

Convective conditions on Sakiadis flow of magnetohydrodynamic Casson fluid with heat sink

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Abstract

Theoretical analysis is performed for studying the flow of heat transfer characteristics of Sakiadis flow of magnetohydrodynamic Casson fluid with convective conditions and heat sink. Numerical solutions are carried out by using Runge-Kutta based Shooting technique. The effects of various governing parameters on the flow quantities are demonstrated graphically. Also computed the friction factor coefficient and local Nusselt numbers for the Sakiadis with Newtonian and non-Newtonian fluid cases. It is found that the rate of heat transfer is very high in sakiadis non-Newtonian fluid case when compared with Sakiadis Newtonian fluid case.

AMS Classification: 76A05, 80A20, 76W05, 65L05.

Key Words and Phrases: MHD, Sakiadis flow, Casson fluid, Convective condition, heat source/sink and Porosity parameter.

1 Introduction

It is well experienced that the most of the fluids are not satisfies the common assumption of a linear relationship between the rate of strain and shear stress, therefore these types of fluids called as non-Newtonian fluids. The fluid which holds the Newton's law of viscosity is called Newtonian fluid. The Newton's law of viscosity is where the kinematic viscosity of the fluid is and the shear stress is . All the fluids are not following the stress-strain relation. Those are not obeys the Newton law of viscosity is known as non-Newtonian fluids. The kinematic viscosity of the non-Newtonian fluids is a function of strain rate. These types of fluid flows have presented exciting the challenges the mathematicians, numerical engineering simulators and physicists. The non-Newtonian fluids equations are more difficult than the second order Navier- Stoke's equations. Such types of fluids have been modelled by constitutive equations. The resultant governing non-linear equations are not possible to solve analytically, due to this reason recently many researchers are trying to solving the non-Newtonian fluid models by using numerical techniques. It also has various science and engineering, industrial applications such as thermal insulation of buildings, cooling of electronic devices, food processing system, enhanced oil recovery system, drying of solid systems, geophysical systems and geothermal energy systems.

The laminar boundary layer flow driven by a continuously moving solid surface has received considerable attention since the pioneering investigations by [1, 2] partly because of its practical relevance in various extrusion processes but also because of its role as a canonical flow problem in the boundary layer theory of Newtonian and non-Newtonian fluid mechanics. By keeping this into view Further, [3, 4] analysed a viscoelastic fluid flow through a stretching sheet. Later on, [5] elucidated non- Newtonian fluid flows with various effects and flow geometries. Free convective heat transfer of MHD Cu-kerosene nanofluid over a cone with temperature dependent viscosity studied by [6]. [7] elaborated the boundary layer stagnation point flow of a Casson fluid and heat transfer over a stretching or shrinking surface. The radiative Casson fluid over a wedge with microorganisms was studied by [8].

Motivated by the above mentioned studies and challenges, but

no studies have been described yet up to the author's knowledge on Sakiadis flow of magnetohydrodynamic Casson fluid with convective conditions and heat generation/absorption. The transformed ordinary differential equations are solved numerically. Comparisons of some exceptional cases are made with the previous results and found a better agreement.

2 Mathematical analysis

We consider the steady two-dimensional boundary layer flow of Sakiadis flow filled with Casson fluid properties. The heat sink also considered into account. The flow is considered in x -direction and y -axis is normal to it. It is assume that the flow takes place at $y \geq 0$. It is also assumed that the constant temperature of the sheet is T_w and that of the ambient fluid is T_∞ . Under the boundary layer approximations, the fact is that the flow is of zero pressure gradients. Based on the assumptions the basic equations written in the Cartesian coordinates x and y as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = v \left(1 + \frac{1}{\beta} \right) \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} u - \frac{v}{k_0} u + g \beta_T (T - T_\infty), \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} - Q_0 (T - T_\infty), \quad (3)$$

We assume that these equations are subject to the boundary conditions,

$$\text{Sakiadis problem} \quad \left. \begin{aligned} v &= 0, \quad u = U, \quad k \frac{\partial T}{\partial y} = -h(T_w - T_\infty) \quad \text{at } y = 0, \\ u &= 0, \quad T = T_\infty \quad \text{as } y \rightarrow \infty, \end{aligned} \right\} \quad (4)$$

Where u and v the velocity components along the x and y -directions, β is the Casson fluid parameter, σ is the electric conductivity, k_0 is the permeability parameter, g is the acceleration due to gravity, β_T is the thermal expansion coefficient, α is thermal diffusivity and Q_0 is the heat generation coefficient. U is the constant velocity of the free stream (inviscid flow) or that of a moving flat sheet.

