

Analysis of Power Spectrum Density on Earthquake Data Using Modified Covariance Algorithm

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Abstract

Seismic signals have very low Signal to Noise Ratio (SNR) due to ground roll noise. These signals undergo filtering and can be extracted by using seismogram. In this paper, FIR band pass filter is used to reduce noise. Modified Covariance is used to analyze the seismic signal spectrum and also for estimating traces of seismic signals.

Keywords: Stochastic signal processing, Adaptive Signal processing, Seismology, Applied statistics.

1. INTRODUCTION:

I.I. Seismology

Study of Earthquakes is known as Seismology. They occur when the rocks in the subsurface bursts along the plane surface, thus seismic waves are produced [6]. These are elastic in nature and propagate through fluid, solid materials and are not direction specific. They are interpreted by seismogram, which records the earth's motion as the function of time. There are many seismographic stations that are located in many parts of the world.

Seismic waves are categorized into three types. They are body waves, surface waves and normal modes [10]. Body waves propagate through the interior of materials; they can be again divided into two types, primary waves and

wave is travelling. Secondary waves, as they are the transverse waves move normal to the direction of propagation of waves. Surface waves propagate along the material interfaces. They are of two types, Rayleigh waves and Love waves. Rayleigh waves satisfy all the boundary conditions as they were formed by the interaction of both vertically polarized S and P waves. Love waves do not have compression motion as Rayleigh waves, when compared to P and S waves, surface waves are slower but stronger. Normal modes are like stationary waves.

1.2. Seismic Signal Processing

Signal processing is used to obtain the power spectrum that helps in analyzing of signals. There are mainly three algorithms that are used for the prediction of seismic traces. They are parametric and nonparametric algorithms in which signals in time domain are converted into the frequency domain and in frequency algorithms, directly the signals are available in frequencies only. Seismic data consists of high noise that is raised from many different sources. The main two sources are coherent and random noise [1]. Coherent noise is the undesirable energy that is added to the primary signals. One of the coherent noises is the ground roll noise that travels along the ground surface with very high amplitudes and low velocity whereas random noise is the additive white Gaussian noise. But mostly real seismic data consists of ground roll noise.

Primary objective of seismic data processing is to increase the SNR. The waves that are produced during this process are random in nature. Random process is based on probability conditions [7], thus FFT is not applicable for seismic signal processing. As FIR filters have a property of linear phase, the delay times for all the signals are same so that phase distortions can be eliminated. In this paper, one of the parametric methods, modified covariance is explained.

1.3. Parametric Method

Parametric method gives very high resolution. As the actual data that is originated can't be determined in nonparametric, accuracy of data is more in parametric. It mainly consists of 3 steps as follows [5, 9]

1. Appropriate model is to be selected for the process.
2. Parameters should be determined.
3. Finally, power spectrum is to be estimated. Depends upon the process generated, there are several models like MA, AR and ARMA.

Moving average (MA) model consist only zeros. Autoregressive moving average (ARMA) model is to predict

auto regressive and the other is the moving average model, thus this model has both poles and zeros which are located at unit circle [3]. When the noise ratio in the signal is high then this method is adapted. It filters the white noise that is present in the system.

AR model uses previous outputs and consists of poles only. MA and ARMA filters are very difficult for implementation as they mostly consist of nonlinear equations, whereas in AR model the data is uniformly sampled. Input is given in time samples that are in column vector and gives frequency points as output which is used to estimate the PSD [2], when compared to other estimation techniques of AR process, modified covariance gives the result with high resolution.

Parameters of AR model can be determined by various algorithms like covariance, modified covariance and burg.

Section 2 deals with Mathematical modeling, section 3 deals with Simulation and Results, section 4 deals with results, section 5 deals with conclusion.

2. MATHEMATICAL MODELLING:

Parameters of AR model are estimated by using autocorrelation, covariance method and modified covariance method. Finding the order of the model is very important because if it is too low then the output of the spectrum is smooth or else it consists of very low peaks. Thus order of the model must be less than the length of data but it should be comparable.

