirements. Common examples are pumps in standby mode in pharmaceutical industries which are supposed to pump chemical or water for continuous running plants.

Periodic testing is an important measure of ensuring systems operability and reliability enabling timely discover and repair of possible hidden failures. However, during the test and repair the equipment cannot perform its intended function; therefore often testing decreases the availability of the system. This work focuses on the availability analysis of the testable systems with an
intension of deriving inspection policies and optimal inspection rates for maximizing availability.

Most of the machines and equipments in industry are used regularly. However there are some machines operated rarely only when need arises. Therefore the machinery can be classified into working systems and standby systems. The status of the system can be good, degraded or failed. The machine can fail due to both deterioration and random failure. If the machine has failed either due to deterioration or random failure, maintenance is essential to bring it back to operable state. If the machine is in degraded state, Predictive Maintenance can be done to find the condition of the machine and decide the maintenance actions required. The above maintenance activities lead to production delays which can be avoided by performing maintenance activities periodically, known as PM. In standby systems, failures if present are noticed either when test is conducted on the machine or on next activation. During test if degraded failure is found, maintenance is required to be performed to reduce critical failure rate. Therefore tests must be conducted periodically to find hidden failures. It then becomes important to decide the frequency of test interval. If test is carried out frequently it will occupy the system, leading to increased unavailability. Frequent tests also involve increased cost. On the other hand if the test is carried out after too long a period, it may lead to failure of the system by deterioration. Therefore the optimal inspection rate and test interval for working and standby systems need to be obtained for addressing this problem.

A. Standby Systems

Consider a system in standby mode. As the system is standby, tests need to be conducted in order to find the current state of the system viz. Good (G), Degraded (D), Critical shock type failure (Cs) or Critical degraded type failure (Cd). After identifying the state of the machine, maintenance can be carried out to place the equipment in working condition. Accordingly the machine can be put in good working state due to perfect maintenance, remain in present state or go to further inferior state due to imperfect maintenance. During tests and maintenance the machine will be unavailable for active use.

The State Transition Diagram (STD) for the system is shown in Fig.1. In the STD the maintenance state is represented by M. T_G, T_s and T_D represent testing in good state, in the state of shock type critical failure at degraded state respectively. The rates of transition are represented on the STD.

It is assumed that transition from state G to state D is modeled by Weibull distribution such that on using method of stages, the process can be replaced by two stages each one following Exponential distribution.

The machine is available for operation only in state G, D_1 and D. State D_1 is a virtual state used to transform non-exponential transition from G to D by
dividing it into sub states between G to D₁ and D₁ to D using Method of Stages.

The State Transition Matrix (STM) for the system is shown in Eqn. (1) and the dependence matrix in Eqn. (2). The numbers of the states used in STM are tabulated in Table 1.

Table 1: Numbers for states-standby system

<table>
<thead>
<tr>
<th>No.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>State</td>
<td>G</td>
<td>D₁</td>
<td>D</td>
<td>Cₛ</td>
<td>Cₜ</td>
<td>Tₕ</td>
<td>Tₖ</td>
<td>Tₘ</td>
<td>Tₙ</td>
<td>M</td>
<td>M</td>
<td>M</td>
</tr>
</tbody>
</table>

The STM for the system is shown below.

Where

\[ a_{11} = -\lambda t - \lambda x - \lambda y \]

\[ a_{33} = -\lambda t - \lambda y - \lambda z \]

\[ a_{10} = -\lambda a - \lambda b - \lambda c \]

\[ a_{12} = -\lambda f - \lambda g - \lambda h \]

The dependence matrix is shown below.
To find the optimal test interval based on maximum availability of the machine, the transition probabilities must be found. The sum of \( P_1 + P_2 + P_3 \) will give the availability in terms of test rate \( \lambda t \). Taking the derivative of availability and setting it equal to zero and solving for test rate will give the optimal test rate.

The number of deteriorated states in the method of stages used in the model was 2. However this approach can be conveniently used to handle cases with any number of deteriorated states.

**B. Working Systems**

One of the objectives of inspection-repair system is maximization of availability of the system. Any system which begins operating from good state (G) may deteriorate to state D or fail by shock failure to reach state Cs. The deterioration process is assumed to follow Weibull distribution with shape parameter greater than 1. Therefore using method of stages, this process can be replaced by number of stages each following exponential distribution. The model presented assumes 4 stages and states \( D_1, D_2 \) and \( D_3 \) are the virtual states of degradation. From the deteriorated state D it can go to deteriorated failure state (Cd) or shock type critical failure (Cs). From the failed states, the system can be repaired to put it back in state G. It is assumed that the repair processes follow exponential distribution with rates \( \mu_1 \) from state C, to G and \( \mu_2 \) from State C to G. When the system is in the deteriorated state (D), inspection is carried out which is represented by state (I) to assess the type of maintenance actions required which are classified as minor (MM) or Major (M).

As the system performs its intended function satisfactorily when it is in state G, the inspection and maintenance activities are not required in this stage. STD of the proposed model is shown in Fig. 2. The transition rates between various states are shown in the STD.

![STD of proposed model for working systems](image.png)

The STM for the system is shown in Eqn. (3) and the dependence matrix in Eqn. (4).
The numbers of the states used in STM are tabulated in Table 2.

Table 2: Numbers for states-working system

<table>
<thead>
<tr>
<th>No.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>State</td>
<td>G</td>
<td>D_1</td>
<td>D_2</td>
<td>D</td>
<td>C_s</td>
<td>C_d</td>
<td>I</td>
<td>D</td>
<td>MM</td>
<td>D_M</td>
</tr>
</tbody>
</table>

Where

\[ a_{55} = -\lambda_n - \lambda_y - \lambda_z \]

\[ a_{88} = -\lambda_y - \lambda_y - \lambda_{so} \]

\[ a_{99} = -\lambda_e - \lambda_y - \lambda_g \]

\[ a_{1010} = -\lambda_h - \lambda_i - \lambda_{sj} \]

The dependence matrix is shown below.

To find the optimal inspection rate based on maximum availability of the system, the transition probabilities must be found. The sum of \( P_1 + P_2 + P_3 + P_4 + P_5 \) will give the availability in terms of inspection rate (\( \lambda n \)). Taking the derivative of availability and setting it equal to zero and solving for inspection rate will give the optimal inspection rate.
The number of deteriorated states in the method of stages used in the model was 4. However this approach can be conveniently used to handle cases with any number of deteriorated states.

4. Conclusion

In mechanical and electrical standby equipments like pumps/generators the degradation process often follows Weibull distribution with shape parameter greater than one. In this paper method of stages is incorporated to convert the Semi-Markov model into Markov model for modeling and analyzing these failures. The failures in standby equipments if present are detected only during tests. The model presented can be conveniently used to find the optimal test rate for maximizing availability of the standby systems.

The equipments subjected to degradation can fail due to deterioration and random failures. Inspection is necessary to decide on the type of maintenance activities to be performed in order to restore the system in operable state. When a working system is in good state inspection is not necessary as it works satisfactorily. Therefore eliminating inspection from good state and performing it only in the degraded state leads to higher availability. The model presented can be conveniently used to find the optimal inspection rate for maximizing availability of the working systems.

References


[6] Moura M.C., Droguett E.L., Mathematical formulation and numerical treatment based on transition frequency...


