Centralized Inventory Model with Shortages and Screening Process Using Analytical Geometry and Algebraic Method

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Abstract

In this paper, we develop an economic order quantity inventory model with screening cost and disposable cost at two part coordination scheme. Shortages are allowed for buyer in non coordination scheme and no shortages for coordination scheme. In coordination scheme, vendor provides quantity discount to the buyer. Hence, the buyer orders more quantity than regular quantity. The damaged products are screened or disposed by the vendor for coordination scheme and by the buyer for non coordination scheme. In addition, centralized model is developed for system optimization. We provide the basic analytical geometry and algebraic method for deriving the optimal solutions. The proposed model is illustrated through numerical example and the sensitivity analysis is also performed.

Key Words: Inventory, order quantity, shortage cost, screening cost, disposable cost.
1. Introduction

Now days, the production quality assurance is one of the most important factor in manufacturing industries. Product quality depends upon the customer expectation on products. Hence, the manufacturing industries concentrated on production of high quality products. In real world production, defective items may be produced which can be screened or disposed. Defective products can be reworked which assuring production quality and also minimize the total inventor production cost.


The reminder of the paper is organized as follows. In section 2 assumption, notations and in section 3, centralized model with and without coordination are given. Numerical examples are given in section 4. Finally conclusion and summary are presented.

2. Assumptions and Notations

The model use the following assumptions and notations

**Assumptions**

1. Demand rate is uniform and constant.
2. Shortages are allowed for buyer only.
3. Buyer screened or disposed the damaged items in coordination scheme and vendor screened or disposed the damaged items in non coordination
scheme.

4. For benefits of buyer and vendor the system cost is formed. The system cost can be written as $T_{CS} = T_{CB} + T_{CV}$

**Notations**

- $D$: Demand rate per time unit
- $r_1$: Buyer’s unit setup cost per order
- $r_2$: Vendor’s unit ordering cost per order
- $B$: Backorders level
- $s_1$: Shortage cost per order per unit
- $Q$: Economic Order quantity
- $h_b$: Buyer’s unit holding cost per order per unit
- $h_v$: Vendor’s unit holding cost per order per unit
- $s_c$: Screening cost per order per unit
- $d_c$: Disposal cost per order per unit
- $p_c$: Purchase cost per order per unit
- $a$: Percentage of defective items
- $b$: Percentage of scrap items
- $n$: Vendor’s multiples of order with coordination
- $m$: Vendor’s multiples of order without coordination
- $K$: Buyer’s multiples of order with coordination
- $d(K)$: Discount factor

**3. Formulation of the Model**

In this section, centralized model is developed for non coordination and coordination strategies. i.e., In centralized model, integrated system cost is developed for system optimization. The integrated system cost is addition of buyer’s total cost and vendor’s total cost. In non coordination strategy, the vendor screened or disposed the damaged product for resale and the buyer having shortages. In coordination strategy, the vendor offers quantity discount
for the buyer for large purchase. Hence, the buyer has no shortage as well as the buyer himself screened or disposed the damaged products for resale.

**Centralized Model with Non Coordination**

Now the buyer’s total cost is addition of ordering cost, holding cost and shortage cost.

\[
TC_{B1} = \frac{r_1D}{Q} + \frac{h_bB^2}{2Q} + \frac{s_1(Q-B)^2}{2Q}
\]

The vendor’s total cost is addition of setup cost, holding cost, screening cost and disposal cost

\[
TC_{V1} = \frac{r_2D}{mQ} + \frac{h_v(m-1)Q}{2} + \frac{s_c(m-1)Q}{2} + \frac{d_{cab}(m-1)Q}{2}
\]

The integrated system cost is expressed as

\[
TC_{S1} = TC_{B1} + TC_{V1} = \frac{D}{Q}\left(\frac{r_1 + r_2}{m}\right) + \frac{h_bB^2 + s_1(Q-B)^2}{2Q} + \frac{(m-1)(h_v + s_c + d_{cab})Q}{2}
\]

Equation (1) can be written as

\[
TC_{S1} = \left(\frac{h_b + s_1}{2Q}\right)B^2 - s_1B + \frac{s_1Q}{2} + \frac{D}{Q}\left(\frac{r_1 + r_2}{m}\right) + \frac{(m-1)(h_v + s_c + d_{cab})Q}{2}
\]

It is of the form \(c_1x^2 + c_2x + c_3\). By Cardenas Barron (2011),

\[
B = \frac{-c_2}{2c_1} = \frac{Qs_1}{h_b + s_1}
\]

i.e., \(B^* = \frac{Qs_1}{h_b + s_1}\) (2)

