

A new numerical approach for Soret effect on mixed convective boundary layer flow of a nanofluid over vertical frustum of a cone

Ch.RamReddy and Ch.Venkata Rao*

Department of Mathematics,

National Institute of Technology Warangal-506004, India.

chittetiram@gmail.com, *venki003@gmail.com

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Abstract

This paper analyzes the problem of mixed convective heat and mass transfer over a vertical frustum of a cone in a nanofluid under the Soret effect. The system of governing equations is transformed into the non-dimensional form using suitable non-dimensional transformations and then solved by employing Bivariate Pseudo-Spectral Local Linearisation Method. The present numerical results are compared with the existing results in some special cases and found to be in good agreement. The effects of physical parameters on skin-friction, heat, regular mass and nanoparticle mass transfer characteristics over vertical frustum of a cone are given and the salient features are discussed.

AMS Subject Classification: 76E06, 76A05, 76N20, 76R50, 65M70.

Key Words and Phrases: Nanofluid, Vertical frustum of a cone, Soret effect, Bivariate pseudo-spectral local linearisation method.

1 Introduction

The enhancement of thermal conductivity in heat transfer analysis is important due to several engineering and industrial applications. In view of this, several researchers have been proposed various heat transfer enhancers for enhancing the rate of heat transfer. One such mechanism is heat transfer enhancement using nanofluids. These fluids are prepared by suspending solid nanometer-sized particles with typical lengths of $1 - 100nm$ and proposed by Choi [1]. One can find a detailed review of nanofluids in Buongiorno [3] and Das *et al.* [2]. In the recent days, the fluid flow problems involving thermal-diffusion in nanofluids attracted by many researchers owing to its wide range of industrial applications. The cross-diffusion effects over a vertical truncated cone in a Newtonian fluid has been investigated by Awad *et al.* [4] and Cheng [5]. Recently, the Soret effect on double-diffusive convection flow over the inclined plate in a nanofluid saturated non-Darcy porous medium has been given by RamReddy *et al.* [6]. From the above studies, it is observed that the similarity solutions admitted for the problems over full cone, but not for problems over vertical frustum of a cone. To deal such complex flow problems, several authors employed various numerical methods like finite difference, finite element, local non-similarity, etc. But, in this article, we used a new spectral collocation method based on local linearization to investigate the effects of thermal-diffusion, Brownian motion and thermophoresis on mixed convective flow of a nanofluid over a vertical frustum of a cone.

2 Formulation of the problem

Consider the two-dimensional laminar, steady mixed convective boundary-layer flow over a vertical frustum of a cone embedded in a nanofluid. The geometry is chosen such that x -axis is along the surface of the full cone and y -axis is normal to the surface of a vertical frustum of a cone with the origin O at the vertex of the full cone (see [4] and [5]). The temperature and solutal concentration on the surface of the vertical frustum of a cone are taken to be uniform and are given by T_w and C_w , while the velocity, temperature, solutal concentration and nanoparticle volume fraction at

ambient medium are taken as U_∞ , T_∞ , C_∞ and ϕ_∞ . Considering the boundary layer assumptions and Oberbeck-Boussinesq approximations, the two-dimensional boundary layer equations are given by

$$\frac{\partial(ur)}{\partial x} + \frac{\partial(vr)}{\partial y} = 0 \quad (1)$$

$$\rho_{f\infty} \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = \mu \frac{\partial^2 u}{\partial y^2} + \rho_{f\infty} g (1 - \phi_\infty) [\beta_T(T - T_\infty) + \beta_C(C - C_\infty)] \cos A \quad (2)$$

$$-(\rho_p - \rho_{f\infty})g(\phi - \phi_\infty) \cos A$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_m \frac{\partial^2 T}{\partial y^2} + \frac{(\rho c)_p}{(\rho c)_f} \left[D_B \frac{\partial \phi}{\partial y} \frac{\partial T}{\partial y} + \frac{D_T}{T_\infty} \left(\frac{\partial T}{\partial y} \right)^2 \right] \quad (3)$$

$$u \frac{\partial \phi}{\partial x} + v \frac{\partial \phi}{\partial y} = D_B \frac{\partial^2 \phi}{\partial y^2} + \frac{D_T}{T_\infty} \frac{\partial^2 T}{\partial y^2} \quad (4)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_S \frac{\partial^2 C}{\partial y^2} + D_{CT} \frac{\partial^2 T}{\partial y^2} \quad (5)$$

where u and v are the components of velocity along the x and y axes respectively, T is the temperature, ϕ is the nanoparticle volume fraction, C is the solutal concentration and g is the gravitational acceleration. Further, $\nu = \mu/\rho_{f\infty}$, $\alpha_m = k/(\rho c)_f$, $(\rho c)_f$, $\rho_{f\infty}$, k , μ are the coefficient of kinematic viscosity, thermal diffusivity, heat capacity, density, thermal conductivity, viscosity while ρ_p , $(\rho c)_p$, β_C , β_T are the density, effective heat capacity, volumetric solutal and thermal expansion coefficients of the nanofluid. Finally, D_S , D_B , D_{CT} and D_T are the coefficients of solutal diffusivity, Brownian diffusion, Soret-type diffusivity and thermophoretic diffusion.

