

Centralized Inventory Model with Shortages and Screening Process Using Analytical Geometry and Algebraic Method

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Abstract

In this paper, we develop an economic order quantity inventory model with screening cost and disposable cost at two part coordination scheme. Shortages are allowed for buyer in non coordination scheme and no shortages for coordination scheme. In coordination scheme, vendor provides quantity discount to the buyer. Hence, the buyer orders more quantity than regular quantity. The damaged products are screened or disposed by the vendor for coordination scheme and by the buyer for non coordination scheme. In addition, centralized model is developed for system optimization. We provide the basic analytical geometry and algebraic method for deriving the optimal solutions. The proposed model is illustrated through numerical example and the sensitivity analysis is also performed.

Key Words: Inventory, order quantity, shortage cost, screening cost, disposable cost.

1. Introduction

Now days, the production quality assurance is one of the most important factor in manufacturing industries. Product quality depends upon the customer expectation on products. Hence, the manufacturing industries concentrated on production of high quality products. In real world production, defective items may be produced which can be screened or disposed. Defective products can be reworked which assuring production quality and also minimize the total inventor production cost.

Muniappan and Uthayakumar [4] derived optimal replenishment polices by using mathematical techniques. Ravithammal et al. [7] concentrated buyer-manufacturer model involving shortages and quantity discount for fixed life time product. Ravithammal et al. [6] studied buyer - vendor inventory model for fixed lifetime product using fixed and linear back orders. Ravithammal et al. [8] developed production inventory model for perishable items. Muniappan et al. [5] analyzed vendor-buyer coordination model for fixed life time products involving quantity discount, backordering and rework.

Cardenas-Barron [1] analyzed an easy method to derive EOQ and EPQ inventory models with backorders. Cardenas-Barron [2] derived EOQ/EPQ inventory models with two backorders costs using analytic geometry and algebra. Teerapabolarn and Khamrod [10] developed inventory models with backorders and defective items derived algebraically and AGM. Chang et al. [3] studied impacts of inspection errors and trade credits on the economic order quantity model for items with imperfect quality. Sarkar et al. [9] developed EPQ model with random defective rate, rework process and backorders for a single stage production system. Zhou et al. [11] considered a synergic economic order quantity model with trade credit, shortages, imperfect quality and inspection errors.

The reminder of the paper is organized as follows. In section 2 assumption, notations and in section 3, centralized model with and without coordination are given. Numerical examples are given in section 4. Finally conclusion and summary are presented.

2. Assumptions and Notations

The model use the following assumptions and notations

Assumptions

1. Demand rate is uniform and constant.
2. Shortages are allowed for buyer only.
3. Buyer screened or disposed the damaged items in coordination scheme and vendor screened or disposed the damaged items in non coordination

scheme.

4. For benefits of buyer and vendor the system cost is formed. The system cost can be written as $TC_S = TC_B + TC_V$

Notations

D Demand rate per time unit

r_1 Buyer's unit setup cost per order

r_2 Vendor's unit ordering cost per order

B Backorders level

s_1 Shortage cost per order per unit

Q Economic Order quantity

h_b Buyer's unit holding cost per order per unit

h_v Vendor's unit holding cost per order per unit

s_c Screening cost per order per unit

d_c Disposal cost per order per unit

p_c Purchase cost per order per unit

a Percentage of defective items

b Percentage of scrap items

n Vendor's multiples of order with coordination

m Vendor's multiples of order without coordination

K Buyer's multiples of order with coordination

$d(K)$ Discount factor

3. Formulation of the Model

In this section, centralized model is developed for non coordination and coordination strategies. i.e., In centralized model, integrated system cost is developed for system optimization. The integrated system cost is addition of buyer's total cost and vendor's total cost. In non coordination strategy, the vendor screened or disposed the damaged product for resale and the buyer having shortages. In coordination strategy, the vendor offers quantity discount

for the buyer for large purchase. Hence, the buyer has no shortage as well as the buyer himself screened or disposed the damaged products for resale.

