

Modeling and Vibration analysis of shaft misalignment

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Abstract

In all kind of rotating machinery application misalignment is the major concern with machinery health monitoring. Usually, misalignment effects are seen at coupling, bearing and at support. The lateral responses along with torsional vibrations of misalignment have considered for modeling the equations. The general equation of motions are derived using Lagrangian approach. The misalignment magnitude is derived using frequency and transient responses in dimensionless form. Newton – Raphson method is used to obtain from dimensionless form. The analysis of signal using FFT is possible up to single level of decomposition as it loses time domain information. Therefore, to analyze vibration signals at different level of decomposition in both time and frequency domain Discrete Wavelet Transform (DWT) is proposed in this paper. Different mother wavelets are available in DWT which shows different result of analysis for same signals. In this work, under healthy and misalignment conditions vibration signals are collected to extract features using suitable mother wavelet. It is observed that selected mother wavelet, level of decomposition and extracted features are most useful for prediction of misalignment.

Key Words and Phrases: Misalignment, vibration signal, discrete wavelets transform, OVL.

1 Introduction

In literature survey, different methods are available for detecting misalignment of shaft. But these methods are generally based on FFT analysis. Ashish Darpe, et.al [16] modeled coupled rotor system using Timoshenko beam elements with six degree of freedom (DOF). The responses for misalignment at coupling and vibrations in bending, longitudinal and torsional modes are studied. Marangoni, et.al has presented studies for misaligned shaft with universal joint [19]. They concluded with observations that harmonics observed with misalignment are even multiple frequencies of motor rotational speed. P.N. Savedra et.al [7] calculated and obtained vibration pattern with misaligned condition for different coupling types viz. three pin and love joy type respectively and shown experimental results depicting 1X and 3X vibration signals as misalignment fault frequency. A.W. Lees proposed kinematics of the connecting bolt of coupling with 3-pin and obtained 2X vibration response for introduced misalignment [4]. I. Redmond, et.al [5] have been developed a mathematical model representing two rotor-misaligned system considering lateral and torsion vibration. They have developed entire force analysis of system with Lagrange's method. This study does not claim any second order harmonics present which is a common claim of similar kind of other work. Prabhu, et. al [17] have been proposed theoretical model with two rotor consisting flexible diaphragm coupling. The analysis revealed 2X response for combined unbalance and misalignment and 1X response only for unbalance present in system. The effects are verified for both type of misalignment. They ascertained proportional increase in harmonics with misalignment by FEM analysis.

The above mentioned methods are based on FFT analysis. In FFT, only one level of decomposition is possible as it losses information in time domain. Different methods of transformations are used to convert original time signal from time to frequency domain, dq plane, etc [9]-[15]. For detail analysis of vibration signals during misalignments, multiple level of decomposition is proposed in this paper. Multiple level of decomposition also helps in effective segregation of combined mode of fault diagnosis. In this article, mathematical modeling for parallel misalignments of flexibly coupled shaft and analysis of vibration signal using DWT to select effective mother wavelet to predict degree of misalignment has been discussed.

2 Mathematical Modeling

Mathematical modeling for parallel misalignment is introduced assuming misalignment in only X-direction. Model of parallel misalignment is as shown in Figure 1. Mathematical model is referred

from I.Redmond [20]

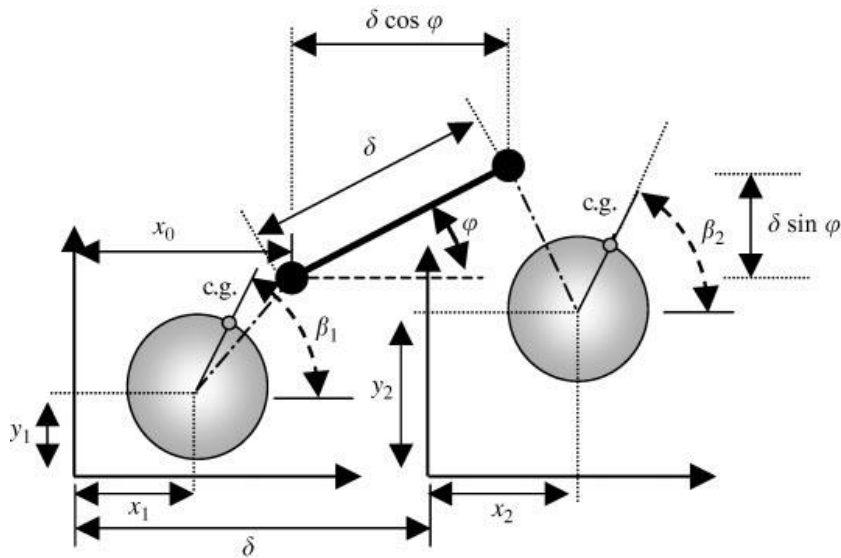


Figure 1. Position of misaligned rotors

Let, β_1 and β_2 be displacement of rotors in parallel misalignment. The coordinate of systems for analysis is referred as mentioned below.