Power spectrum for the m^{th} order AR process is

$$P_y(e^{j\omega}) = \frac{|b(0)|^2}{|1 + \sum_{n=1}^m a_m(n)e^{-jn\omega}|^2} \quad (1)$$

when $b(0)$ and $a_m(n)$ are known from the data then power spectrum can be estimated by using

$$P_{AR}^{\wedge}(e^{j\omega}) = \frac{|b^{\wedge}(0)|^2}{|1 + \sum_{n=1}^m a_m^{\wedge}(n)e^{-jn\omega}|^2} \quad (2)$$

Depending upon the estimation of model parameters, accuracy of $P_{AR}^{\wedge}(e^{j\omega})$ will be determined and also data that is produced during the process of AR can be used to find whether it was consistent or not.

By solving autocorrelation equations, AR coefficients are calculated without performing the windowing of data[4]

$$\begin{bmatrix} r_x(0) & r_x^*(1) & r_x^*(2) & \dots & r_x^*(m) \\ r_x(1) & r_x(0) & r_x^*(1) & \dots & r_x^*(m-1) \\ r_x(2) & r_x(1) & r_x(0) & \dots & r_x^*(m-2) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ r_x(m) & r_x(m-1) & r_x(m-2) & \dots & r_x(0) \end{bmatrix} \begin{bmatrix} 1 \\ a_m(1) \\ a_m(2) \\ \vdots \\ a_m(m) \end{bmatrix} = \epsilon_m \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad (3)$$

where $r_x(n) = \frac{1}{N} \sum_{l=0}^{N-1-n} x(l+n)x^*(l)$; $n=0, 1, \dots, m$

By finding the coefficients of $a_m(n)$ through $|b(0)|^2 = \epsilon_q = r_y(0) + \sum_{k=1}^q a_m(n) r_y^*(n)$ and substituting these parameters in $P_{AR}^\wedge(e^{j\omega})$ power spectrum accuracy can be estimated.

AR parameters are also found by using covariance method. No windowing of data is required in covariance but the linear equations do not consist of constant diagonal matrix like the autocorrelation method. In this only forward prediction takes place. When the data length is short this method is used but spectral line splitting takes place [4]. In this set of linear equations are found by using

$$\begin{bmatrix} r_x(1,1) & r_x(2,1) & \cdots & r_x(m,1) \\ r_x(1,2) & r_x(2,2) & \cdots & r_x(m,2) \\ \vdots & \vdots & \cdots & \vdots \\ r_x(1,m) & r_x(2,m) & \cdots & r_x(m,m) \end{bmatrix} \begin{bmatrix} a_p(1) \\ a_p(2) \\ \vdots \\ a_p(m) \end{bmatrix} = - \begin{bmatrix} r_x(0,1) \\ r_x(0,2) \\ \vdots \\ r_x(0,m) \end{bmatrix} \quad (4)$$

where, $r_y(n, k) = \sum_{l=q}^{S-1} x(l-1)x^*(l-n)$.

Modified covariance predicts sum of squares of both forward and backward error, in this algorithm the data is less corrupted and gives an accurate result. In this algorithm, by solving linear equations AR parameters are found thus by

$$r_y(n, m) = \sum_{l=q}^{S-1} x(t-1)x^*(t-n) + x(t-q+m)x^*(t-q+n) \quad (5)$$

Input is taken in the form of sinusoidal waves that are present in white Gaussian noise, thus peaks are shifted from their original positions but they are less sensitive to phase when compared to AR estimation process. Finally power spectrum is obtained with high resolution.

3. SIMULATION AND RESULTS:

Step1: In the parametric method, estimation of signals can be done from known data. The data used for analysis is obtained [1]. The file name is Book_Seismic_Data.mat of a land line from east Texas.

Before applying modified covariance parametric algorithm, find out the tonals of the seismic signal, the algorithm's performance is evaluated with known synthetic signal. The synthetic signal can be generated by convolving Earth's wavelets with Earth's reflectivity and it can be displayed on seismogram. Assume Synthetic signal be Auto Regressive and find poles from the transfer function and then power spectral density is estimated.

