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TWO NEW OPERATORS DEFINED ON NON NEGATIVE TRIANGULAR FUZZY NUMBER MATRICES

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Abstract. Fuzzy matrix is a very important topic of fuzzy algebra. Fuzzy matrices are successfully used when fuzzy uncertainty occurs in a problem. Determining the arithmetic operations of fuzzy numbers is a very important issue in fuzzy sets theory, decision process, data analysis, and applications. In 1985, Chen formulated the arithmetic operations between generalized fuzzy numbers by proposing the function principle. In this paper, we present two new binary fuzzy operators $\oplus$ and $\odot$ are introduced for non negative triangular fuzzy number matrices. Several properties on $\oplus$ and $\odot$ are presented here.

1. Introduction

In 1965, Zadeh [13] introduce the concept of fuzzy set theory. The fuzzyness can be represented by different ways. One of the most useful representation is membership function. Also depending the nature and shape of the membership function. A fuzzy number is a generalization of regular, real number in the sense that it does not refer to one single value but rather to a connected set of possible values. Where each possible value has its own weight between 0 and 1. This weight is called the membership function. A fuzzy number is thus a special case of a convex, normaliszed fuzzy set of the real line. Fuzzy numbers are an extension of real numbers. In 1985, Chen formulated the arithmetic operations between generalized fuzzy numbers by proposing the function principle [4]. The fuzzy number can be classified in different forms such as triangular fuzzy number (TFNs), trapezoidal fuzzy number, L-R type fuzzy number, etc., Fuzzy matrices play an important role in scientific development. Fuzzy matrices were introduced by M.G.Thomasan

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[12] A.K.Shymal and M.Pal introduced triangular fuzzy number matrices [1], [2]. Two new operators on fuzzy matrices are given in [1] α-cuts of triangular fuzzy numbers and α-cuts of triangular fuzzy number matrices [8]. Fuzzy numbers positive and negative are given in [5].

The paper is organized as follows: Section 2 introduces the preliminaries of Triangular fuzzy number and operations of triangular fuzzy number using function principle. Section 3 deals with some operations on triangular fuzzy number matrices. Section 4 based on some special types of triangular fuzzy number matrices. Section 5 discusses some properties of non negative triangular fuzzy number matrices over these new operators $\oplus$ and $\odot$ are presented.

2. Preliminaries

Definition 1. A fuzzy set $\tilde{A}$ on R must process atleast the following three properties to qualify as a fuzzy number,

(i) $\tilde{A}$ be a normal fuzzy set

(ii) ${}^{\alpha}\tilde{A}$ must be closed interval for every $\alpha \in [0,1]$

(iii) the support of $A,^{0+}\tilde{A}$ must be bounded.

Definition 2. Triangular fuzzy number is a fuzzy number represented with three points as follows $\tilde{A} = \langle a_1, a_2, a_3\rangle$ such that $(a_1 \leq a_2 \leq a_3)$ and defined on R with membership function defined as

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & \text{for } x < a_1 \\ \dfrac{x-a_1}{a_2-a_1} & \text{for } a_1 \leq x \leq a_2 \\ \dfrac{a_3-x}{a_3-a_2} & \text{for } a_2 \leq x \leq a_3 \\ 0 & \text{for } x > a_3. \end{cases}$$

Example 1. $\tilde{A} = \langle -2, 6, 7\rangle$ is a triangular fuzzy number.

Definition 3. A positive triangular fuzzy number $\tilde{A}$ is denoted as $\tilde{A} = \langle a_1, a_2, a_3\rangle$ where all a_i's > 0 for all $i = 1, 2, 3$.

Example 2. $\tilde{A} = \langle 2, 6, 7\rangle$ is a positive triangular fuzzy number.

Definition 4. A negative triangular fuzzy number $\tilde{A}$ is denoted as $\tilde{A} = \langle a_1, a_2, a_3\rangle$ where all a_i's < 0 for all $i = 1, 2, 3$.

Example 3. $\tilde{A} = \langle -2, -6, -7 \rangle$ is a negative triangular fuzzy number.

Definition 5. A non negative triangular fuzzy number $\tilde{A}$ is denoted as $\tilde{A} = \langle a_1, a_2, a_3 \rangle$ where all a_i's ≥ 0 for all $i = 1, 2, 3$.

Example 4. $\tilde{A} = \langle 0, 6, 7 \rangle$ is a nonnegative triangular fuzzy number.