Also, Equation (1) can be written as

\[
TC_{S1} = \left\{\frac{s_1 + (m-1)(h_v + s_c + d_{cab})}{2}\right\}Q + \left\{D\left(\frac{r_1 + r_2}{m}\right) + \frac{(h_b + s_1)B^2}{2}\right\} - s_1B
\]

It is of the form \(c_1x^2 + c_2x + c_3\). By Cardenas Barron (2011),

\[
Q = \sqrt{\frac{c_2}{c_1}} = \sqrt{\frac{2D\left(\frac{r_1 + r_2}{m}\right)(h_b + s_1)}{\sqrt{(h_b + s_1) + (m-1)(h_b + s_1)(h_v + s_c + d_{cab})}}}
\]
Centralized Model with Coordination

In coordination strategy, buyer’s order size is greater than the regular size because the vendor offers quantity discount at discount factor $d(K)$. Hence, the buyer has no shortage and also the buyer screened or disposed the damaged products for resale. Now the buyer’s order size is $KQ_0$ and the vendor’s order size is $KnQ_0$.

Now the buyer’s total cost is addition of ordering cost, holding cost, screening cost and disposal cost.

\[
\text{i.e., } TC_{B2} = \frac{r_1D}{Q_0} + \frac{h_bQ_0}{2} + \frac{s_cQ_0}{2} + \frac{d_{c}abQ_0}{2}
\]

The vendor’s total cost is addition of setup cost, holding cost and buyer’s quantity discount.

\[
\text{i.e., } TC_{V2} = \frac{r_2D}{KnQ_0} + \frac{h_vK(n-1)Q_0}{2} + p_cDd(K)
\]

The integrated system cost is expressed as

\[
TC_{S2} = TC_{B2} + TC_{V2}
= \frac{D}{Q_0} \left( r_1 + \frac{r_2}{Kn} \right) + \frac{(h_b+n-1)h_v+s_c+d_cab)Q_0}{2} + p_cDd(K)\tag{4}
\]

Equation (4) can be written as

\[
TC_{S1} = \left( \frac{(h_b+n-1)h_v+s_c+d_cab)Q_0}{2} \right) + \left( D \left( r_1 + \frac{r_2}{Kn} \right) \right) \frac{1}{Q_0} + p_cDd(K)
\]

It is of the form $c_1x + \frac{c_2}{x} + c_3$. By Cardenas Barron (2011),

\[
Q_0 = \sqrt{\frac{c_2}{c_1}} = \sqrt{\frac{2D \left( r_1 + \frac{r_2}{Kn} \right)}{h_b + K(n-1)h_v + s_c + d_cab}}
\]

i.e.,

\[
Q_0^* = \sqrt{\frac{2D \left( r_1 + \frac{r_2}{Kn} \right)}{h_b + K(n-1)h_v + s_c + d_cab}}\tag{5}
\]
4. Numerical Example

Let $D = 2000$ units per year, $r_1 = 300$ $ per order, r_2 = 100$ $ per order, hv = 10$ $, hb = 20$ $, s_1 = 25, sc = 2, dc = 1.5$, $a = 0.1, b = 0.5, p = 3, m = 8, n = 2, K = 3, d(K) = 5\%$. The optimal solutions are

$$Q^* = 114, B^* = 64, TC_{S1} = 1.0934 \times 10^4, Q^*_0 = 156, TC_{S2} = 8.4217 \times 10^3$$

**Sensitivity Analysis**

The sensitivity analysis is performed by changing the value of each parameter by $-50\%$, $-25\%$, $+25\%$, $+50\%$, taking one parameter at a time and keeping the remaining parameters unchanged. The solutions are highly sensitive to change in $r_1$, $D$, $h_v$ and moderately sensitive to change in $h_b$ and slightly sensitive to change in $r_2$, $p$, $s_1$, $s_c$, $d_c$. The results are shown in Table 1.

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5. Conclusion

In this paper, we have developed centralized inventory model for buyer–vendor with shortages and screening process. Moreover, it was assumed that shortages are allowed for buyer in non-coordination and no shortages for coordination. Furthermore, the damaged products are disposed or screened by buyer for non-coordination and by vendor for coordination scheme. In coordination scheme, quantity discount is offered by the vendor at some discount rate. It proves that quantity discount coordination scheme achieve system optimization. Finally, numerical example is presented to illustrate the solution procedure. For future work the model can extended by temporary discount or price discount, delay in payments etc.,

References