Centralized Model with Non Coordination

Now the buyer's total cost is addition of ordering cost, holding cost and shortage cost.

$$\text{i.e., } TC_{B1} = \frac{r_1 D}{Q} + \frac{h_b B^2}{2Q} + \frac{s_1 (Q-B)^2}{2Q}$$

The vendor's total cost is addition of setup cost, holding cost, screening cost and disposal cost

$$\text{i.e., } TC_{V1} = \frac{r_2 D}{mQ} + \frac{h_v (m-1)Q}{2} + \frac{s_c (m-1)Q}{2} + \frac{d_c ab (m-1)Q}{2}$$

The integrated system cost is expressed as

$$\begin{aligned} TC_{S1} &= TC_{B1} + TC_{V1} \\ &= \frac{D}{Q} \left(r_1 + \frac{r_2}{m} \right) + \frac{h_b B^2 + s_1 (Q-B)^2}{2Q} + \frac{(m-1)(h_v + s_c + d_c ab)Q}{2} \quad (1) \end{aligned}$$

Equation (1) can be written as

$$TC_{S1} = \left(\frac{h_b + s_1}{2Q} \right) B^2 - s_1 B + \frac{s_1 Q}{2} + \frac{D}{Q} \left(r_1 + \frac{r_2}{m} \right) + \frac{(m-1)(h_v + s_c + d_c ab)Q}{2}$$

It is of the form $c_1 x^2 + c_2 x + c_3$. By Cardenas Barron (2011),

$$B = \frac{-c_2}{2c_1} = \frac{Qs_1}{h_b + s_1}$$

$$\text{i.e., } B^* = \frac{Qs_1}{h_b + s_1} \quad (2)$$

Also, Equation (1) can be written as

$$TC_{S1} = \left\{ \frac{s_1 + (m-1)(h_v + s_c + d_c ab)}{2} \right\} Q + \left\{ D \left(r_1 + \frac{r_2}{m} \right) + \frac{(h_b + s_1)B^2}{2} \right\} \frac{1}{Q} - s_1 B$$

It is of the form $c_1 x + \frac{c_2}{x} + c_3$. By Cardenas Barron (2011),

$$Q = \sqrt{\frac{c_2}{c_1}} = \sqrt{\frac{2D \left(r_1 + \frac{r_2}{m} \right) (h_b + s_1)}{(h_b + s_1) + (m-1)(h_b + s_1)(h_v + s_c + d_c ab)}}$$

$$\text{i.e., } Q^* = \sqrt{\frac{2D\left(r_1 + \frac{r_2}{m}\right)(h_b + s_1)}{(h_b + s_1) + (m-1)(h_b + s_1)(h_v + s_c + d_c ab)}} \quad (3)$$

Centralized Model with Coordination

In coordination strategy, buyer's order size is greater than the regular size because the vendor offers quantity discount at discount factor $d(K)$. Hence, the buyer has no shortage and also the buyer screened or disposed the damaged products for resale. Now the buyer's order size is KQ_0 and the vendor's order size is KnQ_0 .

Now the buyer's total cost is addition of ordering cost, holding cost, screening cost and disposal cost.

$$\text{i.e., } TC_{B2} = \frac{r_1 D}{Q_0} + \frac{h_b Q_0}{2} + \frac{s_c Q_0}{2} + \frac{d_c ab Q_0}{2}$$

The vendor's total cost is addition of setup cost, holding cost and buyer's quantity discount.

$$\text{i.e., } TC_{V2} = \frac{r_2 D}{KnQ_0} + \frac{h_v K(n-1)Q_0}{2} + p_c Dd(K)$$

The integrated system cost is expressed as

$$\begin{aligned} TC_{S2} &= TC_{B2} + TC_{V2} \\ &= \frac{D}{Q_0} \left(r_1 + \frac{r_2}{Kn} \right) + \frac{(h_b + (n-1)h_v + s_c + d_c ab)Q_0}{2} + p_c Dd(K) \end{aligned} \quad (4)$$

Equation (4) can be written as

$$TC_{S1} = \left\{ \frac{(h_b + (n-1)h_v + s_c + d_c ab)}{2} \right\} Q_0 + \left\{ D \left(r_1 + \frac{r_2}{Kn} \right) \right\} \frac{1}{Q_0} + p_c Dd(K)$$

It is of the form $c_1 x + \frac{c_2}{x} + c_3$. By Cardenas Barron (2011),

$$Q_0 = \sqrt{\frac{c_2}{c_1}} = \sqrt{\frac{2D \left(r_1 + \frac{r_2}{Kn} \right)}{h_b + K(n-1)h_v + s_c + d_c ab}}$$

$$\text{i.e., } Q_0^* = \sqrt{\frac{2D \left(r_1 + \frac{r_2}{Kn} \right)}{h_b + K(n-1)h_v + s_c + d_c ab}} \quad (5)$$