The boundary conditions are

$$u = 0, \quad v = 0, \quad T = T_w, \quad D_B \frac{\partial \phi}{\partial y} + \frac{D_T}{T_\infty} \frac{\partial T}{\partial y} = 0, \quad C = C_w \quad \text{at} \quad y = 0 \quad (6a)$$

$$u = U_\infty, \quad T = T_\infty, \quad \phi = \phi_\infty, \quad C = C_\infty \quad \text{as} \quad y \rightarrow \infty \quad (6b)$$

Now, the stream function is interpreted as $u = \frac{1}{r} \frac{\partial \psi}{\partial y}$, $v = -\frac{1}{r} \frac{\partial \psi}{\partial x}$

and the non-similarity variables are introduced as

$$\left. \begin{aligned} \xi &= \frac{\bar{x}}{x_0}, \eta = Re_{\bar{x}}^{1/2} \frac{y}{\bar{x}}, f(\xi, \eta) = \frac{\psi}{r\nu Re_{\bar{x}}^{1/2}}, \theta(\xi, \eta) = \frac{T - T_{\infty}}{T_w - T_{\infty}}, \\ \gamma(\xi, \eta) &= \frac{\phi - \phi_{\infty}}{\phi_{\infty}}, S(\xi, \eta) = \frac{C - C_{\infty}}{C_w - C_{\infty}} \end{aligned} \right\} \quad (7)$$

where $\bar{x} = x - x_0$ and $Re_{\bar{x}} = \frac{U_{\infty} \bar{x}}{\nu}$ is the local Reynolds number.

Using (7), the Eqs. (1) - (5) reduces to the following form:

$$f''' + \left(R + \frac{1}{2}\right) f f'' + \lambda \xi (\theta + Nc S - Nr \gamma) = \xi \left(f' \frac{\partial f'}{\partial \xi} - \frac{\partial f}{\partial \xi} f''\right) \quad (8)$$

$$\frac{1}{Pr} \theta'' + \left(R + \frac{1}{2}\right) f \theta' + Nb \gamma' \theta' + Nt (\theta')^2 = \xi \left(f' \frac{\partial \theta}{\partial \xi} - \frac{\partial f}{\partial \xi} \theta'\right) \quad (9)$$

$$\frac{1}{Le} \gamma'' + \left(R + \frac{1}{2}\right) f \gamma' + \frac{1}{Le} \frac{Nt}{Nb} \theta'' = \xi \left(f' \frac{\partial \gamma}{\partial \xi} - \frac{\partial f}{\partial \xi} \gamma'\right) \quad (10)$$

$$\frac{1}{Sc} S'' + \left(R + \frac{1}{2}\right) f S' + S_T \theta'' = \xi \left(f' \frac{\partial S}{\partial \xi} - \frac{\partial f}{\partial \xi} S'\right) \quad (11)$$

where $R = \xi/(1 + \xi)$, $Sc = \nu/D_S$ is the Schmidt number, $Pr = \nu/\alpha_m$ indicate the Prandtl number, $Le = \nu/D_B$ is the Lewis number, $Nc = [\beta_C(C_w - C_{\infty})]/[\beta_T(T_w - T_{\infty})]$ and $Nr = [(\rho_p - \rho_{f\infty})\phi_{\infty}]/[\rho_{f\infty}\beta_T(T_w - T_{\infty})(1 - \phi_{\infty})]$ are the regular and nanofluid buoyancy ratios. Further, $\lambda = Gr_{x_0}/Re_{x_0}^2$ is the mixed convection parameter, $S_T = [D_{CT}(T_w - T_{\infty})]/[\nu(C_w - C_{\infty})]$ is the Soret number, $Nt = [(\rho c)_p D_T(T_w - T_{\infty})]/[(\rho c)_f \nu T_{\infty}]$ and $Nb = [(\rho c)_p D_B \phi_{\infty}]/[(\rho c)_f \nu]$ are the thermophoresis and Brownian motion parameters.