$$\{q\} = [x_1, y_1, \beta_1, x_2, y_2, \beta_2] \quad (1)$$

Total kinetic energy (K.E.) associated with system is sum of K.E. of input side and K.E. of output side. Let, T be total K.E, which can be expressed as below

$$T = K.E_{input} + K.E_{output}$$

$$T = \frac{1}{2}m_1[(\dot{x}_1 - e_1\dot{\beta}_1\sin\beta_1)]^2 + \frac{1}{2}m_1[(\dot{y}_1 + e_1\dot{\beta}_1\cos\beta_1)]^2 + \frac{1}{2}m_2[(\dot{x}_2 - e_2\dot{\beta}_2\sin\beta_2)]^2 + \frac{1}{2}m_2[(\dot{y}_2 + e_2\dot{\beta}_2\cos\beta_2)]^2 + \frac{1}{2}J_1\dot{\beta}_1^2 + \frac{1}{2}J_2\dot{\beta}_2^2. \quad (2)$$

The potential energy is

$$V = \frac{1}{2}k_1x_1^2 + \frac{1}{2}k_1y_1^2 + \frac{1}{2}k_1'(x_1 - x_a)^2 + \frac{1}{2}k_1'(y_1 - y_a)^2 + \frac{1}{2}k_{t1}(\beta_1 - \varphi)^2 + \frac{1}{2}k_2x_2^2 + \frac{1}{2}k_2y_2^2 + \frac{1}{2}k_2'(x_a + \delta\cos\varphi - x_2 + \delta)^2 + \frac{1}{2}k_2'(y_a + \delta\sin\varphi - y_2)^2 + \frac{1}{2}k_{t2}(\varphi - \beta_2)^2 + m_1g(y_1 + e_1\sin\beta_1) + m_2g(y_2 + e_2\sin\beta_2). \quad (3)$$

The generalized equation of vibration motion analysis is given as,

$$[m]\{\ddot{q}\} + [k]\{q\} = \{F\}. \quad (4)$$

Where m, k and F are mass, stiffness and force matrix respectively.

Effect of misalignment is observed on force vector because of lateral

vibration. Preload is seen which is because of assumption of x direction misalignment.

$$k_e' = k_1'k_2'/(k_1' + k_2'), k_{te} = k_{t1}k_{t2}/(k_{t1} + k_{t2}), \omega_{n1}^2 = k_e'/m_1, \omega_{n2}^2 = k_e'/m_2 \quad (5)$$

The Eq. 4 is converted in dimensionless form dividing by $k_e'\delta$.

Dimensionless space vector can be expressed as

$$\{q^*\} = \{x_1/\delta, y_1/\delta, \beta_1, x_2/\delta, y_2/\delta, \beta_2\}^T = \{x_1^*, y_1^*, \beta_1, x_2^*, y_2^*, \beta_2\}^T, \quad (6)$$

The generalized equation of motion in dimensionless form considering damping can be expressed as,

$$[m^*]\{\ddot{q}^*\} + [c^*]\{\dot{q}^*\} + [k]\{q^*\} = \{F^*\} \quad (7)$$

The damping factor is introduced in Eq. (16) which are given as below.

$$c = 2\zeta m_1 \omega_{n1} / \delta k_e' \quad (8)$$

Following equations are obtained which determines coupling lateral deflections.

$$x_a^* = (k_e'/k_2')x_1^* + (k_e'/k_1')x_2^* - (k_e'/k_2')\cos\varphi + k_2'/(k_1' + k_2') \quad (9)$$

$$y_a^* = (k_e'/k_2')y_1^* + (k_e'/k_1')y_2^* - (k_e'/k_{12}')\sin\varphi \quad (10)$$

