DENSITY MATRIX APPROACH: 
ROLE OF GROUND STATE 
POPULATION DISTRIBUTION IN A 
THREE-LEVEL ATOMIC SYSTEM

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Abstract

In this work we have considered the three- level atomic system and solved the optical Bloch equations for the steady state approximation. Density matrix formalism is adopted to write the equations for various schemes. Electromagnetically induced transparency and the role of ground state population distribution have been studied for different Rabi frequencies. 

AMS Subject Classification: 81V80, 81V45 
Key Words and Phrases: density matrix, optical Bloch equations, three level system 

1 Introduction

In quantum optics, electromagnetically induced transparency is a fascinating discovery demonstrated by Harris and his co-workers in 1990 [1]. The phenomena of electromagnetically induced transparency (EIT), coherent population trapping (CPT) and non-linear magneto optical rotation (NMOR) in alkali atoms have been explained with the understanding of different atomic level schemes and the relaxation mechanisms. Density matrix formalism has been
adopted to explain these effects. The physical interpretation of various elements in the density matrix description allows the inclusion of various processes directly in the equation of motion \[2,3,4\]. The equation of motion of the density matrix can be obtained from the Schrödinger equation. In the present work, we have considered different atomic sublevels and solved the density matrix equations appropriately. In particular we have studied the absorption and dispersion effects by varying the probe detuning frequency. Ground state population distribution has also been studied for different Rabi frequencies.

2 Theoretical Formalism

2.1 Optical Bloch Equations

In quantum mechanics, wave function describes the complete system. Atomic ensembles can be more appropriately described in terms of a Liouville’s equation \[5\],

\[
\frac{d\rho}{dt} = \frac{i}{\hbar} [\rho, H] - \Gamma \rho
\] (1)

In a three level system the Hamiltonian can be written as

\[
\begin{pmatrix}
\hbar \omega_1 & \hbar \Omega \frac{1}{2} e^{-i\omega_1 t} & \hbar \Omega \frac{2}{2} e^{-i\omega_2 t} \\
\hbar \Omega \frac{1}{2} e^{i\omega_1 t} & \hbar \omega_2 & 0 \\
\hbar \Omega \frac{2}{2} e^{i\omega_2 t} & 0 & \hbar \omega_3
\end{pmatrix}
\] (2)

The ijth matrix element of equation (1) is:

\[
\frac{d\rho_{ij}}{dt} = \frac{i}{\hbar} \sum_k (\rho_{ik} H_{kj} - H_{ik} \rho_{kj}) - (\Gamma \rho)_{ij}
\] (3)

where \( \Gamma_{ij} = \frac{\Gamma_i + \Gamma_j}{2} \)

The following equations correspond to the coherences in the
The following equations represent the population distribution of a three-level system.

\[
\begin{align*}
\dot{\rho}_{12} &= -(i(\omega_1 - \omega_2) + \Gamma_{12})\rho_{12} + \frac{i\Omega_1}{2} e^{-i\omega_1 t}(\rho_{11} - \rho_{22}) - \frac{i\Omega_2}{2} e^{-i\omega_2 t}\rho_{32} \\
\dot{\rho}_{21} &= (i(\omega_1 - \omega_2) - \Gamma_{21})\rho_{21} - \frac{i\Omega_1}{2} e^{i\omega_1 t}(\rho_{11} - \rho_{22}) + \frac{i\Omega_2}{2} e^{i\omega_2 t}\rho_{23} \\
\dot{\rho}_{13} &= -(i(\omega_1 - \omega_3) + \Gamma_{13})\rho_{13} + \frac{i\Omega_2}{2} e^{-i\omega_2 t}(\rho_{11} - \rho_{33}) - \frac{i\Omega_1}{2} e^{-i\omega_1 t}\rho_{23} \\
\dot{\rho}_{31} &= (i(\omega_1 - \omega_3) - \Gamma_{31})\rho_{31} - \frac{i\Omega_2}{2} e^{i\omega_2 t}(\rho_{11} - \rho_{33}) + \frac{i\Omega_1}{2} e^{i\omega_1 t}\rho_{32} \\
\dot{\rho}_{32} &= -(i(\omega_3 - \omega_2) + \Gamma_{32})\rho_{32} + \frac{i\Omega_1}{2} e^{-i\omega_1 t}\rho_{31} - \frac{i\Omega_2}{2} e^{-i\omega_2 t}\rho_{12} \\
\dot{\rho}_{23} &= (i(\omega_3 - \omega_2) - \Gamma_{23})\rho_{23} - \frac{i\Omega_1}{2} e^{i\omega_1 t}\rho_{13} + \frac{i\Omega_2}{2} e^{i\omega_2 t}\rho_{21}
\end{align*}
\]
three-level system.

\[
\rho_{11} = \frac{i\Omega_1}{2} (\rho_{12} e^{i\omega_1 t} - \rho_{21} e^{-i\omega_1 t}) + \frac{i\Omega_2}{2} (\rho_{13} e^{i\omega_2 t} - \rho_{31} e^{-i\omega_2 t}) - \Gamma_1 \rho_{11}
\]

\[
\rho_{22} = \frac{i\Omega_1}{2} (\rho_{21} e^{-i\omega_1 t} - \rho_{12} e^{i\omega_1 t}) - \Gamma_2 \rho_{22} + \frac{\Gamma_1}{2} \rho_{11} + \frac{\Gamma_2}{2}
\]

\[
\rho_{33} = \frac{i\Omega_2}{2} (\rho_{31} e^{-i\omega_2 t} - \rho_{13} e^{i\omega_2 t}) - \Gamma_3 \rho_{33} + \frac{\Gamma_1}{2} \rho_{11} + \frac{\Gamma_2}{2}
\]