We look for a similarity solution of Eqs. (1)-(3) with the boundary conditions (4), (5) and (6) of the following form:

$$\psi = (U\nu x)^{1/2} f(\zeta), \quad \theta(\zeta) = (T - T_\infty)/(T_w - T_\infty), \quad \zeta = (U/\nu x)^{1/2} y, \quad (5)$$

where ν is the kinematic viscosity of the fluid fraction and ψ is the stream function that is defined in the usual way as $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$. On substituting Eqs. (6) into (2) and (3), we obtain the following uncoupled ordinary differential equations:

$$\left(1 + \frac{1}{\beta}\right) f''' + \frac{1}{2} f f'' - (M + \delta) f' + \lambda \theta = 0, \quad (6)$$

$$\frac{1}{\text{Pr}} \theta'' + \frac{1}{2} f \theta' - Q \theta = 0, \quad (7)$$

Subject to the boundary conditions

$$f(0) = 0, \quad f'(0) = 1, \quad \theta'(0) = -Bi(1 - \theta(0)), \quad f'(\infty) = 0, \quad \theta(\infty) = 0, \quad (8)$$

Where a prime denotes the differentiation with respect to ζ the similarity variable and $\text{Pr} = \nu/\alpha$ is the Prandtl number, $Q = Q_0/c$ is heat generation parameter, $M = \sigma B_0^2/\rho$ is the magnetic field parameter, $\delta = v/k_0 U$ is porosity parameter and $\lambda = g\beta_T(T_w - T_\infty)/U^2 x$ is the buoyancy parameter. The physical quantities of engineering interest are Skin-friction coefficient (C_f) and Nusselt number (Nu_x), which are given by

$$C_f = \frac{\tau_w}{\rho_f U^2}, \quad Nu = \frac{x q_w}{k_f (T_w - T_\infty)}, \quad (9)$$

where τ_w is the skin friction or the shear stress and q_w is the heat flux from the plate which are given by

$$\tau_w = \mu \left(\frac{\partial u}{\partial y} \right)_{y=0}, \quad q_w = -k \left(\frac{\partial T}{\partial y} \right)_{y=0}, \quad (10)$$

Substituting Eq. (6) into (13), we obtain

$$\text{Re}_x^{1/2} C_f = \left(1 + \frac{1}{\beta}\right) f''(0), \quad \text{Re}_x^{-1/2} Nu = -\theta'(0), \quad (11)$$

where $Re_x = \frac{Ux}{\nu_f}$ is the local Reynolds number.

3 Result and discussion

The non-dimensional governing equations (7) and (8) subjected to boundary conditions (9) and (10) are solved numerically using shooting technique. The numerical solutions are found for various values of non-dimensional governing parameters on the flow and heat transfer characteristics of Blasius and Sakiadis Casson fluid with heat generation. For numerical calculations we considered the non-dimensional values as $\delta = 0.5, Q = 0.2, Pr = 0.7, \lambda = 3, M = 0.5, Bi = 0.2$. These values are preserved as common in the entire examination except the different values are displayed in specific figures. In this analysis, solid lines indicate the Newtonian Sakiadis flow and dashed lines indicates the non-Newtonian Sakiadis flow.

Figs. 1 and 2 displayed to show that the influence of porosity parameter on velocity and temperature fields. It is evident that the velocity field decreases rapidly near the boundary with increase in porosity parameter and reverse trend can be observed in temperature fields as shown in Fig 2. Fig. 3 depicts that influence of Biot number on the velocity field. It is observed that the velocity field increases rapidly near the boundary with increase in Biot number. In Fig. 4, we present the behavior of Biot number Bi on the thermal boundary layer Biot number is defined as the ratio of convection heat transfer to conduction heat transfer. Biot number larger temperature at the plate which creates thicker thermal boundary layer. The non-dimensional velocity and temperature for different values of magnetic field for Newtonian and non-Newtonian fluids are shown in Figs. 5 and 6. It is observed that the velocity declines with rise in magnetic field, but reverse trend has been observed with increase in the magnetic field for temperature fields, the magnetic field opposes the rate of transportation. Generally, an increase in the magnetic field results a strong reduction in the dimension less velocity. This due to the fact that the temperature field introduces a retardant force is called Lorentz's force. This happens because of the interaction of the magnetic field and electric fields to the transport phenomena. As the Lorentz's force is a

resistive force which opposes the fluid motion, so heat is produced and result the thermal boundary layer thickness hence thicker than magnetic field Figs. 7 and 8 illustrates that the effect of thermal buoyancy parameter on dimensionless velocity and temperature. It is noticed that the temperature reduces as increasing in δ . It is very well agreed with the fact that buoyancy force increases the fluid velocity which in turn causes the temperature of the fluid to decrease. The heat source parameter on velocity and temperature profiles from Figs. 9 and 10, we can observed that as increase in heat source parameter decrease in velocity and temperature fields. From Figs. 11 and 12 we observed that the variation of rate of heat transfer in terms of local Nusselt number decelerates with increasing the values of governing parameters heat source parameter and Biot number over a magnetic field parameter for both Sakiadis flow with Newtonian and non-Newtonian fluid.