3 Methodology

The proposed method of shaft misalignment is shown in [Figure 2](#). Motor shaft and output are coupled by means of rubber bush coupling. An artificially created misalignment in set up induces forces in system. These forces are measured in terms of vibration at bearing position 2 using accelerometers. The vibration signals are sensed by accelerometer in x, y and z coordinates. The obtained signal is processed and analyzed using DWT. Vibration signals are non-stationary in nature. Non-stationary signals are generally analyzed using DWT [1]. It is required to know whether and also when an incident was happened. Therefore, detailing of non-stationary vibration signals is essential. Such detailing of signals is carried out using DWT. In DWT, analysis of the information signal is converted into scaled and translated version of mother wavelet which is very irregular in nature. Hence, DWT is more suitable for stationary and non-stationary signal. In DWT, original vibration signals are passed through low pass filter (LPF) and high pass filter (HPF). The output of LPF is called as detail coefficients (C_D) and output of high pass filter is called as approximate coefficients (C_A). C_D carries information in frequency domain and C_A carries information in time domain. For further level of decomposition C_A are used. In DWT the C_D and C_A can be expressed as [Eq. 1](#) and [Eq. 2](#).

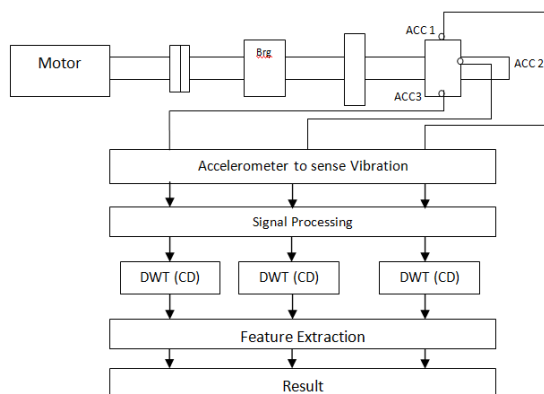


Figure 2. Methodology for processing vibration signal using DWT

$V(n)$ are vibration signals in three direction x , y and z , therefore $n = x, y$ and z . DWT gives complete analysis of original information in time and frequency domain. Hence, in common behavior of different operating conditions it may be faulty or healthy. For locating exact fault DWT is more suitable. In this proposed system suitable mother wavelet and decomposition levels is selected by analyzing maximum feature of CD.

4 Experimental setup

The experimental setup is shown in Figure 3. A three phase, A.C. induction, 1 H.P motor is used in experimental set up. A shaft with central load is supported between two bearings to simulate condition with rotating machinery. An arrangement is done at base plate to facilitate an artificial misalignment in set up.



Figure 3. Methodology for processing vibration signal using DWT

This section provides details of full-bridge forward converter with respect to FC power conversion besides other relevant information.

A suitable range of speed on industrial application based is considered. The maximum permitted misalignment is verified from shaft and machinery alignment handbook. These limits are

taken as permissible range for variation of misalignment in the experimental set up artificially. Motor side arrangement is given scope for introducing misalignment artificially. (Offset and angular misalignment). The experimental set up and its result obtained is validated with known harmonics pattern (1X, 2X) standard referred in Rath-bone chart. A 4-channel digital power scope is used to read and store accelerometer signals in three directions. It also records three phase current data. A VFD (50 Hz, 1.5 Amp) is used to have precise control on speed parameter of motor. Vibration isolator pad are used to damp unwanted structure vibration.

5 Results

Actual vibration signal obtained from experimental set up are shown in Figure 3. In this, 2500 samples are collected for different operating conditions of motor. It is seen that overall vibration level is more in the plane of offset misalignment i.e. in lateral direction i.e. 'Z' direction. Sample vibration signal of 1800 rpm with 0.03 misalignments are shown in Figure 3. The FFT is applied for validation of collected vibration signals under healthy and misalignment conditions. As accepted the result of FFT highest amplitude at 1X is obtained as shown in Figure 4. Hence the results of FFT are authenticating that collected data under various conditions is correct.