2.1. **Arithmetic Operations of triangular fuzzy numbers using S.H.Chen** [4] **function principle.** Let $\tilde{A} = \langle a_1, a_2, a_3 \rangle$ and $\langle b_1, b_2, b_3 \rangle$ be triangular fuzzy numbers, then

(i) The addition of $\tilde{A} + \tilde{B} = \langle a_1 + b_1, a_2 + b_2, a_3 + b_3 \rangle$, where $a_1, a_2, a_3,\ b_1, b_2 and b_3$ are real numbers.
(ii) The product of $\tilde{A}$ and $\tilde{B}$ is $\tilde{A} \times \tilde{B} = \langle a_1 b_1, a_2 b_2, a_3 b_3 \rangle$, where a_1, a_2, a_3 $,b_1, b_2$ and b_3 are all non zero positive real numbers.
(iii) $-\tilde{B} = \langle -b_3, -b_2, -b_1 \rangle$ then the subtraction of $\tilde{B}$ from $\tilde{A}$ is $\tilde{A} - \tilde{B} = \langle a_1 - b_3, a_2 - b_2, a_3 - b_1 \rangle$ where a_1, a_2, a_3 $,b_1, b_2$ and b_3 are real numbers.

Definition 6. A triangular fuzzy number matrix of order $m \times n$ is defined as $\tilde{M} = (m_{ij})$, where $m_{ij} = \langle \alpha_{ij}, \beta_{ij}, \gamma_{ij} \rangle$.

3. Two new operations $\oplus$ and $\odot$ on triangular fuzzy number matrices and some operations on triangular fuzzy number matrices [1]

Let $\tilde{M} = (M_{ij})$ and $\tilde{N} = (N_{ij})$ be two fuzzy matrices of order $m \times n$ then

(i) $M \oplus N = [(m_{ij} + n_{ij}) - (m_{ij} \cdot n_{ij})]$
(ii) $M \odot N = [m_{ij} \cdot n_{ij}]$
(iii) $M' = [m_{ji}]$
(iv) $M \leq N$ if and only if $m_{ij} \leq n_{ij}$ for all i, j.

4. Based on [1] We define Some special types of triangular fuzzy number matrices

Let $\tilde{R} = (r_{ij})$ be an $n \times n$ triangular fuzzy number matrix. Then,

(i) R is reflexive if and only if $r_{ii} = 1$ for all $i = 1, 2, \ldots, n$
(ii) R is irreflexive if and only if $r_{ii} = 0$ for all $i = 1, 2, \ldots, n$
(iii) R is nearly irreflexive if and only if $r_{ii} \leq r_{ij}$ for all $i = 1, 2, \ldots, n$.

(iv) R is symmetric if and only if R' $= R$
(v) R is constant if and only if $r_{ij} = r_{jk}$ for all $i, j, k = 1, 2, \ldots, n$
(vi) R is identity if and only if $r_{ii} = 1$ and $r_{ij} = 0$ $(i \neq j)$ for all i, j
The identity matrix of order $n \times n$ is generally denoted by I_n
(vii) R is weakly reflexive if $r_{ii} \geq r_{ij}$ for all i, j
(viii) R is diagonal if $r_{ii} \geq 0, r_{ij} = 0$ $(i \neq j)$ for all i, j

5. Properties based on two new operators $\oplus$ and $\odot$ and special types of non negative triangular fuzzy number matrices

Proposition 7. *Let M and N be two non negative triangular fuzzy number matrices, then*

(i) $M \oplus N \leq M \odot N$
(ii) If M and N are symmetric ,then $M \oplus N$ and $M \odot N$ are symmetric.
(iii) If M and N are nearly irreflexive, then $M \odot N$ is nearly irreflexive.

Proof. (i) The ij th element of $M \oplus N$ is $[(m_{ij} + n_{ij}) - (m_{ij} \cdot n_{ij})]$ and $M \odot N$ is $m_{ij} \cdot n_{ij}$
$m_{ij} (\langle 1,1,1 \rangle - n_{ij}) + n_{ij} (\langle 1,1,1 \rangle - m_{ij}) \leq 0$ and $m_{ij} \cdot n_{ij} \geq 0$
Hence $M \oplus N \leq M \odot N$

(ii) Let $M = [m_{ij}]$ and $N = [n_{ij}]$ be two symmetric triangular fuzzy number matrices
Therefore $m_{ij} = m_{ji}$ and $n_{ij} = n_{ji}$ Let c_{ij} be the ij th element of $M \oplus N$
$c_{ij} = (m_{ij} + n_{ij}) - (m_{ij} \cdot n_{ij}) = (m_{ji} + n_{ji}) - (m_{ji} \cdot n_{ji}) = c_{ji}$. Hence $M \oplus N$ is symmetric.
Let d_{ij} be the ij th element of $M \odot N$
$d_{ij} = m_{ij} \cdot n_{ij} = m_{ji} \cdot n_{ji} = d_{ji}$. Hence $M \odot N$ is symmetric.