Synthetic signal is generated by the source that is dynamite in 80-100 ft. depth holes. One trace consisting of

Regressive process with poles $0.98\exp\pm j\pi/5$ and $0.98\exp\pm j0.3\pi$. This means that the normalized frequencies are 0.2π and 0.3π . The poles are at $0.5760\pm j0.7928$ and $0.7928\pm j0.5760$. The signal generated is shown in Fig.1.

Step2: Power Spectral Density (PSD) of the synthetic signal with peaks at 0.3 and 0.2 normalized frequencies are shown in Fig.2. This means the modified covariance algorithm is working fine.

Step3: By extracting traces from the synthetic signal, raw seismic signal is generated and is shown in Fig.3.

Step4: The detrended seismic signal is obtained by eliminating synthetic signal from raw seismic signal and it is shown in Fig.4.

Step5: Modified covariance algorithm is applied on detrended seismic signal and the PSD obtained is shown Fig.5. The max peak is at 0.958 normalized frequency.

$$\begin{aligned} w &= \frac{2\pi f}{f_s} = 0.0958\pi \\ &= \frac{2\pi f}{500} \\ &= \frac{2\pi}{f_s} f = 0.0958\pi \end{aligned}$$

$$\begin{aligned} \text{Tonal Frequency } f &= \frac{500}{2} * 0.0958 \\ &= 250 * 0.0958 \\ &= 25 * 0.0958 \\ &= 23.950 \text{ Hz} \end{aligned}$$

Step6: The data is band pass filtered in the range [15Hz, 60Hz] from [1]. For ensuring purpose, a Band Pass Filter (BPF) [8] with Finite Impulse Response (FIR) order 8 is realized. The transfer function of the same is shown in Fig.6.

Step7: The detrended seismic signal is convolved with FIR BPF and the output is shown in Fig.7. The same PSD, as shown in Fig.6, is obtained. So the seismic signal tonal is 23.95Hz.

$$\begin{aligned} \text{Another insignificant tonal is } w_1 &= 0.1505\pi \\ \therefore f &= 250 * 0.1505 \\ &= 37.625 \text{ Hz.} \end{aligned}$$