4. Numerical Example

Let $D = 2000$ units per year, $r_1 = 300\$$ per order, $r_2 = 100\$$ per order, $h_v = 10\$$, $h_b = 20\$$, $s_1 = 25$, $s_c = 2$, $d_c = 1.5\$$, $a=0.1$, $b= 0.5$, $p = 3$, $m = 8$, $n = 2$, $K = 3$, $d(K) = 5\%$. The optimal solutions are

$$Q^* = 114, B^* = 64, TC_{S_1}^* = 1.0934 \times 10^4, Q_0^* = 156, TC_{S_2}^* = 8.4217 \times 10^3$$

Sensitivity Analysis

The sensitivity analysis is performed by changing the value of each parameter by -50%, -25%, +25%, +50%, taking one parameter at a time and keeping the remaining parameters unchanged. The solutions are highly sensitive to change in r_1 , D , h_v and moderately sensitive to change in h_b and slightly sensitive to change in r_2 , p , s_1 , s_c , d_c . The results are shown in Table 1.

Table 1

Parameters		Q^*	B^*	$TC_{S_1}^*$	Q_0^*	$TC_{S_2}^*$
D	-50	81	45	7.7313×10^3	110	5.8929×10^3
	-25	99	55	9.4688×10^3	135	7.2586×10^3
	+25	128	71	1.2224×10^3	174	9.4553×10^3
	+50	140	78	1.3391×10^4	191	1.0397×10^3
r_1	-50	82	46	7.8844×10^3	113	6.1921×10^3
	-25	100	55	9.5318×10^3	136	7.3950×10^3
	+25	127	70	1.2175×10^4	174	9.3324×10^3
	+50	139	77	1.3301×10^4	189	1.0159×10^3
r_2	-50	113	63	1.0824×10^4	154	8.3141×10^3
	-25	114	63	1.0879×10^4	155	8.3681×10^3
	+25	115	64	1.0988×10^4	157	8.4749×10^3
	+50	116	64	1.1042×10^4	158	8.5278×10^3
h_v	-50	144	80	8.7060×10^3	185	7.1529×10^3
	-25	127	70	9.8828×10^3	169	7.8141×10^3
	+25	105	58	1.1892×10^4	146	8.9869×10^3
	+50	98	54	1.2779×10^4	137	9.5175×10^3
h_b	-50	117	83	1.0704×10^4	174	7.6003×10^3
	-25	115	72	1.0834×10^4	164	8.0219×10^3
	+25	114	57	1.1013×10^4	149	8.8026×10^3
	+50	113	51	1.1077×10^4	143	9.1673×10^3
s_1	-50	116	45	1.0736×10^4	156	8.4217×10^3
	-25	115	56	1.0851×10^4	156	8.4217×10^3
	+25	114	69	1.0995×10^4	156	8.4217×10^3
	+50	114	74	1.1044×10^4	156	8.4217×10^3
s_c	-50	119	66	1.0526×10^4	158	8.3433×10^3
	-25	117	65	1.0732×10^4	157	8.3826×10^3
	+25	113	62	1.1132×10^4	155	8.4606×10^3
	+50	110	61	1.1327×10^4	155	8.4993×10^3
d_c	-50	115	64	1.0919×10^4	156	8.4188×10^3
	-25	114	64	1.0927×10^4	156	8.4203×10^3
	+25	114	64	1.0941×10^4	156	8.4230×10^3
	+50	114	63	1.0949×10^4	156	8.4246×10^3
p	-50	114	64	1.0934×10^4	156	8.2717×10^3
	-25	114	64	1.0934×10^4	156	8.3467×10^3
	+25	114	64	1.0934×10^4	156	8.4967×10^3
	+50	114	64	1.0934×10^4	156	8.8717×10^3

5. Conclusion

In this paper, we have developed centralized inventory model for buyer – vendor with shortages and screening process. Moreover, it was assumed that shortages are allowed for buyer in non coordination and no shortages for coordination. Furthermore, the damaged products are disposed or screened by buyer for non coordination and by vendor for coordination scheme. In coordination scheme, quantity discount is offered by the vendor at some discount rate. It proves that quantity discount coordination scheme achieve system optimization. Finally, numerical example is presented to illustrate the solution procedure. For future work the model can extended by temporary discount or price discount, delay in payments etc.,

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