(iii) Since M and N are nearly irreflexive $m_{ii} \leq m_{ij}$ and $n_{ii} \leq n_{ij}$ for all i, j
Let d_{ij} be the ij th element of $M \odot N$
$d_{ij} - d_{ii} = m_{ij} \cdot n_{ij} \geq m_{ii} \cdot n_{ii} \geq 0 \Rightarrow d_{ii} \leq d_{ij}$. Hence $M \odot N$is nearly irreflexive. □

Example 5. $M = \begin{bmatrix} \langle 1,2,3\rangle & \langle 2,3,4\rangle & \langle 4,5,6\rangle \\ \langle 3,5,7\rangle & \langle 2,4,6\rangle & \langle 1,3,5\rangle \\ \langle 5,6,7\rangle & \langle 3,5,7\rangle & \langle 1,2,4\rangle \end{bmatrix}$ and
$N = \begin{bmatrix} \langle 1,2,3\rangle & \langle 2,5,6\rangle & \langle 2,3,4\rangle \\ \langle 3,4,5\rangle & \langle 4,6,8\rangle & \langle 1,3,5\rangle \\ \langle 3,4,5\rangle & \langle 5,6,7\rangle & \langle 6,7,8\rangle \end{bmatrix}$

$$M \oplus N \leq M \odot N \ .$$

Example 6. $M = \begin{bmatrix} \langle 1,1.5,2\rangle & \langle 2.5,3,3.5\rangle & \langle 3.5,4,4.5\rangle \\ \langle 2.5.3.3.5\rangle & \langle 2,2.5,3\rangle & \langle 3,3.5,4\rangle \\ \langle 3.5,4,4.5\rangle & \langle 3,3.5,4\rangle & \langle 1,1.5,2\rangle \end{bmatrix}$ and
$N = \begin{bmatrix} \langle 3,3.5,5\rangle & \langle 1,1.5,2.5\rangle & \langle 3,4,5\rangle \\ \langle 1,1.5,2.5\rangle & \langle 4,6,8\rangle & \langle 5,6,6.5\rangle \\ \langle 3,4,5\rangle & \langle 5,6,6.5\rangle & \langle 6,7,8\rangle \end{bmatrix}$
are symmetric then $M \oplus N$ and $M \odot N$ are symmetric.

Example 7. $M = \begin{bmatrix} \langle 1,2,3\rangle & \langle 2,4,6\rangle & \langle 3,4,5\rangle \\ \langle 4,5,7\rangle & \langle 3,4,6\rangle & \langle 6,7,8\rangle \\ \langle 4,5,9\rangle & \langle 5,6,8\rangle & \langle 3,4,7\rangle \end{bmatrix}$ and
$N = \begin{bmatrix} \langle 1,2,4\rangle & \langle 2,3,5\rangle & \langle 3,4,6\rangle \\ \langle 4,5,8\rangle & \langle 5,4,7\rangle & \langle 4,6,8\rangle \\ \langle 3,5,6\rangle & \langle 4,6,7\rangle & \langle 2,4,5\rangle \end{bmatrix}$ are nearly irrreflexive then
$M \odot N = \begin{bmatrix} \langle 1,4,12\rangle & \langle 4,12,30\rangle & \langle 9,16,30\rangle \\ \langle 16,25,56\rangle & \langle 15,16,42\rangle & \langle 24,42,64\rangle \\ \langle 12,25,54\rangle & \langle 2036,58\rangle & \langle 6,16,35\rangle \end{bmatrix}$ is nearly irreflexive.

Proposition 8. *For any non negative triangular fuzzy number matrix $M, M \odot M \geq M$.*

Proof. The ijth element of m_{ij}^2 of $M \odot M$ is greater than m_{ij}. Therefore $M \odot M \geq M$. □

Example 8. $M = \begin{bmatrix} \langle 1,2,3\rangle & \langle 2,3,6\rangle & \langle 0,1,2\rangle \\ \langle 3,4,5\rangle & \langle 5,6,7\rangle & \langle 6,7,8\rangle \\ \langle 2,4,6\rangle & \langle 1,3,5\rangle & \langle 1,4,7\rangle \end{bmatrix}$
$M \odot M = \begin{bmatrix} \langle 1,4,9\rangle & \langle 4,9,36\rangle & \langle 0,1,4\rangle \\ \langle 9,16,25\rangle & \langle 25,36,49\rangle & \langle 36,49,64\rangle \\ \langle 4,16,36\rangle & \langle 1,9,25\rangle & \langle 1,16,49\rangle \end{bmatrix}$.
Therfore $M \odot M \geq M$.