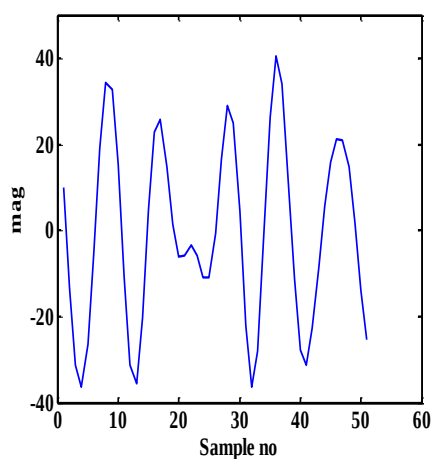


Fig.1 Synthetic Signal

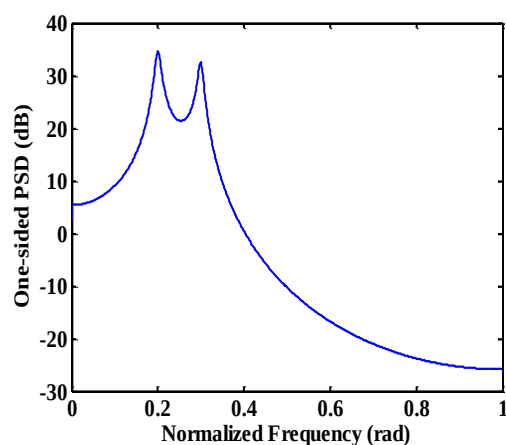


Fig.2 PSD using Modified Covariance

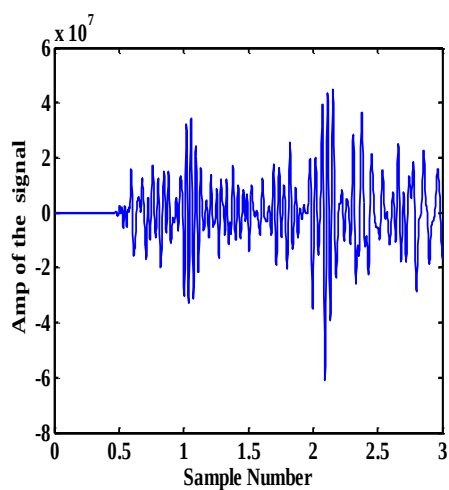


Fig.3 Raw Seismic Signal

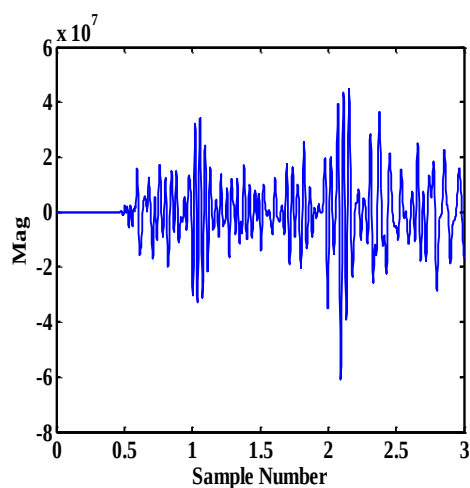


Fig.4 Detrended Raw Seismic signal

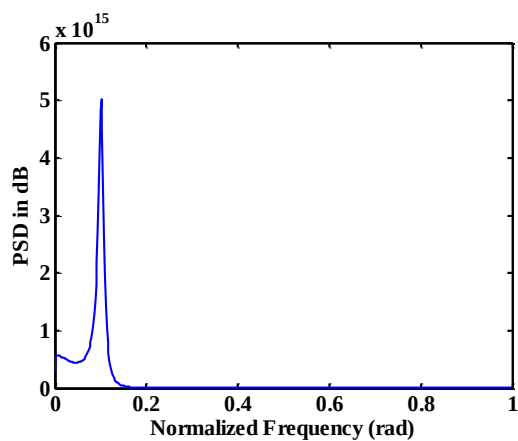


Fig.5 Raw Signal spectrum

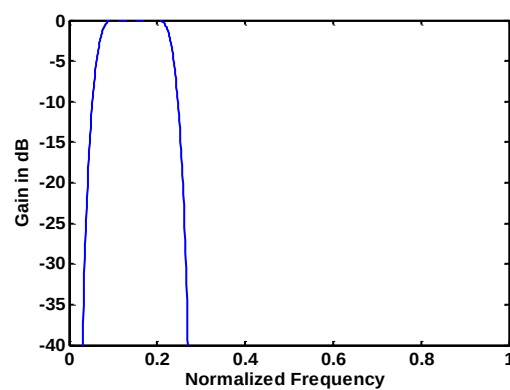


Fig.6 EID PDE

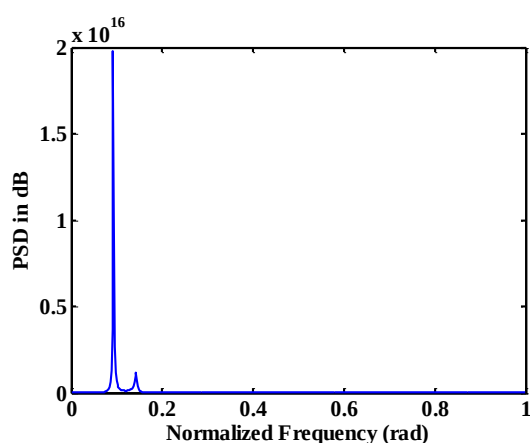


Fig.7 After BPF Signal Spectrum Analysis

4. CONCLUSION:

Spectral estimation technique offers good resolution; modified covariance overcomes and minimizes the sum of squares of the forward prediction errors and also the backward prediction errors without applying the windowing techniques to the data. These characteristics of signal make modified covariance method more efficient. SNR is improved by this method and reducing the noise using the FIR band pass filter and can obtain the desired seismic data.

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