The boundary conditions become

$$\eta = 0 : f(\xi, 0) = \frac{-\xi}{(R + \frac{1}{2})} \frac{\partial f}{\partial \xi}, f'(\xi, 0) = 0, \theta(\xi, 0) = 1, \quad (12a)$$

$$Nb \gamma'(\xi, 0) + Nt \theta'(\xi, 0) = 0, S(\xi, 0) = 1$$

$$\eta \rightarrow \infty : f'(\xi, \infty) = 1, \theta(\xi, \infty) = 0, \gamma(\xi, \infty) = 0, S(\xi, \infty) = 0 \quad (12b)$$

The non-dimensional shear stress, Nusselt, nanoparticle Sherwood and regular Sherwood numbers are given by

$$\begin{aligned} C_f(Re_{\bar{x}})^{1/2} &= 2f''(\xi, 0), Nu_{\bar{x}}(Re_{\bar{x}})^{-1/2} = -\theta'(\xi, 0), \\ NSh_{\bar{x}}(Re_{\bar{x}})^{-1/2} &= -\gamma'(\xi, 0), Sh_{\bar{x}}(Re_{\bar{x}})^{-1/2} = -S'(\xi, 0) \end{aligned} \quad (13)$$

3 Results and Discussion

The system of non-dimensional governing equations (8)-(11) is numerically solved by Bivariate Pseudo-Spectral Local Linearisation Method. Initially, the method start with an innovative linearisation and decoupling technique based on the so-called quasi-linearisation. The idea behind solution method is linearisation of the governing equations about one dependent variable at a time in the sequential order f , θ , γ and S . Next, the pseudo-spectral collocation is employed to discretize both η and ξ domains using Chebyshev-Gauss-Lobatto type collocation points. The approximate solutions are assumed to be defined in terms of bivariate Lagrange interpolation polynomials. Finally, the approximate solutions can be obtained by solving the matrix form of governing equations (8)-(11) by starting with a suitable initial approximations. A detailed explanation about the method has been discussed by Motsa and Animasaun [8] and Canuto *et al.* [9].

The non-dimensional surface drag, heat, regular mass and nanoparticle mass transfer rates have been computed and illustrated graphically. To explore the effects of thermophoresis, Brownian motion, Soret number and Lewis number, the calculations are carried for fixed values $\lambda = 0.5$, $Pr = 1.0$, $Sc = 0.6$, $Nr = Nc = 1.0$. To verify the accuracy of solution method, the current results are compared with the available results reported by Lloyd and Sparrow [7]. Table 1 represents the local Nusselt number $-\theta'(\xi, 0)$ at $\xi = 0$ (Vertical plate case) for $Nt = 0.0$, $S_T = 0.0$, $Nc = 0.0$, $Nr = 0.0$, $Sc = 1.0$, $Le = 1.0$, $Nb \rightarrow 0.0$. The comparison between the values seems to be good and the results are accurate as given in Tab. 1.

Figures 1(a) - 1(d) illustrate, the effects of thermophoresis parameter (Nt) and Brownian motion parameter (Nb) on the dimensionless skin friction coefficient, rate of heat, regular and nanoparticle mass transfer rates versus streamwise coordinate ξ for fixed values of $Le = 10.0$ and $S_T = 1.0$. These figures depict that, an increase of thermophoresis parameter tends to increase the non-dimensional surface drag and regular mass transfer rate but it decrease the heat and nanoparticle mass transfer rates. On the other hand, the surface drag, rate of heat, regular mass transfer rate rises