Proposition 9. *Let M, N, P be any three non negative triangular fuzzy number matrices, then*

(i) $M \oplus N = N \oplus M$
(ii) $M \odot N = N \odot M$
(iii) $(M \odot N) \odot P = M \odot (N \odot P)$

Proof. (i) $M \oplus N = [(m_{ij} + n_{ij}) - (m_{ij} \cdot n_{ij})] = [(n_{ij} + m_{ij}) - (n_{ij} \cdot m_{ij})] = N \oplus M$.
(ii) $M \odot N = m_{ij} \cdot n_{ij} = n_{ij} \cdot m_{ij} = N \odot M$
(iii) $(M \odot N) \odot P = (m_{ij} \cdot n_{ij})\, p_{ij} = m_{ij} \cdot n_{ij} \cdot p_{ij} = m_{ij} \cdot (n_{ij} \cdot p_{ij}) = M \cdot (N \cdot P)$
The operator $\oplus$ do not obey the rule $(M \oplus N) \oplus P = M \oplus (N \oplus P)$. □

Example 9. $M = \begin{bmatrix} \langle 1,2,5 \rangle & \langle 0,1,2 \rangle & \langle 2,3,4 \rangle \\ \langle 1,2,3 \rangle & \langle 2,3,4 \rangle & \langle 3,4,5 \rangle \\ \langle 3,4,5 \rangle & \langle 2,3,6 \rangle & \langle 0,1,3 \rangle \end{bmatrix}$
$N = \begin{bmatrix} \langle 1,2,4 \rangle & \langle 2,3,4 \rangle & \langle 0,1,4 \rangle \\ \langle 1,4,5 \rangle & \langle 1,3,5 \rangle & \langle 2,4,6 \rangle \\ \langle 1,2,5 \rangle & \langle 1,3,4 \rangle & \langle 1,2,4 \rangle \end{bmatrix}$ and $P = \begin{bmatrix} \langle 1,2,4 \rangle & \langle 1,3,5 \rangle & \langle 2,3,6 \rangle \\ \langle 1,3,6 \rangle & \langle 2,3,5 \rangle & \langle 3,5,7 \rangle \\ \langle 1,2,3 \rangle & \langle 2,3,4 \rangle & \langle 0,1,3 \rangle \end{bmatrix}$
(i) $M \oplus N = N \oplus M$ (ii) $M \odot N = N \odot M$ (iii) $(M \odot N) \odot P = M \odot (N \odot P)$

Proposition 10. *Let M, N be any two non negative triangular fuzzy number matrices, then*

(i) $(M \odot N)' = M' \odot N'$
(ii) $(M \oplus N)' = M' \oplus N'$

Proof. (i) Let c_{ij} and d_{ij} be the ijth element of $M \odot N$ and M' N' respectively.
$e_{ij} = c_{ji}$ is the ijth element of (M N)$'$. Then $c_{ij} = m_{ij}.n_{ij}$. Thus $c_{ij} = m_{ji}.n_{ji}$. and $d_{ij} = m_{ji}.n_{ji}$. Therefore $d_{ij} = e_{ij}$ for all i,j. Hence (M N)$'$ = M $'$ N$'$.

(ii) Let c_{ij} and d_{ij} be the ijth element of M $\oplus$ N and M $'\oplus$ N$'$ respectively.
Therefore $e_{ij} = c_{ji}$ is the ijth element of (M $\oplus$ N)$'$.
Then $c_{ij} = (m_{ij} + n_{ij}) - (m_{ij}.n_{ij})$ and $d_{ij} = (m_{ji} + n_{ji}) - (m_{ji}.n_{ji})$; $e_{ij} = (m_{ji} + n_{ji}) - (m_{ji}.n_{ji}) = d_{ij}$. Hence (M $\oplus$ N)$'$ = M $'\oplus$ N'. □

Example 10. $M = \begin{bmatrix} \langle 1,2,3\rangle & \langle 0,1,2\rangle & \langle 2,3,4\rangle \\ \langle 5,6,7\rangle & \langle 6,7,9\rangle & \langle 3,4,5\rangle \\ \langle 1,2,4\rangle & \langle 2,6,7\rangle & \langle 3,5,7\rangle \end{bmatrix}$

$N = \begin{bmatrix} \langle 2,3,5\rangle & \langle 0,1,3\rangle & \langle 2,4,6\rangle \\ \langle 1,2,3\rangle & \langle 3,6,7\rangle & \langle 1,34\rangle \\ \langle 2,4,6\rangle & \langle 3,4,5\rangle & \langle 1,2,4\rangle \end{bmatrix}$

(i) $(M \odot N)' = M' \odot N'$ (ii) $(M \oplus N)' = M' \oplus N'$.

6. Conclusion

In this article two new operators on triangular fuzzy number matrices are defined. Using these operations some properties of non negative triangular fuzzy number matrices are presented.

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