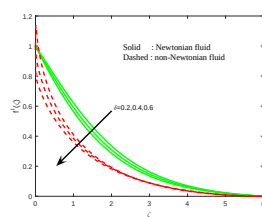


Figure 1: The effect of δ on velocity profiles

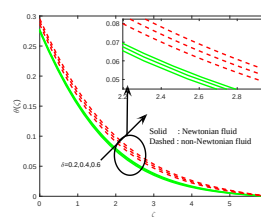


Figure 2: The effect of δ on temperature profiles

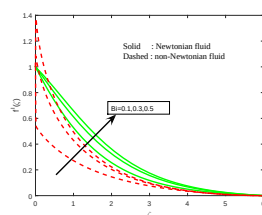


Figure 3: The effect of Bi on velocity profiles

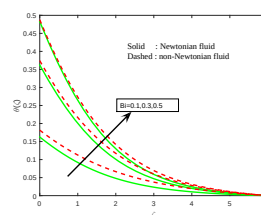


Figure 4: The effect of Bi on θ

Conclusion: The conclusion of this study are as follows. 1.The rising values of Biot number improve the temperature as well as velocity profiles. 2.The non-Newtonian fluid has higher heat transfer

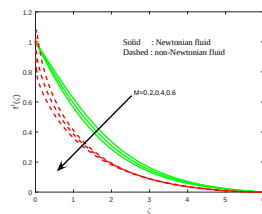


Figure 5: The effect of M on $f'(\zeta)$

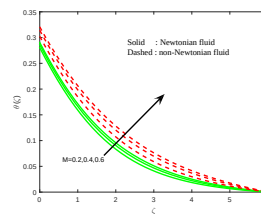


Figure 6: The effect of M on θ

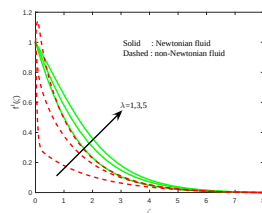


Figure 7: The effect of λ on velocity profiles

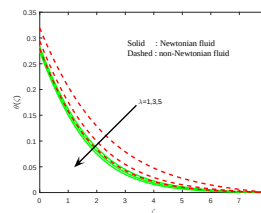


Figure 8: The effect of λ on temperature profiles

rate as compared with Newtonian fluid case. 3. The $f'(\zeta)$ profiles higher in Newtonian fluid case as compared with non-Newtonian fluid case.

References

- [1] Sakiadis, Boundary-layer behavior on continuous solid surfaces: I. Boundary-layer equations for two-dimensional and axisymmetric flow *AIChE Journal*, **7** (1961), 26-28.
- [2] Sakiadis, Boundary-layer behavior on continuous solid surfaces: II. The boundary layer on a continuous flat surface, *AIChE Journal* **7**(1961), 221-225.
- [3] K. R. Rajagopal, A. S. Guptha, An exact solution for the flow of a non-Newtonian fluid past an infinite porous plate, *Mechanica* **19**(1984), 158-160.
- [4] K. R. Rajagopal, T. N. Na, A. S. Guptha, Flow of a viscoelastic fluid over a stretching sheet, *Rheol Acta*, **23** (1984), 213-215.

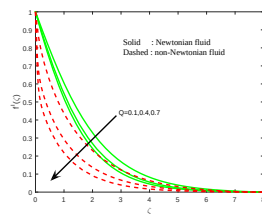


Figure 9: The effect of Q on velocity profiles

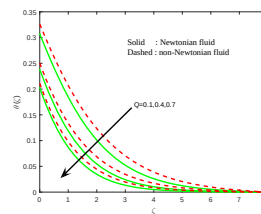


Figure 10: The effect of Q on temperature profiles

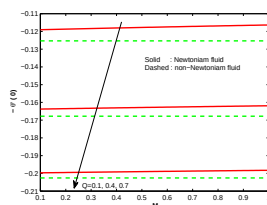


Figure 11: The effect of Q and M on Local Nusselt number

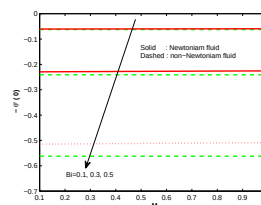


Figure 12: The effect of Bi and M on Local Nusselt number

- [5] Hiral Parmar and M. G. Timol, Deductive group technique for MHD coupled heat and mass transfer natural convective flow of non-Newtonian power law fluid over a vertical cone through porous medium, *International Journal of Applied Mathematics and Mechanics*, **7**(2011), 35-50.
- [6] C.S.K. Raju, N. Sandeep, A. Malvandi, Free convective heat transfer of MHD Cu-kerosene nanofluid over a cone with temperature dependent viscosity, *Acta Astronautica*, **129**(2016), 419-428.
- [7] K. Bhattacharyya, Boundary layer stagnation point flow of Casson fluid and heat transfer towards a shrinking/stretching sheet, *Frontiers in Heat and Mass Transfer*, **04**(2013) 1-9.
- [8] C. S. K. Raju, M. M. Hoque, T. Sivasankar, Radiative flow of Casson fluid over a moving wedge filled with gyrotactic microorganisms, *Advanced Powder Tech.*, **4** (2013), 485-491.