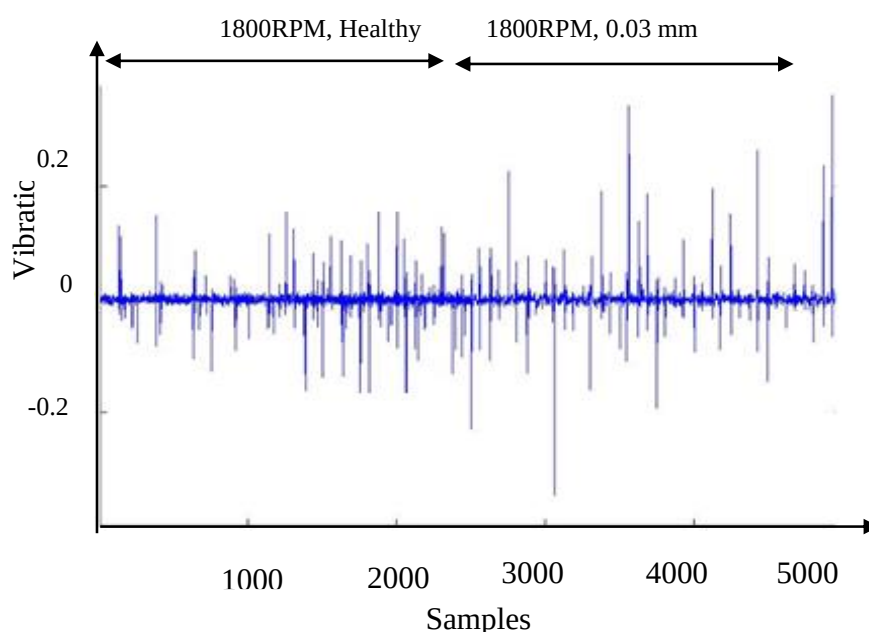


Figure 3. Vibration signal at healthy and 0.03mm offset at 1800RPM.

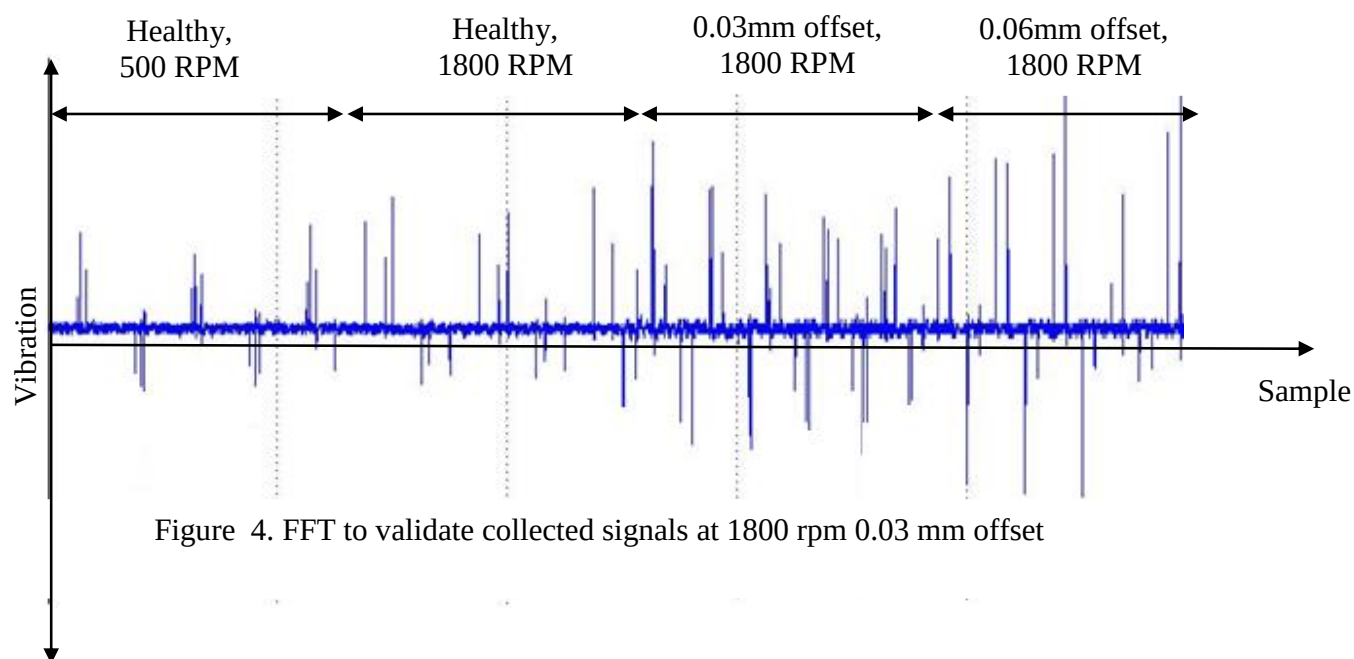


Figure 4. FFT to validate collected signals at 1800 rpm 0.03 mm offset

Figure 5. Vibration signal at different conditions

As level two of decomposition is applied, 625 detailed coefficients are obtained. Packets size of 625 is used to extract features and maximum value is obtained from each packet. The vibration signals under different conditions are obtained as shown in Figure 5. Figure 6 - Figure 15 shows the extracted feature of C_D after applying different mother wavelets for various level of decomposition. For misalignment detection, db2 /sym 2 wavelets are found suitable as their results are verified by extracting maximum feature as shown in Table 1.