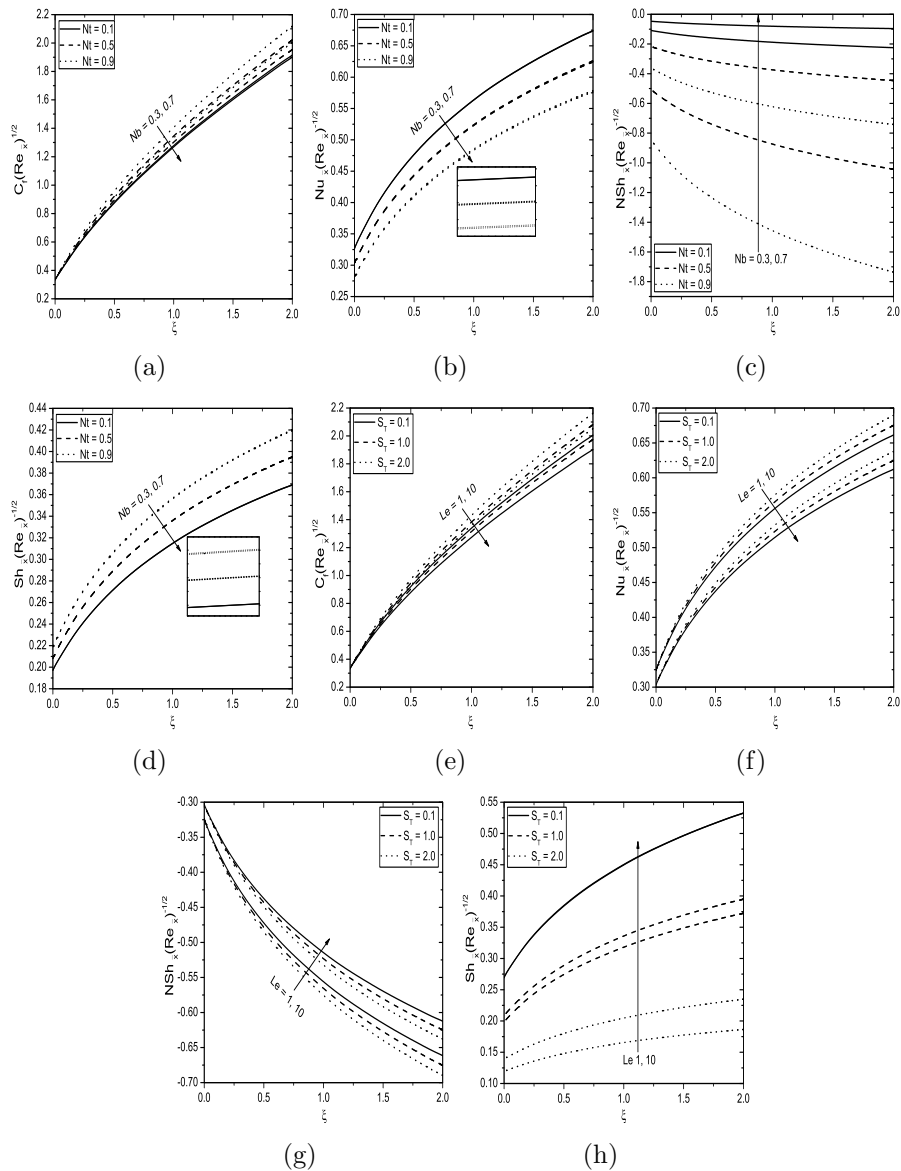


Figure 1: Effect of Nt and Nb on (a) skin friction, (b) heat transfer rate, (c) nanoparticle mass transfer rate, (d) mass transfer rate and Effect of Le and ST on (e) skin friction, (f) heat transfer rate, (g) nanoparticle mass transfer rate, (h) mass transfer rate.

Table 1: Comparison of $-\theta'(0, 0)$ when $Nt = 0.0$, $S_T = 0.0$, $Nc = Nr = 0.0$, $Sc = Le = 1.0$, $Nb \rightarrow 0.0$.

	$-\theta'(0, 0)$	
Pr	Lloyd and Sparrow [7]	Present
0.72	0.2956	0.29563518
10.0	0.7281	0.72814119
100.0	1.572	1.57165763

and nanoparticle mass transfer rate reduces under the increasing values of Brownian motion parameter. Moreover, the values of surface drag, heat transfer and regular mass transfer rates over full cone ($\xi \rightarrow \infty$) are higher than the case of vertical plate ($\xi = 0$).

The streamwise distribution of skin friction, rate of heat, regular and nanoparticle mass transfer rates over streamwise coordinate ξ for different values of Soret number (S_T) and Lewis number (Le) are shown in Figs. 1(e) - 1(h) for fixed values of $Nb = 0.5$, and $Nt = 0.5$. The skin friction coefficient, heat transfer rate increases nonlinearly whereas regular and nanoparticle mass transfer rates decrease nonlinearly with the enhancement of Soret number as displayed in 1(e) - 1(h). An opposite behavior can be found in the case of increasing values of Lewis number. Moreover, the impact of Soret number on the skin friction, heat, and nanoparticle mass transfer rates negligible for $\xi = 0$ and those are rapidly changes as $\xi \rightarrow \infty$. These results show clearly that the emerging parameters have remarkable impact on all the flow, heat and mass transfer characteristics.

4 Conclusions

In this study, we used a numerical approach named as Bivariate Pseudo-Spectral Local Linearisation Method for solving highly nonlinear and coupled system of partial differential equations of mixed convective flow of a nanofluid over a vertical frustum of a cone under the Soret effect. The skin friction, rate of heat, regular and

nanoparticle mass transfer rates are attained for various values of parameters influenced the aiding flow. The primary discoveries are summarized as follows:

- The main conclusion is that the surface drag, rate of heat increase but regular mass and nanoparticle mass transfer rates decrease with the enhancement of Soret number. Further, an opposite behaviour is observed for Lewis number.
- The large values of Brownian motion resulting in a low surface drag, heat and mass transfer rates and, in a high nanoparticle mass transfer rate.
- It is shown that, the surface drag and regular mass transfer rate enhance whereas heat and nanoparticle mass transfer rates diminish with the increase of thermophoresis parameter.

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