Spontaneous emission and optical pumping terms are included in the above equations. Various \(\Gamma\)'s correspond to the decay terms associated with spontaneous emission and optical pumping terms. Slow variables are introduced to eliminate the fast oscillating terms, after the introduction of slow variables

\[
\dot{\rho}_{11} = \frac{i\Omega_1}{2} (\tilde{\rho}_{12} - \tilde{\rho}_{21}) + \frac{i\Omega_2}{2} (\tilde{\rho}_{13} - \tilde{\rho}_{31}) - \Gamma_1 \rho_{11}
\]

\[
\dot{\rho}_{22} = \frac{i\Omega_1}{2} (\tilde{\rho}_{21} - \tilde{\rho}_{12}) - \Gamma_2 \rho_{22} + \frac{\Gamma_1}{2} \rho_{11} + \frac{\Gamma_2}{2}
\]

\[
\dot{\rho}_{33} = \frac{i\Omega_2}{2} (\tilde{\rho}_{31} - \tilde{\rho}_{13}) - \Gamma_3 \rho_{33} + \frac{\Gamma_1}{2} \rho_{11} + \frac{\Gamma_2}{2}
\]

\[
\dot{\rho}_{12} = -(i(\omega_1 - \omega_2 - \omega_a) + \Gamma_{12}) \rho_{12} + \frac{i\Omega_1}{2} (\rho_{11} - \rho_{22}) - \frac{i\Omega_2}{2} \tilde{\rho}_{32}
\]

\[
\dot{\rho}_{21} = (i(\omega_1 - \omega_2 - \omega_a) - \Gamma_{21}) \rho_{21} - \frac{i\Omega_1}{2} (\rho_{11} - \rho_{22}) + \frac{i\Omega_2}{2} \tilde{\rho}_{32}
\]

\[
\dot{\rho}_{13} = -(i(\omega_1 - \omega_3 - \omega_b) + \Gamma_{13}) \rho_{13} + \frac{i\Omega_2}{2} (\rho_{11} - \rho_{33}) - \frac{i\Omega_1}{2} \tilde{\rho}_{32}
\]

\[
\dot{\rho}_{31} = (i(\omega_1 - \omega_3 - \omega_b) - \Gamma_{31}) \rho_{31} - \frac{i\Omega_2}{2} (\rho_{11} - \rho_{33}) + \frac{i\Omega_1}{2} \tilde{\rho}_{32}
\]

\[
\dot{\rho}_{32} = -(i(\omega_3 - \omega_2 - \omega_a + \omega_b) + \Gamma_{32}) \rho_{32} + \frac{i\Omega_1}{2} \tilde{\rho}_{31} - \frac{i\Omega_2}{2} \tilde{\rho}_{12}
\]

\[
\dot{\rho}_{23} = (i(\omega_3 - \omega_2 - \omega_a + \omega_b) - \Gamma_{23}) \rho_{23} - \frac{i\Omega_1}{2} \tilde{\rho}_{31} + \frac{i\Omega_2}{2} \tilde{\rho}_{12}
\]

Detuning can be written as:

\[
\Delta_1 + \Delta_m = \omega_1 - \omega_2; \Delta_2 - \Delta_m = \omega_b - \omega_1 + \omega_3
\]

(4)

\(\Delta_m\) is the magnetic field detuning (related to Zeeman effect) corresponding to the applied magnetic field. \(\Omega\) is the Rabi frequency, the susceptibility of the atomic medium can be written as:

\[
\chi = \frac{-2N\mu^2}{\hbar\varepsilon_0 \Omega_1} \rho_{12}
\]

(5)
The real part of the above equation corresponds to dispersion and imaginary part of the above equation represents the absorption.

3 Results and Discussion

3.1 Three-level closed system

The nine coupled differential equations are numerically solved for a three-level closed system. Here we assume that the total probability for all the population distribution in this three-level system is equal to one. We also assume that the populations do not decay into the neighboring levels (other than the three-levels considered for this calculation) These equations are solved for the steady state conditions. Rabi frequency can be written as $\Omega_R = \Gamma \sqrt{\frac{I}{I_s}}$ where $I$ is the intensity of the laser beam and $I_s$ is the saturation intensity. The
diameter of the laser beam is taken as 1 mm, which can be used to interpret the Rabi frequency with the appropriate laser fields. The computational results for the absorption/dispersion with respect to the probe beam detuning is shown in figure 2. Close to zero probe detuning we observed a reduction in population, which is possibly due to the occurrence of electromagnetically induced transparency.

3.2 Three- level open system

Three-level open system takes into account the population decay of individual levels with respect to the neighboring atomic levels (which are not included in the three-level scheme). Here the absorption/dispersion curve is shown in figure 3. The line shape varies in comparison with figure 2 and the line width of the EIT window broadens here.
3.3 Role of ground state population distribution

In a three-level system it is convenient to assume both of the ground states are equally populated. In the real scenario the ground states may have slightly different population distribution. Here the probe detuning scan with respect to absorption/dispersion have been calculated with different ground state population distribution. The results are shown in figure 4. From this figure we can say that the population distribution may also play a role in the absorption/dispersion spectrum.

4 Summary and Conclusions

We have used the density matrix approach to calculate the absorption/dispersion effects for three-level atomic system. The signature of electromagnetically induced transparency is observed in the
absorption spectrum. It also has been found that the level of absorption can be varied slightly if we manipulate the ground state population distribution.

References