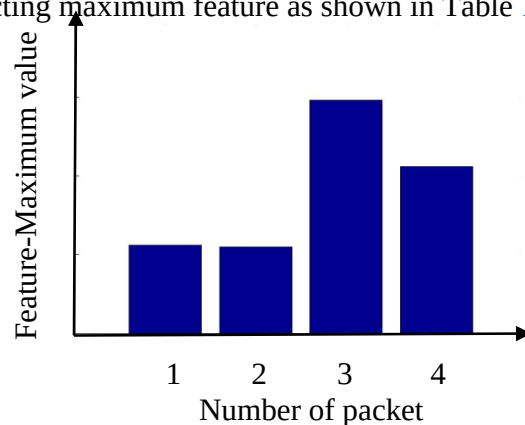


Figure 13. Maximum features of detailed coefficients obtained using RBIO1.3

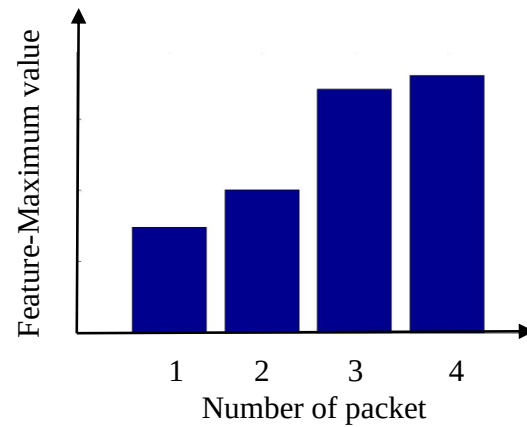


Figure 14. Maximum features of detailed coefficients obtained using SYM2

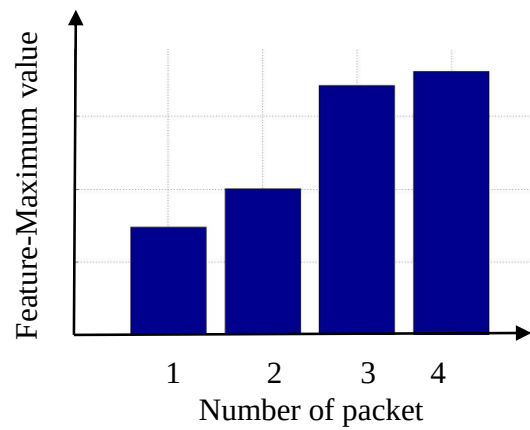


Figure15. Maximum features of detailed coefficients obtained using DB2

Table 1

Max features at different conditions using different mother wavelet

Mother Wavelet	Operating conditions			
	500 RPM	1800 RPM	1800 RPM, 0.03 mm offset	1800 RPM, 0.09 mm offset
BIOR1.1	0.112	0.110	0.296	0.212
BIOR1.3	0.158	0.196	0.382	0.242
COIF1	0.161	0.100	0.400	0.228
COIF2	0.149	0.120	0.368	0.211
COIF3	0.142	0.128	0.350	0.200
DMEY	0.120	0.136	0.295	0.163
HAAR	0.112	0.110	0.296	0.212
RBIO1.3	0.124	0.195	0.342	0.274
SYM2	0.147	0.200	0.341	0.361
DB2	0.147	0.200	0.341	0.361

6 Conclusion

The mathematical modeling of misaligned shaft is presented here. Time-frequency feature of misaligned shaft vibrations for offset condition are considered by DWT time frequency analysis. As per selected range of shaft speed and corresponding limit of misalignment, different vibration signals are obtained by experimental set up. For validation purpose, these signal output is verified for misalignment frequency 1X, 2X on frequency. As these obtained signals are non-stationary, for fault detection DWT analysis is used and different mother wavelet (db, haar, sym etc) are applied. With the results obtained it is clear that proper selection of mother wavelet is done on the basis of different decomposition level. DB 2 and SYM 2 are observed as best suitable mother wavelet based on extracted features.

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